Tuesday
9/24

Test 1
Tuesday 10/8
12.1—12.7

$D>0,\ f_{xx}>0 \Longrightarrow$ local min
$D>0,\ f_{xx}<0 \Longrightarrow$ local max
$D<0 \Longrightarrow$ saddle pt.
$D=0 \Longrightarrow$ no info

Last time

Ex: $f(x,y) = xy(x-2)(y+3)$
$= (x^2-2x)(y^2+3y)$

$f_x = 2y(y+3)(x-1)$
$f_y = x(x-2)(2y+3)$

$$\boxed{\begin{aligned} f_{xx} &= 2y^2+6y \\ f_{yy} &= 2x^2-4x \\ f_{xy} &= (2y+3)(2x-2) \end{aligned}}$$

CAL STATE

critical point	what is it?
$(0,0)$	saddle point
$(2,0)$	saddle point
$(0,-3)$	saddle point
$(2,-3)$	saddle point
$(1,-\frac{3}{2})$	local max

$$D = f_{xx}f_{yy} - [f_{xy}^2]$$

$(a,b) = (2,0)$

$$D(2,0) = (0)(2\cdot 2^2 - 4\cdot 2) - [(3)(2)]^2 = -36 < 0$$

$(a,b) = (0,-3)$

$$D(0,-3) = (2\cdot 9 - 18)(0) - [(-6+3)(2\cdot 0 - 2)]^2 = -36 < 0$$

$(a,b) = (2,-3)$

$$D(2,-3) = (2(-3)^2 + 6(-3))(2(2)^2 - 4(2)) - [((2)(-3)+3)(2(2)-2)]^2 = -36 < 0$$

$(a,b) = (1,-\frac{3}{2})$

$$D(1,-\tfrac{3}{2}) = 9 > 0$$

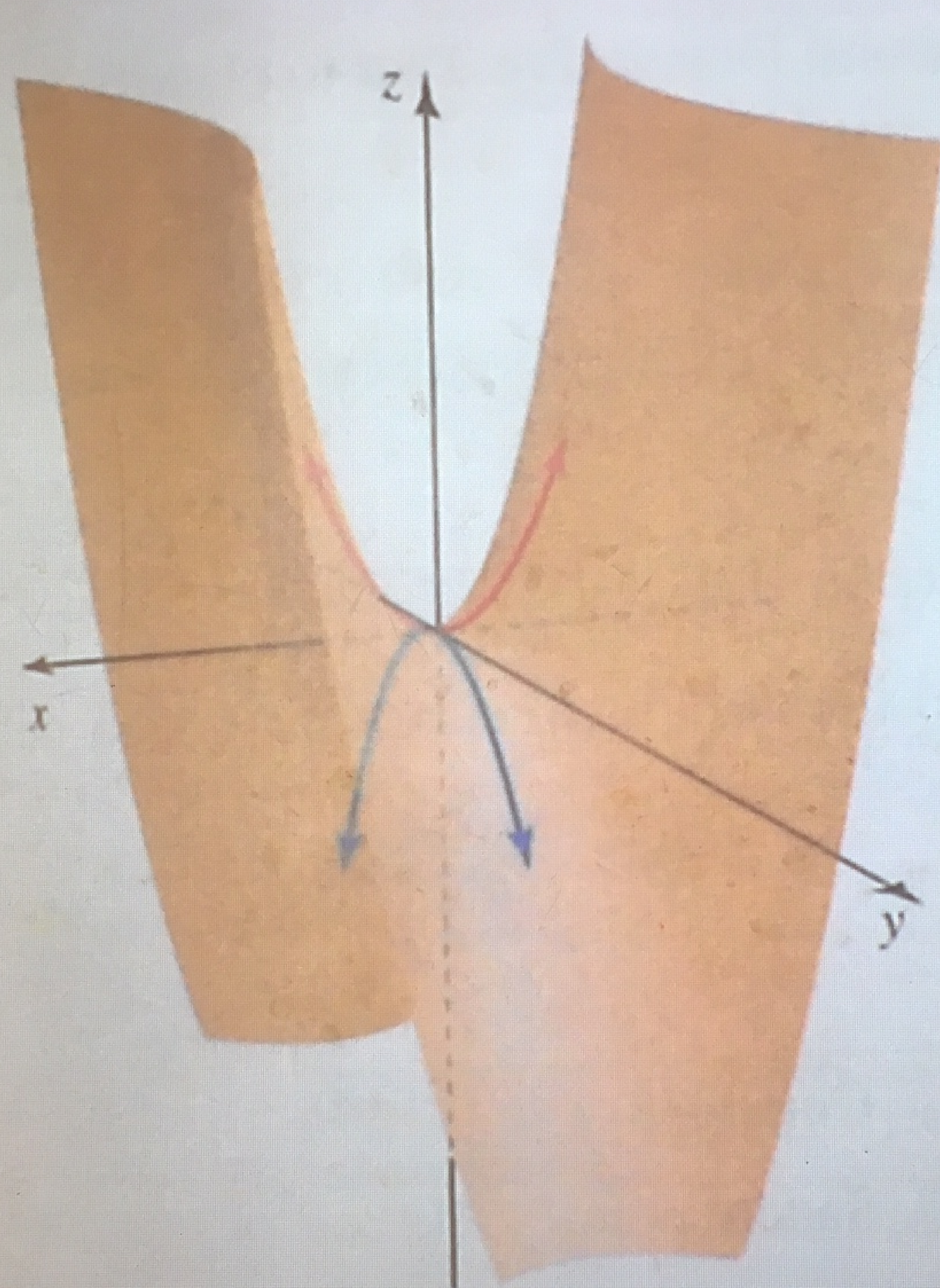
$$f_{xx}(1,-\tfrac{3}{2}) = 2\left(\tfrac{-3}{2}\right)^2 + 6\left(\tfrac{-3}{2}\right) = \frac{9}{2} - \frac{18}{2} = -\frac{9}{2} < 0$$

https://www.google.com/search?source=hp&ei=Z4SKXbyPDcv7-gTMpJqwCg&q=
From
To
x
-10.0000
10.0000
y
-10.0000
10.0000
z
-1779.75
5269.69
More info

= 0 does not guaran-
Theorem 12.13. The
extremum at (a, b),
cal extrema. We call
able. Therefore, the
nd the critical points
local maximum and

either

$a, b)$.



The hyperbolic paraboloid $z = x^2 - y^2$ has a saddle point at $(0, 0)$.

Figure 12.86

If (a, b) is a critic
$(a, b, f(a, b))$, it is po
rections. The function
remember. The surfac
$(0, 0)$ along the y-axi
demonstrating that c
minima.

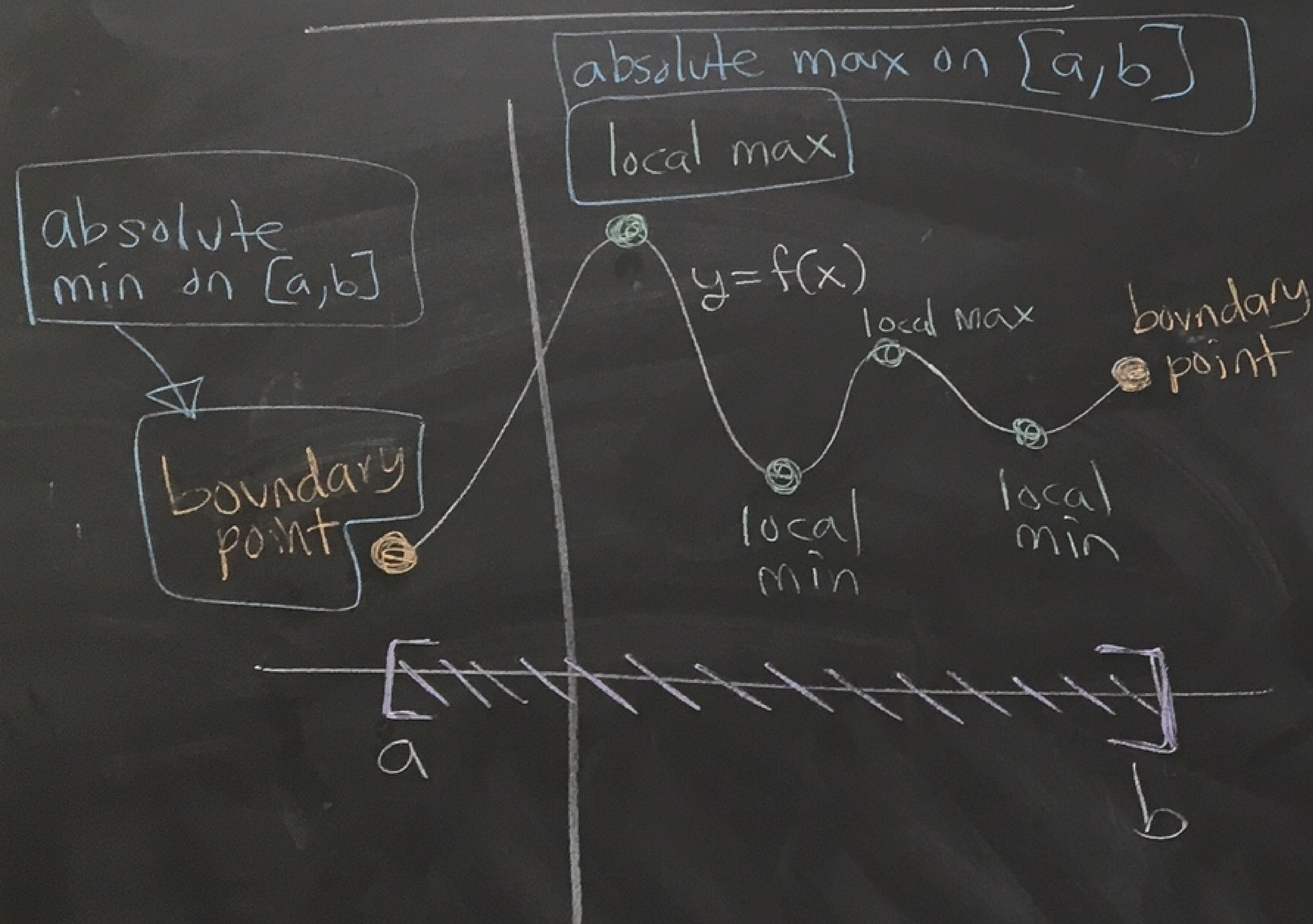
QUICK CHECK 2 Consi
does the normal to th

THEOREM 12.14

Suppose that the s
an open disk cent
$D(x, y) = f_{xx}(x,$

1. If $D(a, b) >$
2. If $D(a, b) >$
3. If $D(a, b) <$

Recall Calc I



To find the max/min of a continuous function $f(x)$ on $[a,b]$

1. Evaluate f at the local max's and min's in $[a,b]$.
2. Evaluate f at the boundary points a & b.
3. Biggest from 1 & 2 is max
   Smallest from 1 & 2 is min

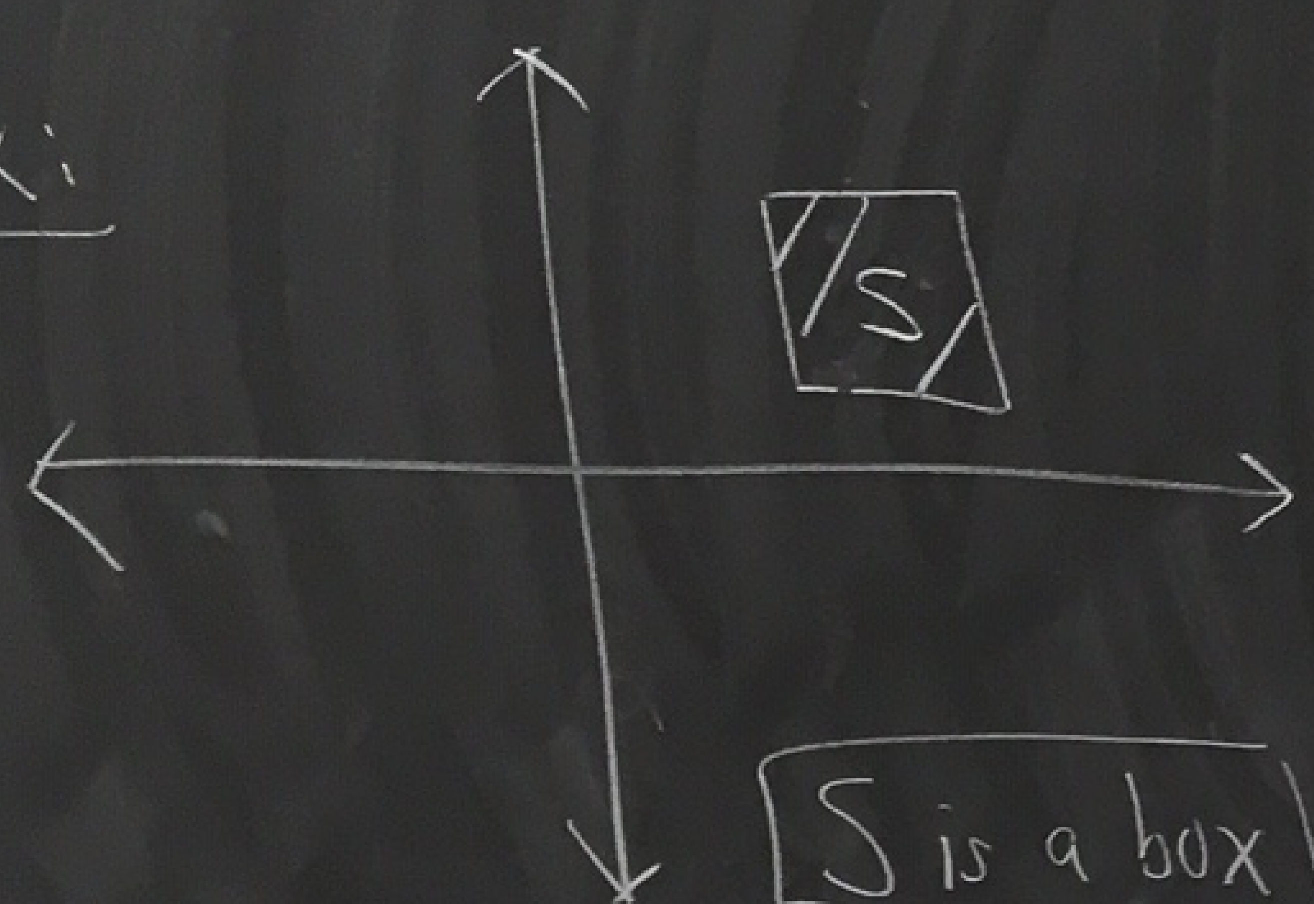
Now Calc III

$\mathbb{R}^2$ means the xy-plane or 2d

Def: A bounded set in $\mathbb{R}^2$ is one that is contained in some disc.

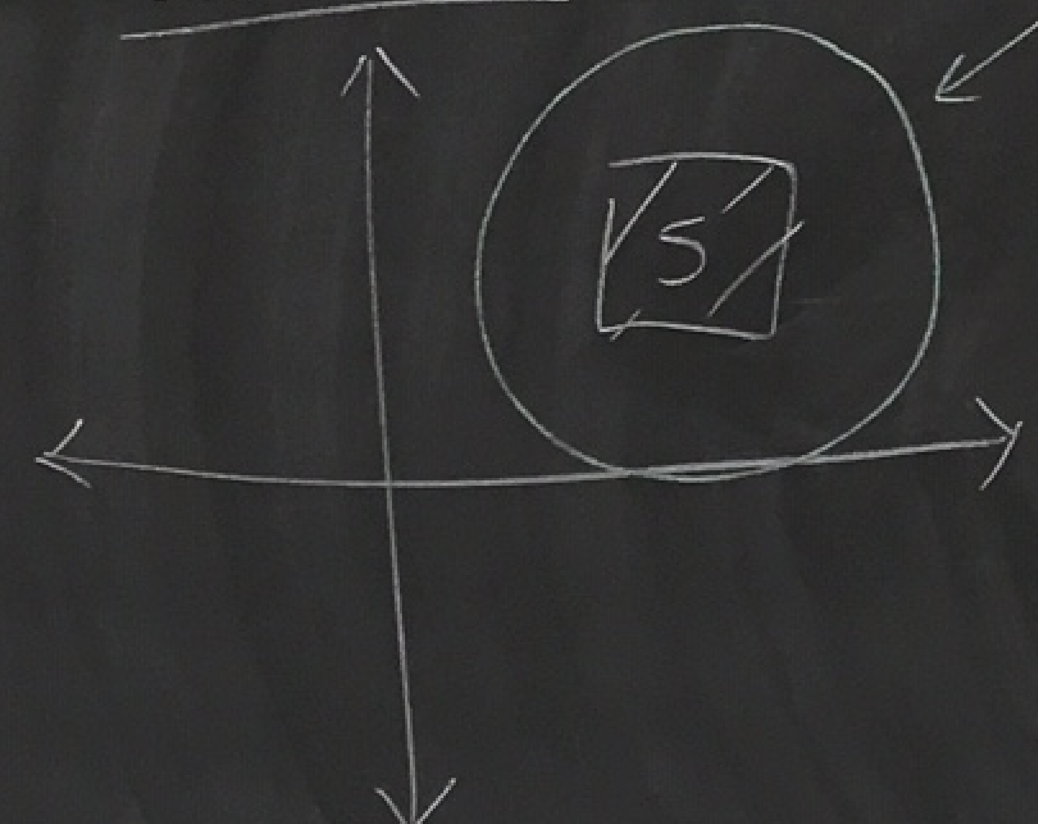
Def: A closed set in $\mathbb{R}^2$ is one that contains all its boundary points.

Ex:



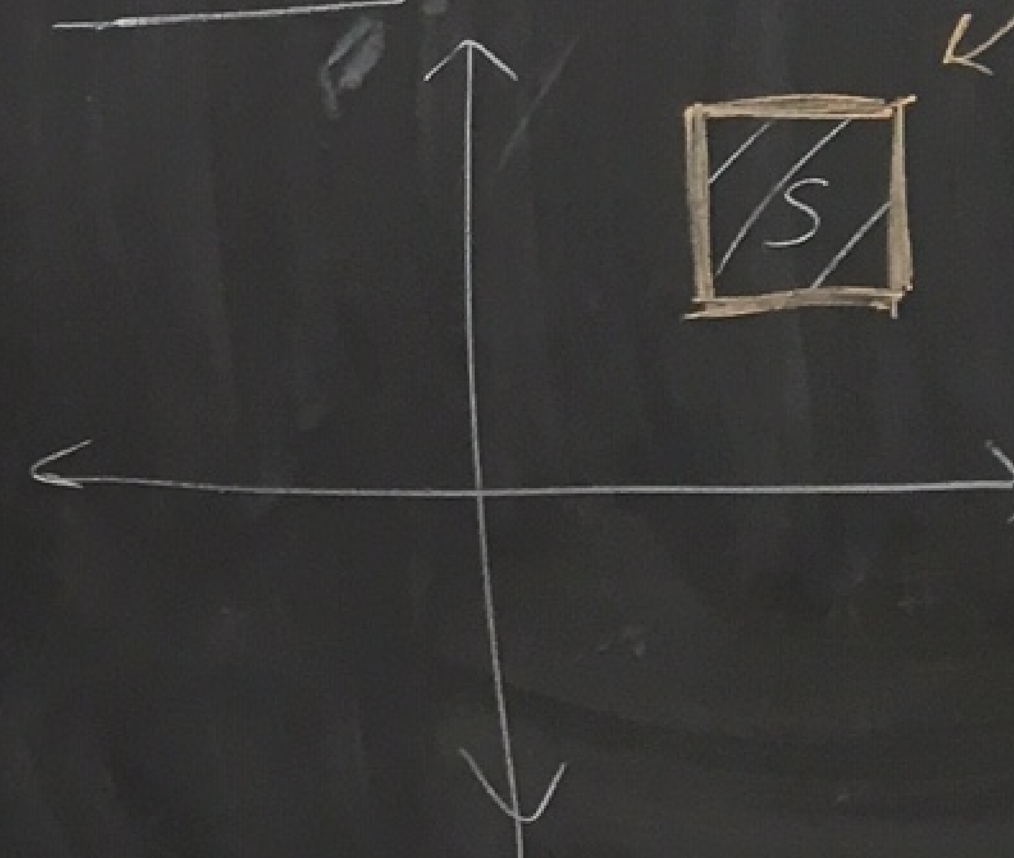
S is a box

bounded? Yes



I can put this disc around it

closed? Yes



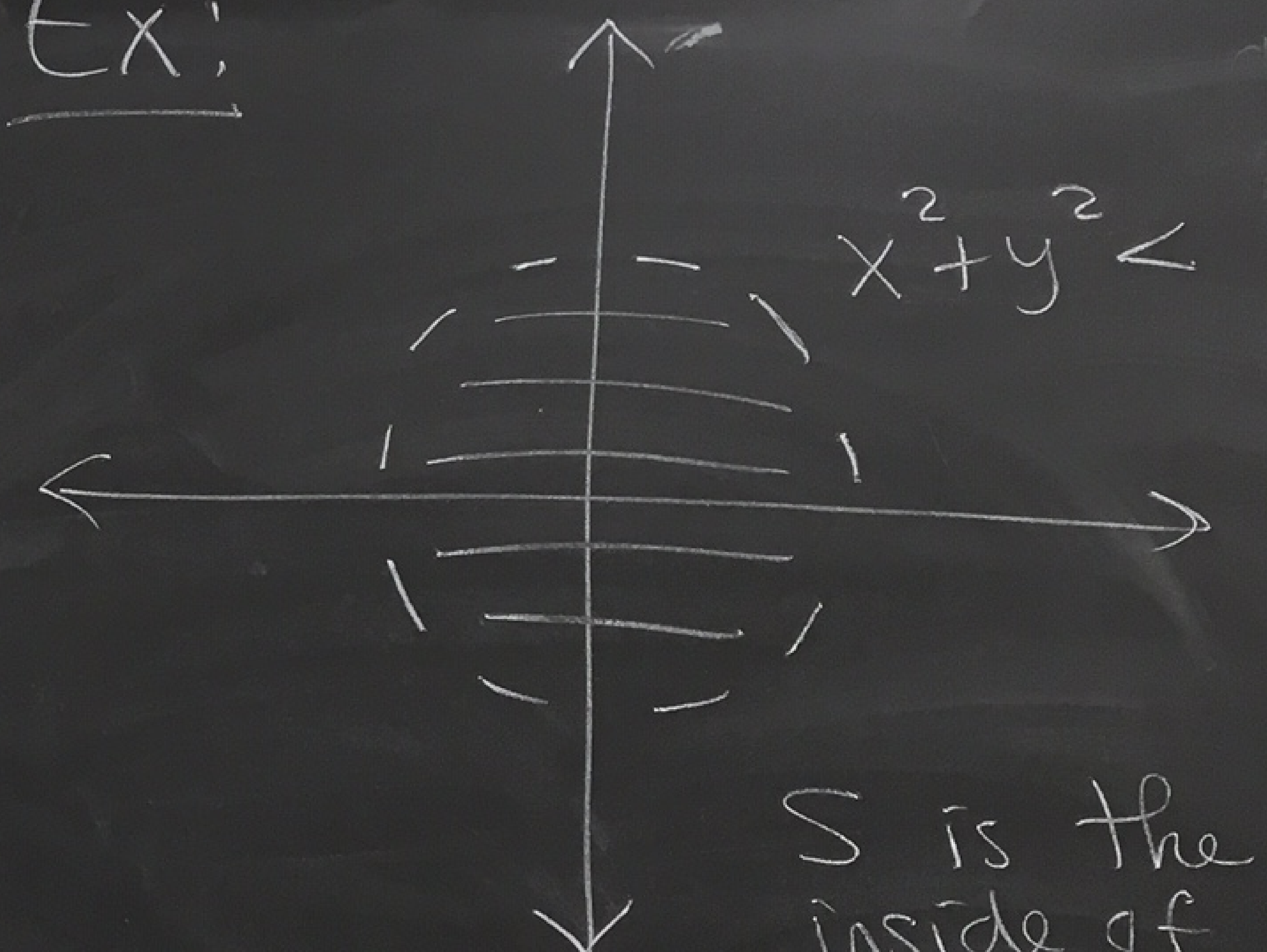
Yes the boundary is included in S

Ex:

S consists of all (x,y) with $y \geq 0$

NOT BOUNDED

Ex:



S is the inside of the unit circle.

NOT CLOSED

__Theorem__ (Extreme Value Theorem)
If $f(x,y)$ is continuous on a closed, bounded set S in $\mathbb{R}^2$ then f attains an absolute maximum and absolute minimum on S.

__How to find such an absolute max/min__

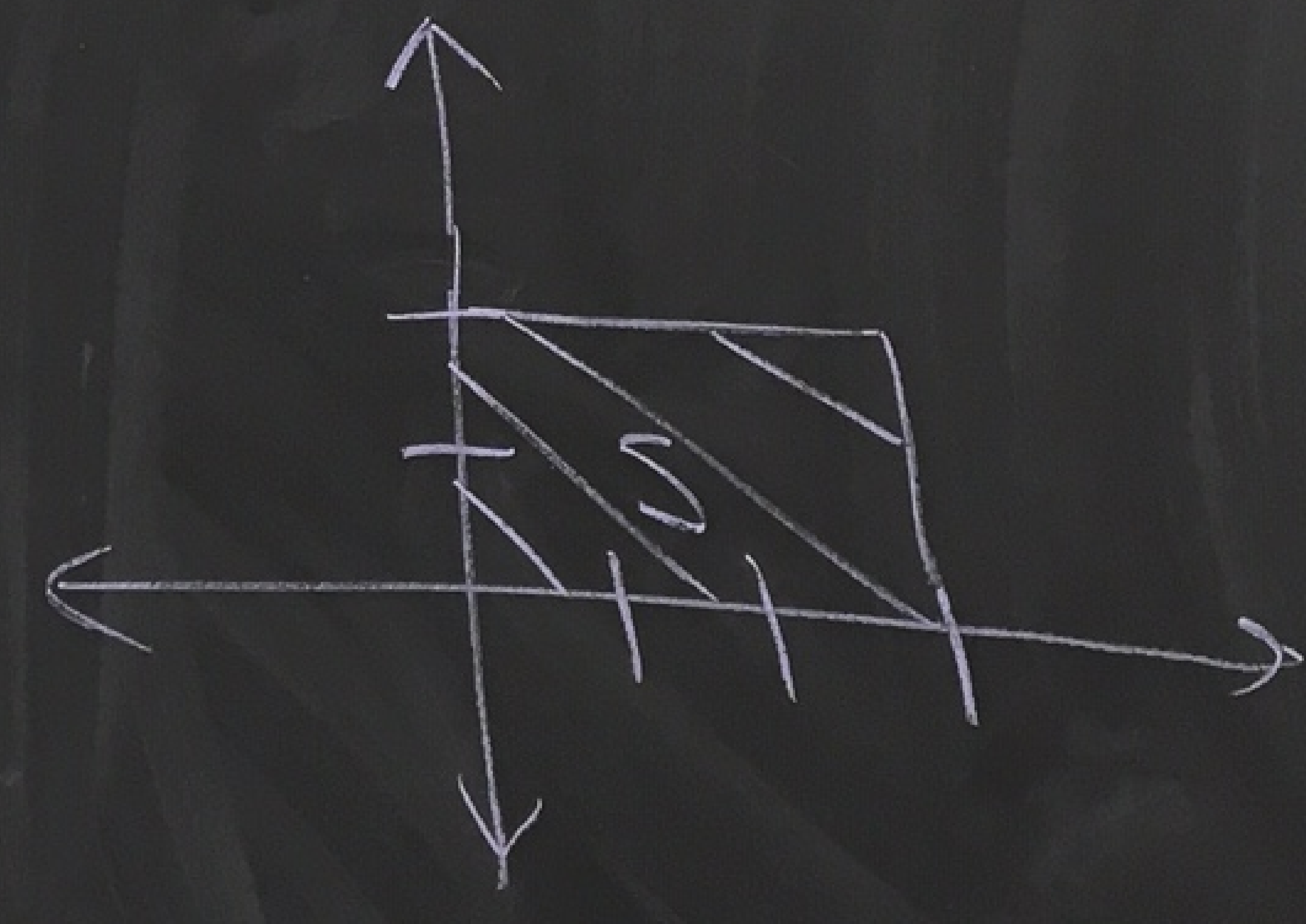
1. Evaluate f at all the critical points of f in S.
2. Find the abs. max/min of f on the boundary of S.
3. The largest of steps 1 and 2 gives the abs. max of f on S.
   The smallest of steps 1 and 2 gives the abs. min of f on S.

Ex: Find the absolute max/min of

$$f(x,y) = x^2 - 2xy + 2y$$

on the rectangle $S = \{(x,y) \mid 0 \le x \le 3,\ 0 \le y \le 2\}$

means: All points (x,y) where $0 \le x \le 3$ and $0 \le y \le 2$.



① Evaluate f at its critical points inside of S.

$$\boxed{\begin{aligned} f_x &= 2x-2y \\ f_y &= -2x+2 \end{aligned}} \longrightarrow \boxed{\begin{aligned} 2x-2y&=0 \\ -2x+2&=0 \end{aligned}} \longrightarrow \boxed{x=1}$$

$$\boxed{x=1} \xrightarrow{\text{plug into } 2x-2y=0} \boxed{2-2y=0} \longrightarrow \boxed{y=1}$$

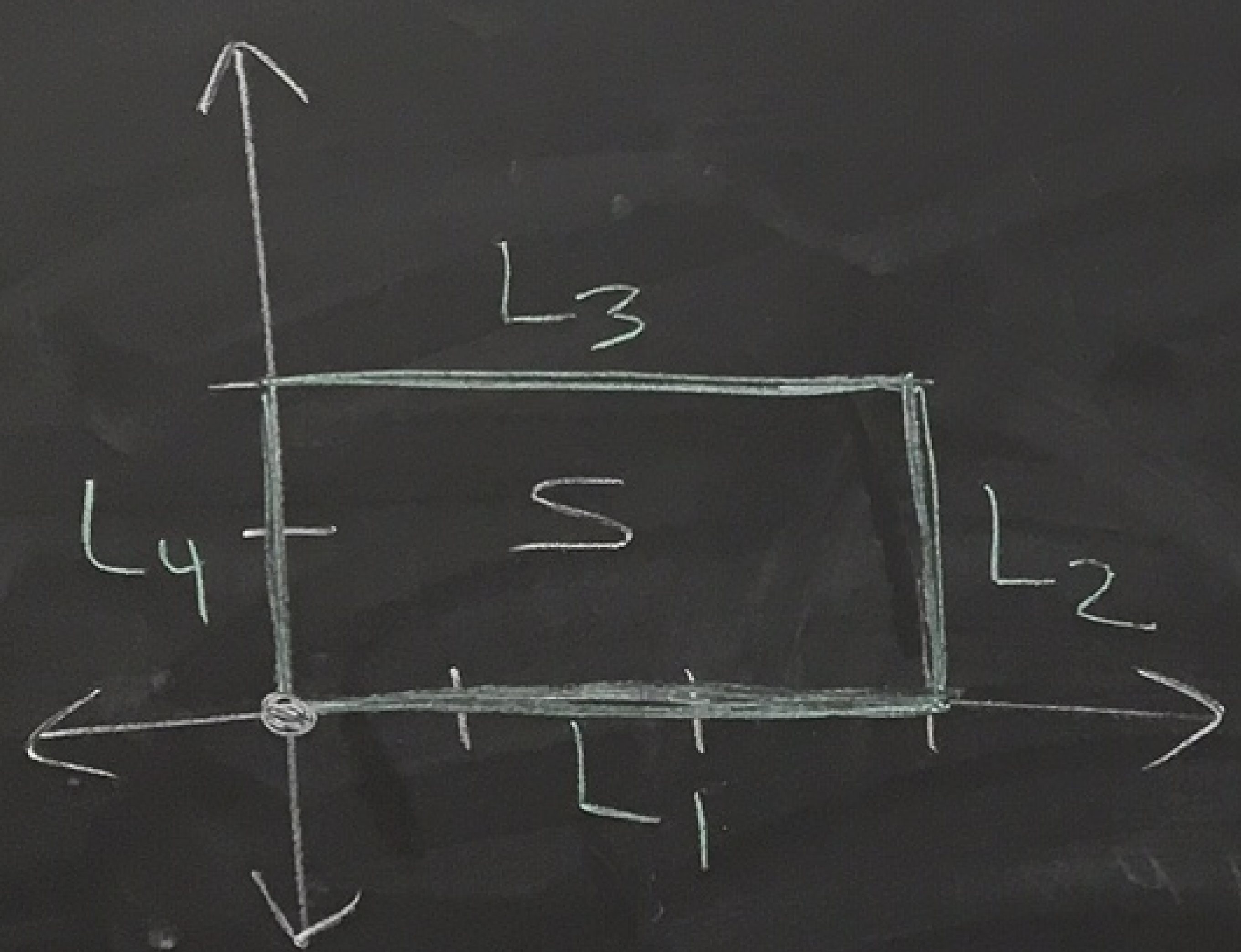
(1,1) is the only critical point.

Plug the critical points into f

$$f(1,1) = (1)^2 - 2(1)(1) + 2(1) = 1$$

(2) Find the max/min of f on the boundary of S.

$$\boxed{f(x,y) = x^2 - 2xy + 2y}$$



Find max/min of f over L_1

Here $y=0$ and $0 \le x \le 3$.

So on L_1, $f(x,0) = x^2$