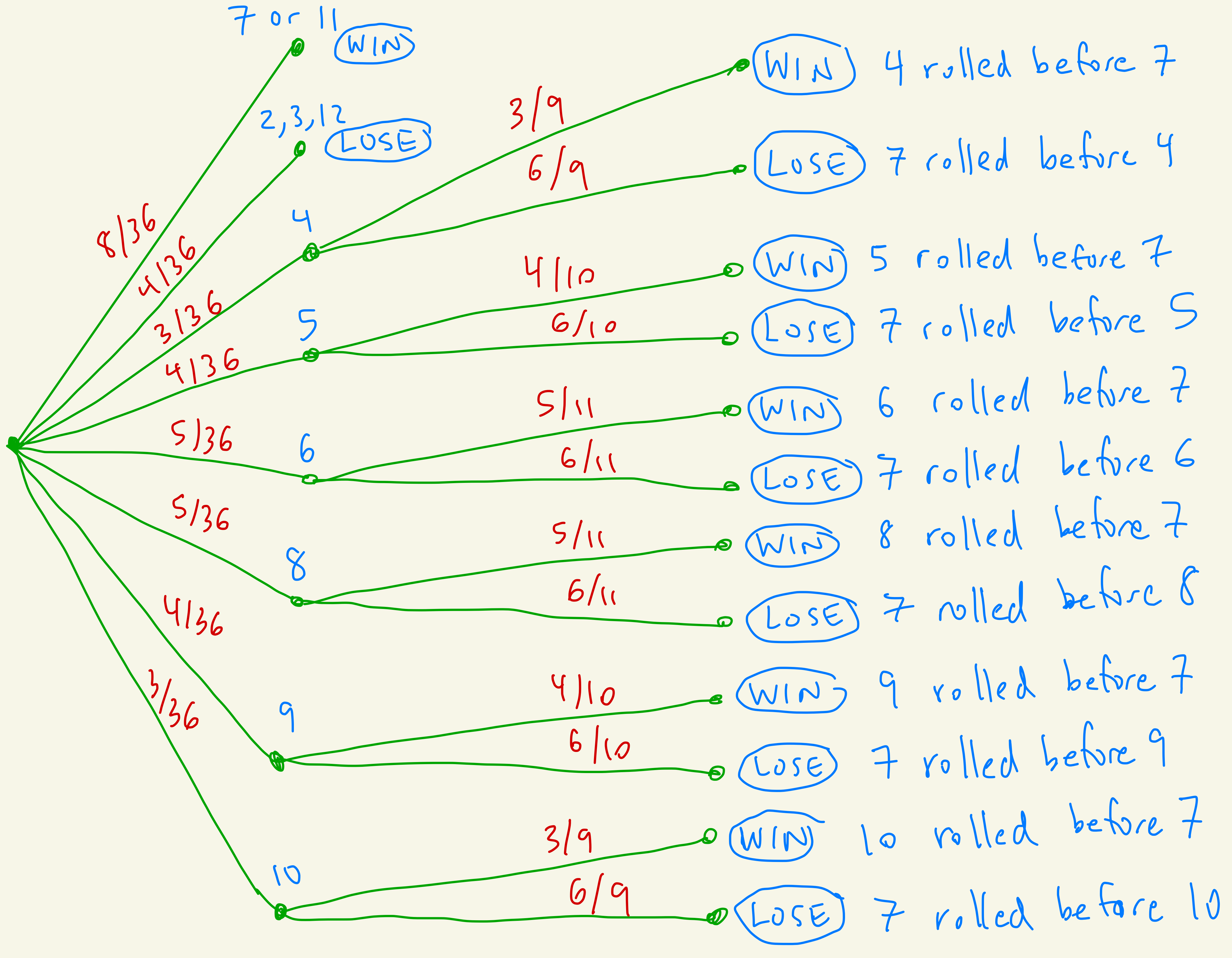
Math 4740
4/16/25



7 or 11
WIN
2,3,12
LOSE
8/36
4/36
3/36
4/36
5/36
5/36
4/36
3/36
4
5
6
8
9
10
3/9
6/9
4/10
6/10
5/11
6/11
5/11
6/11
4/10
6/10
3/9
6/9
WIN
4 rolled before 7
LOSE
7 rolled before 4
WIN
5 rolled before 7
LOSE
7 rolled before 5
WIN
6 rolled before 7
LOSE
7 rolled before 6
WIN
8 rolled before 7
LOSE
7 rolled before 8
WIN
9 rolled before 7
LOSE
7 rolled before 9
WIN
10 rolled before 7
LOSE
7 rolled before 10

The probability of winning the pass line bet is

$$\underbrace{\frac{8}{36}}_{7 \text{ or } 11} + \underbrace{\frac{3}{36} \cdot \frac{3}{9}}_{\text{point 4 and we win}} + \underbrace{\frac{4}{36} \cdot \frac{4}{10}}_{\text{point 5 and we win}}$$

$$+ \underbrace{\frac{5}{36} \cdot \frac{5}{11}}_{\text{point 6 and we win}} + \underbrace{\frac{5}{36} \cdot \frac{5}{11}}_{\text{point 8 and we win}} + \underbrace{\frac{4}{36} \cdot \frac{4}{10}}_{\text{point 9 and we win}}$$

$$+ \underbrace{\frac{3}{36} \cdot \frac{3}{9}}_{\text{point 10 and we win}} = \boxed{\frac{244}{495}} \approx \boxed{0.4929}$$

probability of losing pass line bet is

$$1 - \frac{244}{495} = \boxed{\frac{251}{495}} \approx \boxed{0.5071}$$

Suppose we bet \$1 on the pass line. Let X be the amount won or lost.

$$E[X] = (\$1)\,P(X=\$1) + (-\$1)\,P(X=-\$1)$$

pass line pays 1:1

$$= (\$1)\left(\frac{244}{495}\right) + (-\$1)\left(\frac{251}{495}\right)$$

$$= \boxed{-\$\frac{7}{495}} \approx \boxed{-\$0.01414\ldots}$$

On average if we played this bet <u>many</u> times we would lose about 1.4¢ per \$1 wagered.

The casino allows another type of bet. It's called a "free odds" bet. It's allowed after a point is made.

The free odds bet is paid out at the true odds making it a "fair bet".

That is the expected value of this bet is \$0.

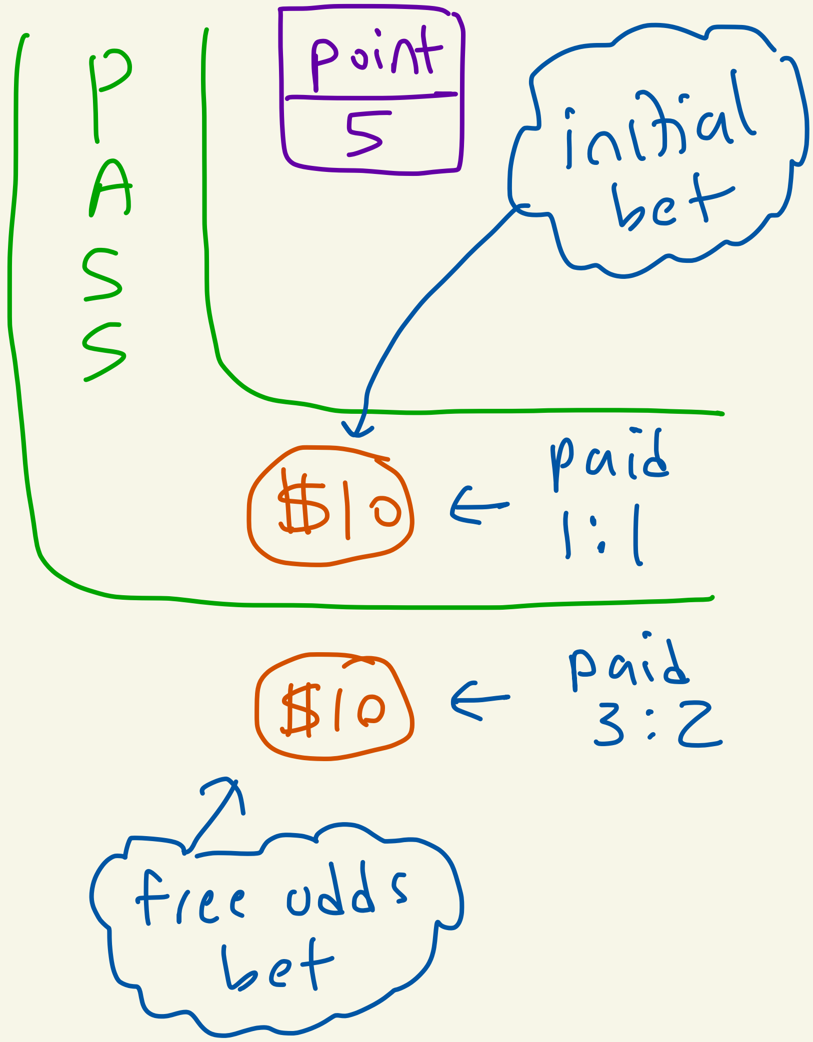
Payouts for "free odds" bet

point	payout
4	2:1
5	3:2
6	6:5
8	6:5
9	3:2
10	2:1

<u>Ex:</u> Suppose we bet \$10 on the pass line. Let's say the come out roll is a 5. Now that a

point is made we can add a "free odds" bet.

Let's say we bet another $10.



come out roll		5
roll 2		3
roll 3		4
roll 4		5

We win

$$\text{Winnings} = (\$10)\left(\frac{1}{1}\right) + (\$10)\left(\frac{3}{2}\right)$$

$$= \$10 + \$15 = \$25$$

How much would we have lost if 7 came before 5?

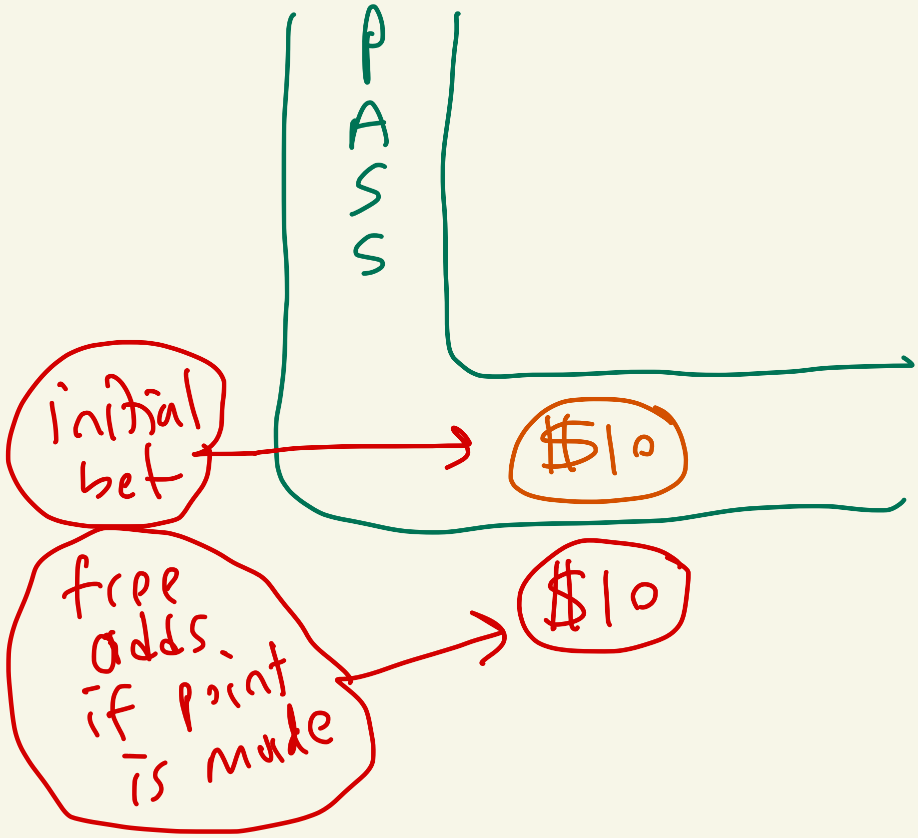
$$\underbrace{\$10}_{\text{initial}} + \underbrace{\$10}_{\text{free odds}} = \$20$$

Let's look at the expected value of the above free odds bet.

Suppose you bet \$10 on the pass line and if a point is made you bet another \$10 as a free odds bet.

Let X be the amount won or lost.

Let's calculate $E[X]$.



$$E[X] = \underbrace{\left(\frac{8}{36}\right)(\$10)}_{\text{come out roll is 7 or 11}} + \underbrace{\left(\frac{4}{36}\right)(-\$10)}_{\text{come out roll is 2, 3, 12}}$$

$$+ \underbrace{2 \cdot \left(\frac{3}{36}\right)\left(\frac{3}{9}\right)(\$30)}_{\text{point is 4 or 10 and we win}} + \underbrace{2 \cdot \left(\frac{3}{36}\right)\left(\frac{6}{9}\right)(-\$20)}_{\text{point is 4 or 10 and we lose}}$$

$10 ← 1:1 initial
$20 ← 2:1 free odds
$30

−$10 ← initial
−$10 ← free odds
−$20

$$+2\cdot\underbrace{\left(\frac{4}{36}\right)\left(\frac{4}{10}\right)(\$25)}_{\text{point is 5 or 9 and we win}} + 2\cdot\underbrace{\left(\frac{4}{36}\right)\left(\frac{6}{10}\right)(-\$20)}_{\text{point is 5 or 9 and we lose}}$$

$\$10 \leftarrow$ 1:1 initial
$\$15 \leftarrow$ 3:2 free odds
$\$25$

$-\$10 \leftarrow$ initial
$-\$10 \leftarrow$ free odds
$-\$20$

$$+2\cdot\underbrace{\left(\frac{5}{36}\right)\left(\frac{5}{11}\right)(\$22)}_{\text{point is 6 or 8 and we win}} + 2\cdot\underbrace{\left(\frac{5}{36}\right)\left(\frac{6}{11}\right)(-\$20)}_{\text{point is 6 or 8 and we lose}}$$

$\$10 \leftarrow$ 1:1 initial
$\$12 \leftarrow$ 6:5 free odds
$\$22$

$-\$10 \leftarrow$ initial
$-\$10 \leftarrow$ free odds
$-\$20$

$$= \boxed{-\$\frac{14}{99}} \approx \boxed{-\$0.1414\ldots}$$

We lose about 14.14 ¢ per game played with this strategy in the long run.

Let's put the above in "per \$1 bet" terms so we can compare it to the first analysis of just betting \$1 on the pass line.

Let's see what the average amount bet is when we do the \$10 initial bet and then the \$10 free odds bet.

$$\begin{pmatrix}\text{average}\\ \text{amount}\\ \text{bet}\end{pmatrix} = \underbrace{(\$10)\left(\frac{12}{36}\right)}_{\substack{\text{come out roll}\\ \text{is } 7, 11, 2, 3, 12}} + \underbrace{(\$20)\left(\frac{24}{36}\right)}_{\substack{\text{come out roll is}\\ 4, 5, 6, 8, 9, 10\\ \text{(point made)}}}$$

$$\approx \boxed{\$16.67}$$

Expected value per dollar bet is $\dfrac{-\$0.1414}{\$16.67} = \boxed{-\$0.0085}$

So we lose about 0.85¢ per dollar wagered. Compare that to losing 1.414¢ per dollar wagered with just \$1 bet on pass line.