Math 4740
4/9/25

Next
page
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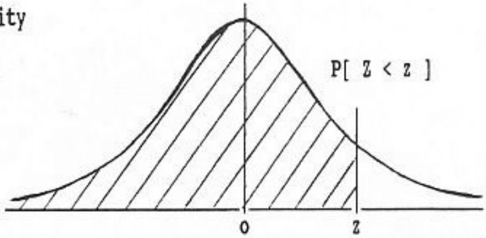
STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z
i.e.

$$P[\ Z < z\ ] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\ dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

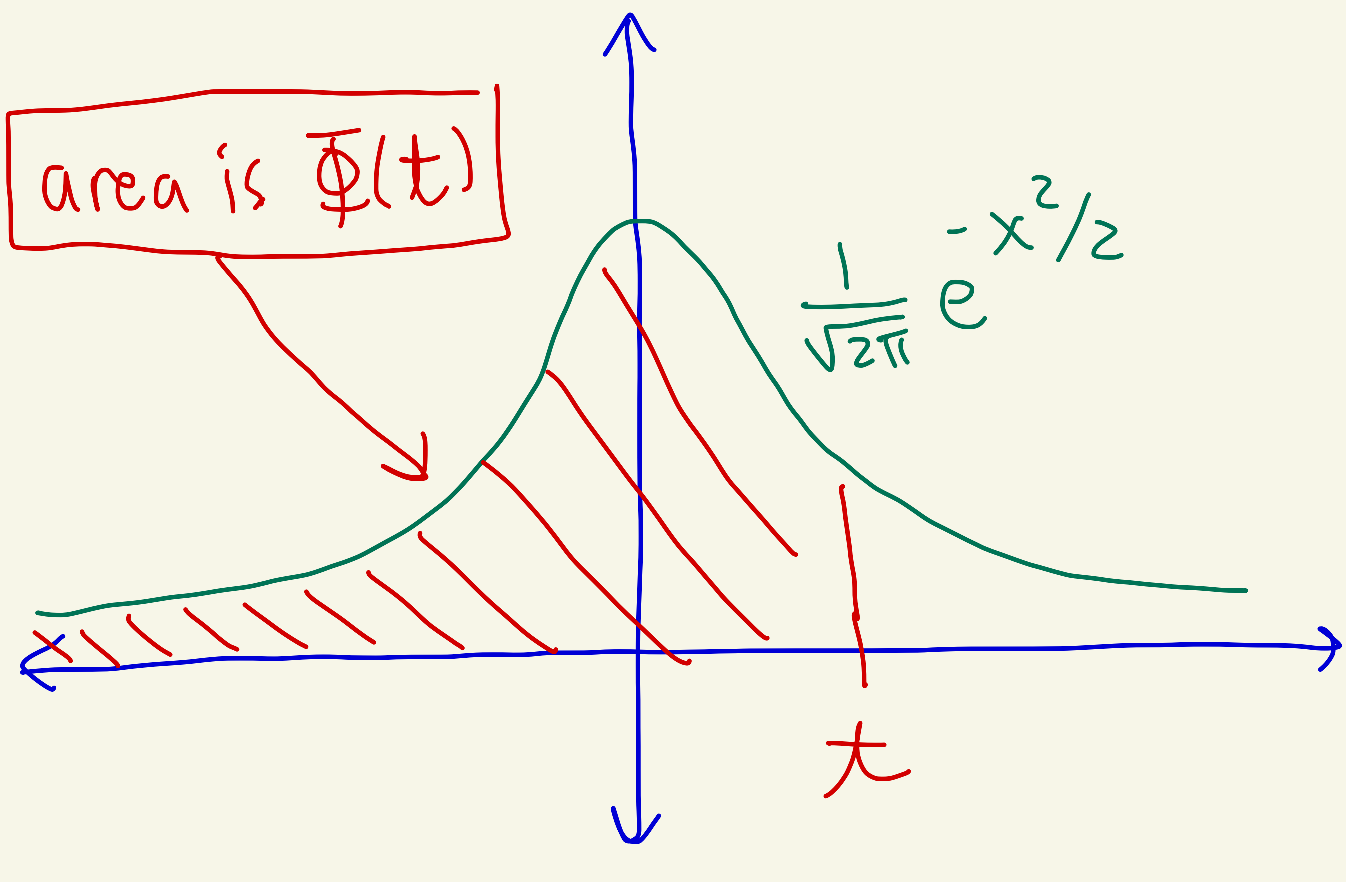
z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

Topic 7 – Normal approximation to binomial random variables

Φ – phi

<u>Def:</u> Let

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-x^2/2}\, dt$$

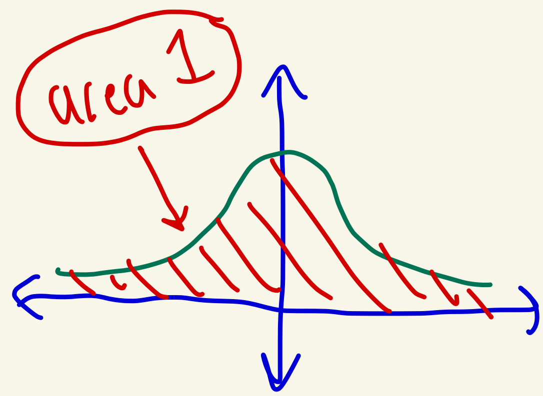


Φ is called the probability

density function of the standard normal random variable (topic 8)

In topic 8, we will show that

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2}\, dx = 1$$



Let's see how to calculate $\Phi(t)$ for $t \geq 0$

Ex: Calculate $\Phi(2.25)$

Look in table

z 0.05

⋮

2.2 → $\boxed{0.9878}$

So, $\underbrace{\Phi(2.25)}_{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{2.25} e^{-x^2/2}\,dx \;\approx\; 0.9878} \approx 0.9878$

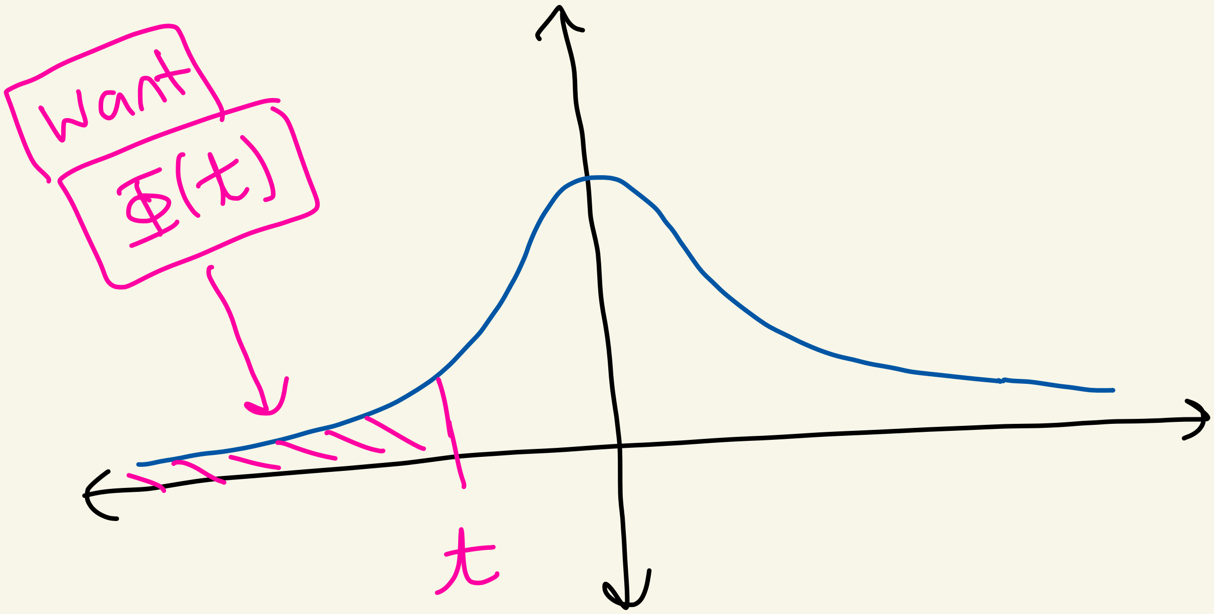
Ex: Calculate $\Phi(1.34)$

Table

z	$\cdots$	0.04
$\vdots$		$\downarrow$
1.3	$\longrightarrow$	$\boxed{0.9099}$

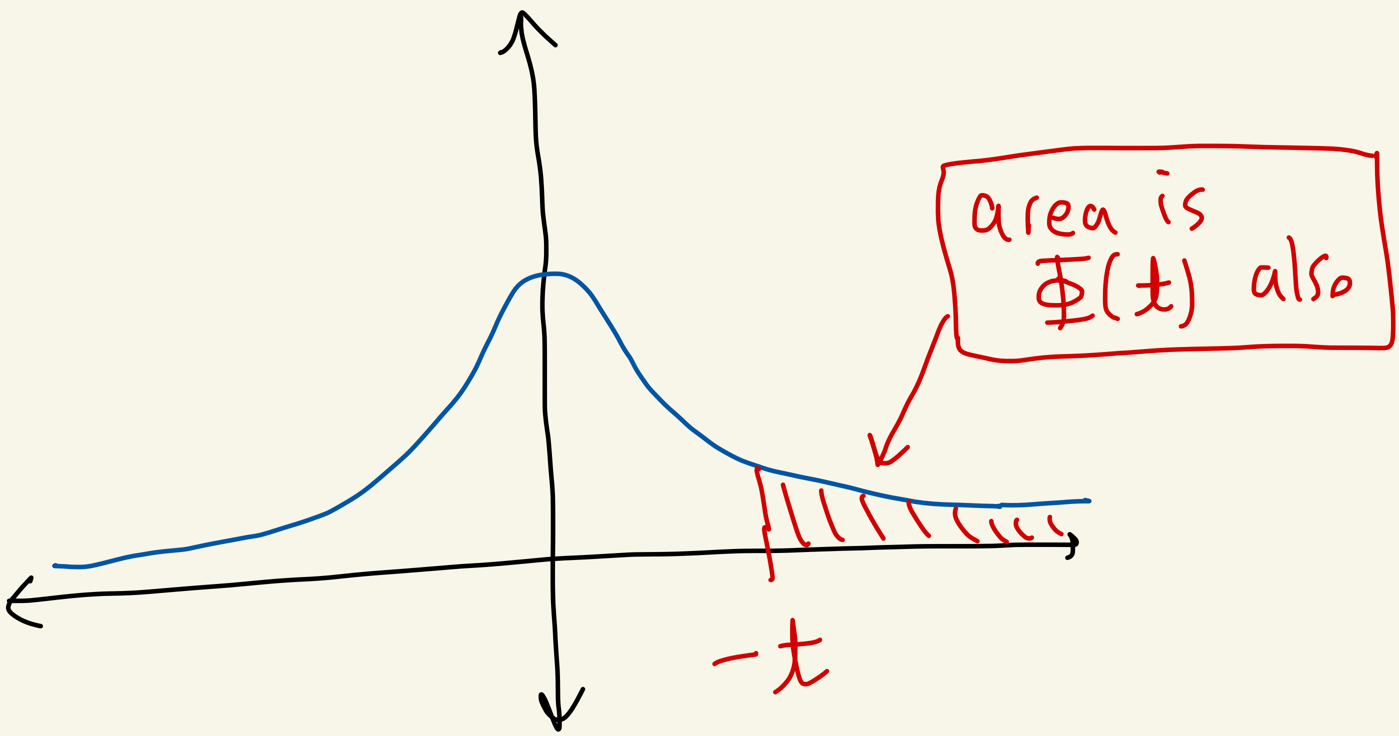
So, $\Phi(1.34) \approx 0.9099$

What about negative t?

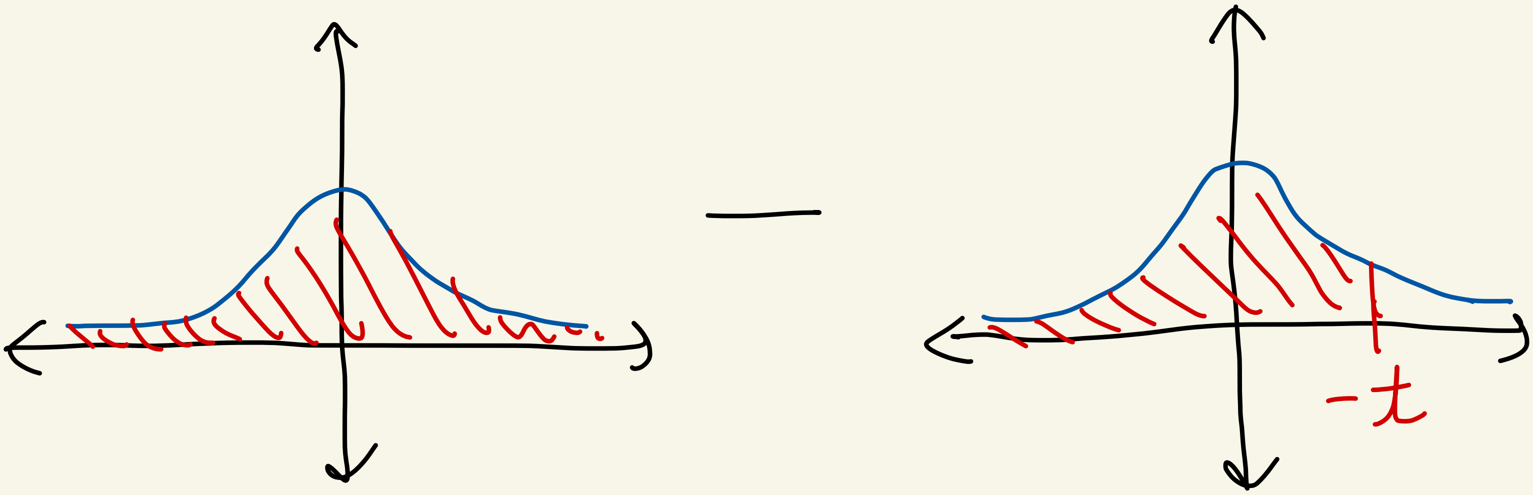


Since $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ is symmetric

this is the same as:



This is the same as:



$$1 - \Phi(-t)$$

$$\boxed{\text{Thus, if } t<0, \text{ then } \Phi(t) = 1 - \Phi(-t)}$$

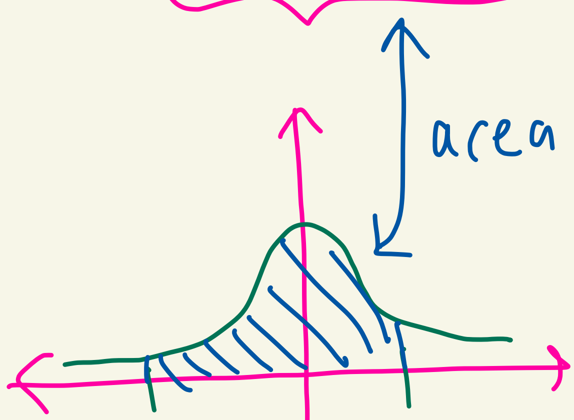
<u>Ex:</u> Calculate $\Phi(-1.27)$

$$\Phi(-1.27) = 1 - \Phi(1.27)$$
$$\approx 1 - 0.8980$$
$$\approx 0.1020$$

DeMoivre – Laplace Theorem

Let X be a binomial random variable with parameters n and p. Then for any real numbers a and b with $a<b$ we get

$$\lim_{n\to\infty} P\left(a \le \frac{X - np}{\sqrt{np(1-p)}} \le b\right) = \Phi(b) - \Phi(a)$$



Note:

$E[X] = np$

$Var(X) = \sqrt{np(1-p)}$

Ex: Suppose you flip a coin 10,000 times. Let X be the number of heads that occur. Approximate the probability that $5000 \leq X \leq 5002$.

$$P(5000 \leq X \leq 5002)$$

$$= P\left(\frac{5000-5000}{\sqrt{2500}} \leq \frac{X-5000}{\sqrt{2500}} \leq \frac{5002-5000}{\sqrt{2500}}\right)$$

$n = 10000$, $p = 1/2$

$np = (10000)\left(\frac{1}{2}\right) = 5000$

$\sqrt{np(1-p)} = \sqrt{(10000)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}$

$= \sqrt{2500}$

$$\frac{X-np}{\sqrt{np(1-p)}}$$

$$= P\left(0 \leq \frac{X-5000}{\sqrt{2500}} \leq 0.04\right)$$

$$\approx \Phi(0.04) - \Phi(0)$$

DL-Theorem
$n = 10000$ is large

$$\approx 0.5159 - 0.5$$

$$\approx 0.0159$$

$$\approx \boxed{1.59\,\%}$$

Real answer

$$P(5000 \le X \le 5002)$$

$$= P(X = 5000) + P(X = 5001) + P(X = 5002)$$

$$= \binom{10{,}000}{5000} \cdot \left(\frac{1}{2}\right)^{5000} \left(1 - \frac{1}{2}\right)^{5000}$$

$$+ \binom{10000}{5001} \cdot \left(\frac{1}{2}\right)^{5001} \left(1 - \frac{1}{2}\right)^{4999}$$

$$+ \binom{10000}{5002} \cdot \left(\frac{1}{2}\right)^{5002} \left(1 - \frac{1}{2}\right)^{4998}$$

$$= \frac{\binom{10000}{5000} + \binom{10000}{5001} + \binom{10000}{5002}}{2^{10000}}$$

$\approx 0.023928\ldots$

$\approx \boxed{2.3928\%}$

Mathematica

<u>Ex:</u> Suppose you flip a coin 40,000 times. Let X be the number of heads that occur. Approximate the probability that we get exactly 20,000 heads.

$n = 40000$

$p = 1/2$

$1-p = 1/2$

$np = 20000$

$\sqrt{np(1-p)} = \sqrt{10000} = 100$

$P(X = 20,000)$

$$= P(19{,}999.5 \le X \le 20{,}000.5)$$

$$= P\left(\frac{19999.5 - 20000}{100} \le \underbrace{\frac{X - 20000}{100}}_{\frac{X - np}{\sqrt{np(1-p)}}} \le \frac{20000.5 - 20000}{100} \right)$$

$$= P\left(-0.005 \le \frac{X - 20000}{100} \le 0.005 \right)$$

$$\approx \Phi(0.005) - \Phi(-0.005)$$

$$= \Phi(0.005) - [1 - \Phi(0.005)]$$

$$= 2\Phi(0.005) - 1$$

$$\approx 2\underbrace{(0.50199)}_{\text{not in table need online calculator}} - 1 \approx \boxed{0.00398}$$

$$\approx \boxed{0.398\%}$$

$\boxed{\text{Real answer:}}$

$$P(X = 20000) =$$

$$\binom{40000}{20000} \cdot \left(\frac{1}{2}\right)^{20000} \left(1 - \frac{1}{2}\right)^{20000}$$

$$= \binom{40000}{20000} \Big/ 2^{40000}$$

$$\approx \boxed{0{,}0039894} \approx \boxed{0{,}39894\ \%}$$