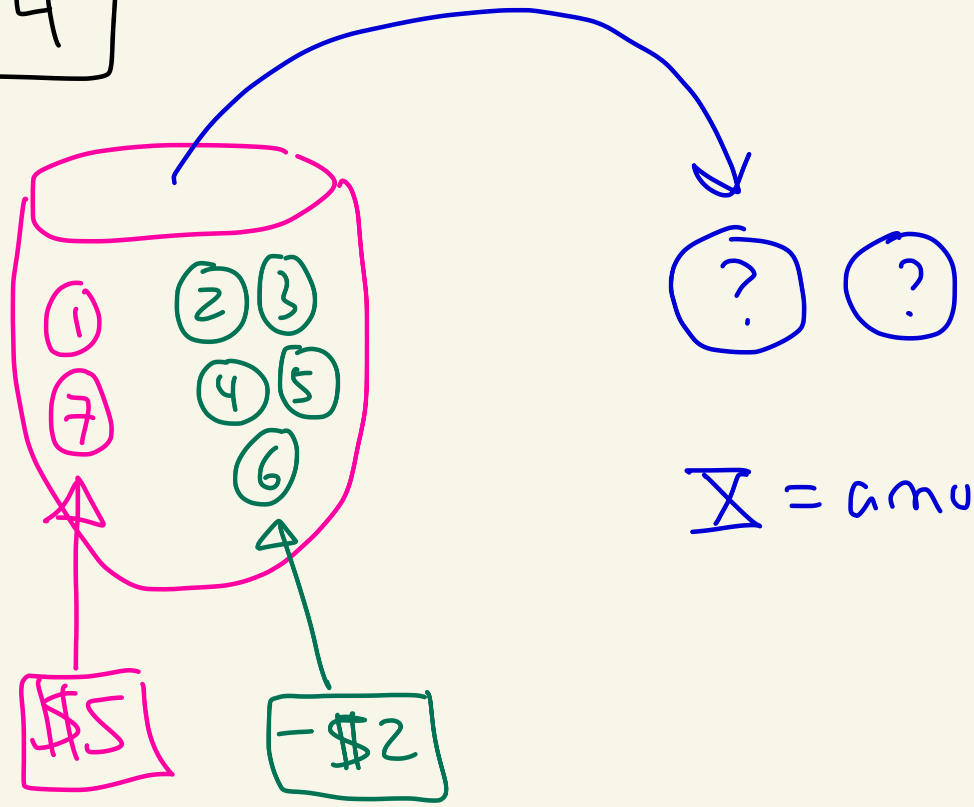
$$\boxed{\begin{array}{l}\text{Math 4740} \\ \text{4/28/25}\end{array}}$$

HW 4

(4)



X = amount won/lost

(a) Draw $P(X=k)$

(b) Calculate $E[X]$

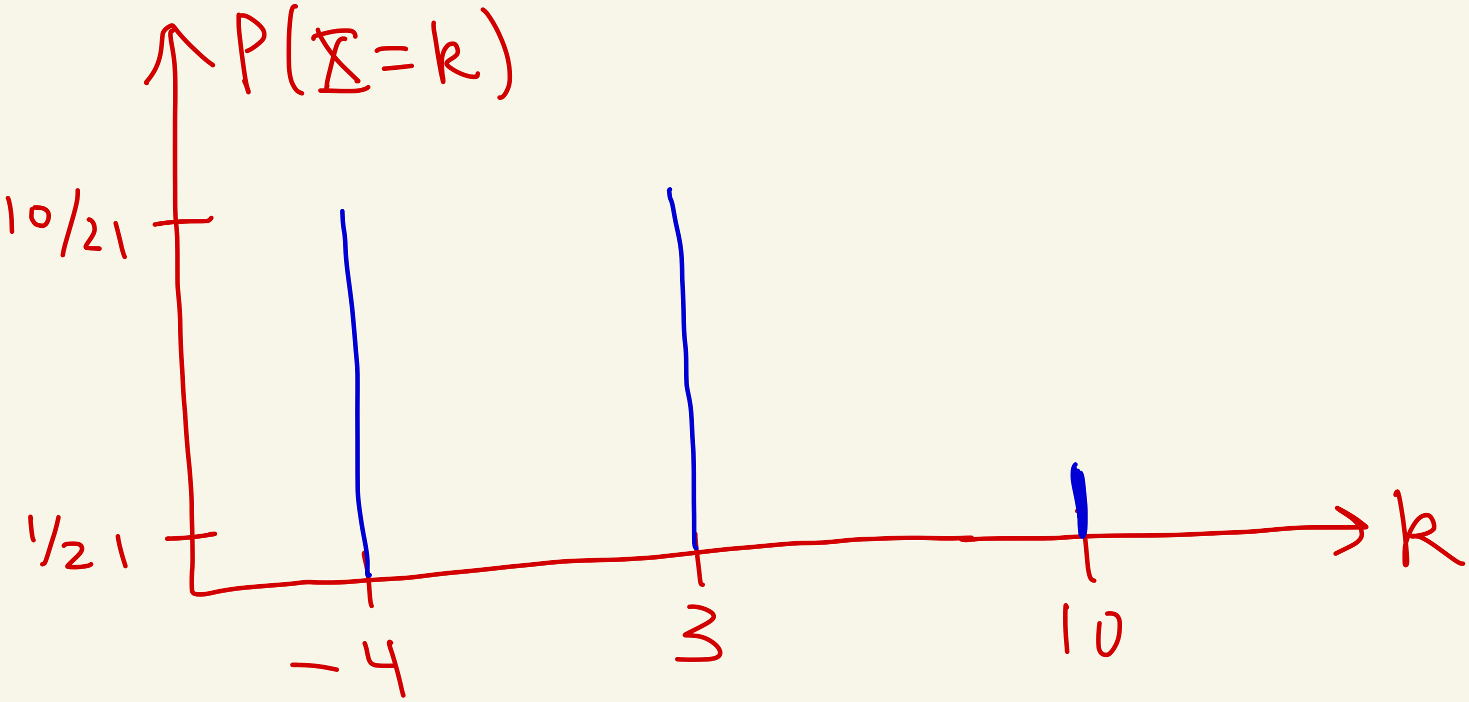
(c) Calculate $Var(X)$, σ.

(a) $|S| = \binom{7}{2} = \frac{7!}{2!5!} = \frac{7\cdot 6}{2} = 21$

$P(X=10) = \frac{1}{21}$

$$P(X=3) = \frac{\binom{2}{1}\binom{5}{1}}{21} = \frac{2\cdot 5}{21} = \frac{10}{21}$$

$$P(X=-4) = \frac{\binom{5}{2}}{21} = \frac{\frac{5!}{2!\,3!}}{21} = \frac{\frac{5\cdot 4\cdot \cancel{3!}}{2\cdot \cancel{3!}}}{21} = \frac{10}{21}$$



(b)

$$E[X] = (-\$4)\left(\frac{10}{21}\right) + (\$3)\left(\frac{10}{21}\right) + (\$10)\left(\frac{1}{21}\right)$$

$$= \$0$$

(c)

$$E[X^2] = (-\$4)^2\left(\frac{10}{21}\right) + (\$3)^2\left(\frac{10}{21}\right) + (\$10)^2\left(\frac{1}{21}\right)$$

$$= \$\,\frac{160 + 90 + 100}{21}$$

$$= \$\,\frac{350}{21}$$

$$Var(X) = E[X^2] - (E[X])^2$$

$$= \$\,\frac{350}{21} - (\$0)^2$$

$$= \$\,\frac{350}{21} = \boxed{\$16.6\overline{6}}$$

$$\sigma = \sqrt{\$\frac{350}{21}} \approx \boxed{\$4.08}$$

HW 5

2 roll two 6-sided dice

success = sum is 7 or 11

X = # successes in $n=10$ rolls

$$\boxed{p = \frac{8}{36}}$$ ← probability of success in one roll

$$P(X=5) = \binom{10}{5} p^5 (1-p)^5$$

$$= \frac{10!}{5!\,5!} \left(\frac{8}{36}\right)^5 \left(\frac{28}{36}\right)^5$$

$$= \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{5!} \frac{8^5\, 28^5}{36^{10}}$$

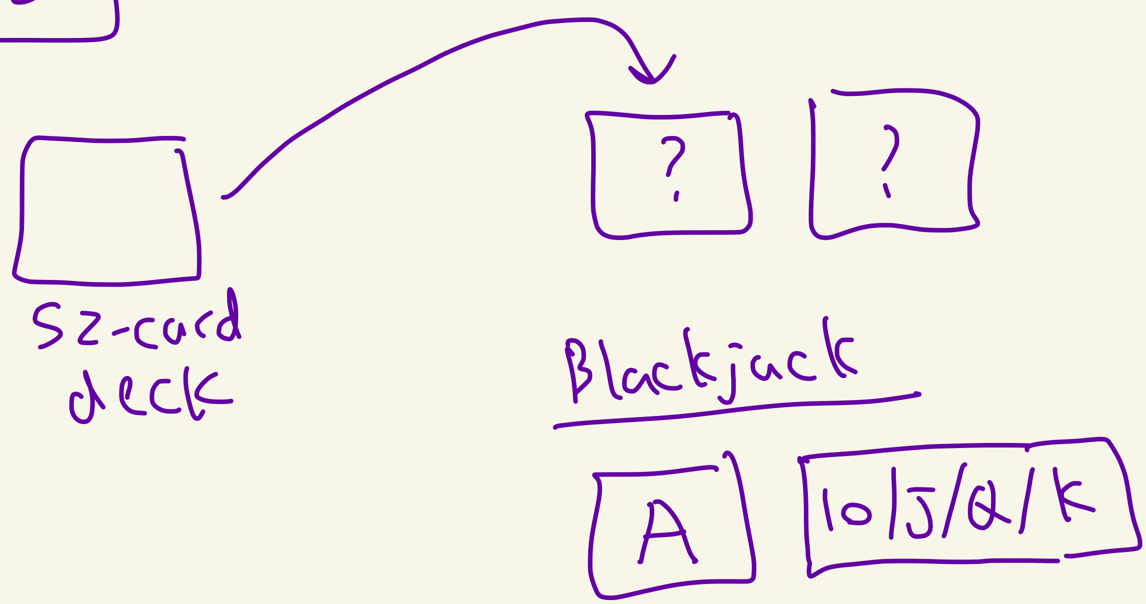
$$= \frac{30{,}240}{120} \cdot \frac{(32{,}768)(17{,}210{,}368)}{36^{10}}$$

$$\approx \boxed{0.03887\ldots}$$

$$\approx \boxed{3.9\%}$$

$\boxed{\text{HW 5}}$

4



Repeat 20 times

X = # blackjacks

$P(X \geq 2)$

probability of blackjack

$$p = \frac{\overbrace{\binom{4}{1}}^{A}\overbrace{\binom{16}{1}}^{10/J/Q/K}}{\binom{52}{2}} = \frac{4 \cdot 16}{\left(\frac{52!}{50!\,2!}\right)}$$

$$= \frac{64}{\left(\frac{52 \cdot 51}{2}\right)} = \frac{64}{1326} = \frac{32}{663}$$

$$\approx \boxed{0.048}$$

$$P(X \geq 2) = 1 - \underbrace{P(X \leq 1)}_{P(X<2)}$$

$$= 1 - P(X=0) - P(X=1)$$

$$= 1 - \binom{20}{0} \cdot (0.048)^0 (0.952)^{20} - \binom{20}{1} \cdot (0.048)^1 (0.952)^{19}$$

$= 1 - 1 \cdot (1)(0.374) - 20(0.048)(0.393)$

$\approx \boxed{0.2487...} \approx \boxed{24.87\%}$

$\boxed{\text{HW 7}}$

1. Roll 6-sided die 100 times.

(a) Estimate probability 4 occurs between 0 and 15 times

X = # of 4's in 100 rolls

Estimate $P(0 \leq X \leq 15)$

$n = 100$

$p = 1/6 \leftarrow$ prob. of 4 on 6-sided die

$np = 100 \cdot \frac{1}{6} = \frac{100}{6}$

$\sqrt{np(1-p)} = \sqrt{100\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} = \sqrt{\frac{500}{36}}$

$$P(0 \leq X \leq 15)$$

$$= P\left(\frac{0-\frac{100}{6}}{\sqrt{500/36}} \leq \underbrace{\frac{X-\frac{100}{6}}{\sqrt{500/36}}}_{\frac{X-np}{\sqrt{np(1-p)}}} \leq \frac{15-\frac{100}{6}}{\sqrt{500/36}}\right)$$

$$= P\left(-4{,}47 \leq \frac{X-np}{\sqrt{np(1-p)}} \leq -0{,}45\right)$$

$$\approx \Phi(-0{,}45) - \Phi(-4{,}47)$$

$$\approx \left(1-\Phi(0{,}45)\right) - \left(1-\Phi(4{,}47)\right)$$

$$\boxed{\Phi(-t) = 1-\Phi(t)}$$

$\approx (1 - 0.6736) - \underbrace{(1 - 1)}_{0}$

$\approx \boxed{0.3264}$

HW 6

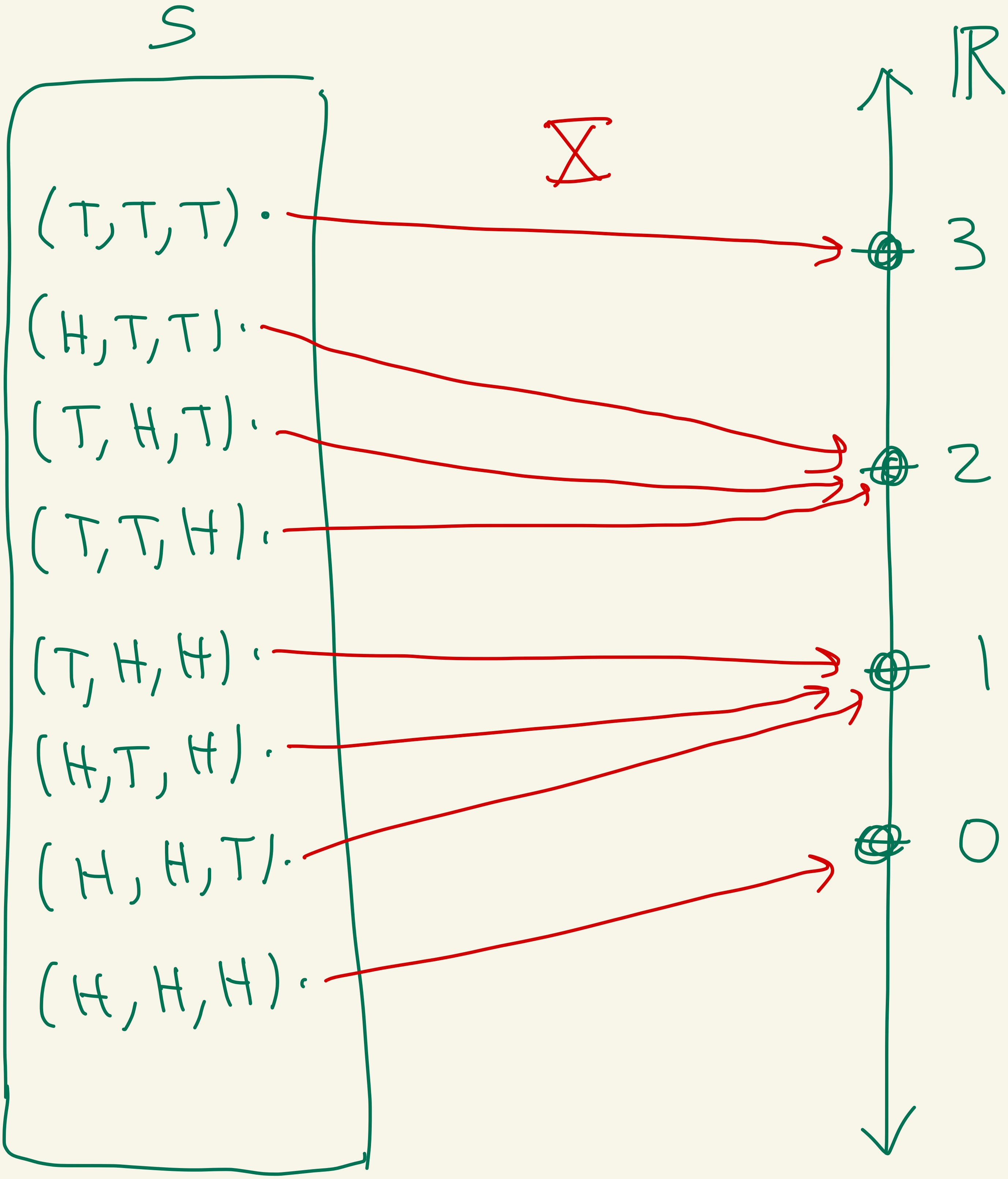
1. Flip a coin 3 times.
   $X = \#$ tails.

Draw X

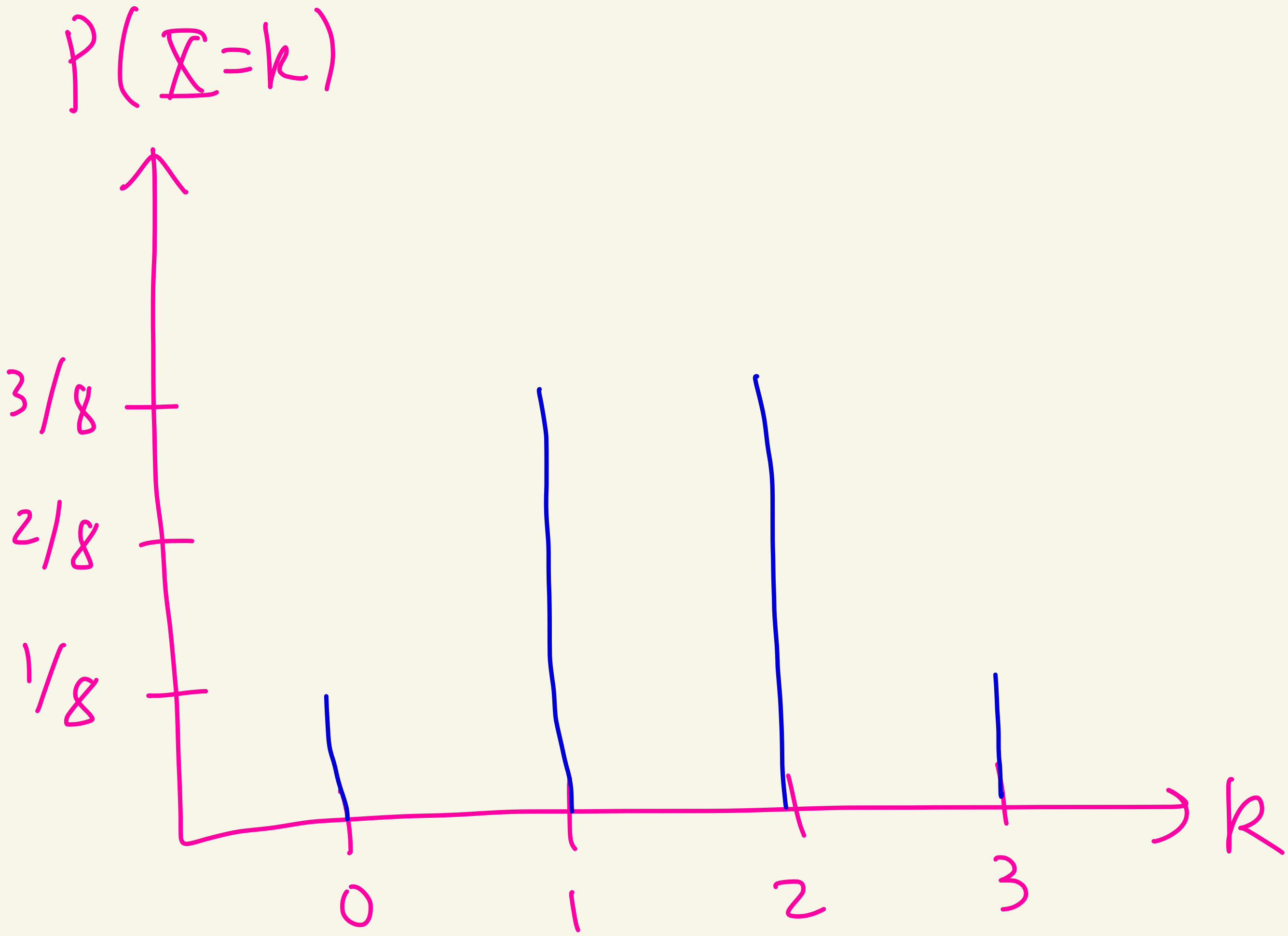
Draw $P(X = k)$

Calculate $E(X)$

Calculate $Var(X)$ and σ.



S
X
$\mathbb{R}$
(T, T, T)
(H, T, T)
(T, H, T)
(T, T, H)
(T, H, H)
(H, T, H)
(H, H, T)
(H, H, H)
3
2
1
0



$$E[X] = (0)\left(\frac{1}{8}\right) + (1)\left(\frac{3}{8}\right) + (2)\left(\frac{3}{8}\right) + (3)\left(\frac{1}{8}\right)$$

$$= \frac{12}{8} = 1.5$$

$$E[X^2] = (0)^2\left(\frac{1}{8}\right) + (1)^2\left(\frac{3}{8}\right) + (2)^2\left(\frac{3}{8}\right) + (3)^2\left(\frac{1}{8}\right)$$

$$= \frac{0+3+12+9}{8} = \frac{24}{8} = 3$$

$$Var(X) = E[X^2] - (E[X])^2$$

$$= 3 - (1.5)^2$$

$$= 3 - 2.25$$

$$= \boxed{0.75}$$

$$\sigma = \sqrt{0.75} \approx \boxed{0.866}$$