Math 4740
4/14/25

Theorem: Let S be the sample space of a repeatable experiment. Let A and B be two events that don't overlap $[A \cap B = \emptyset]$ Suppose further that each time we repeat the experiment S, the results are independent of the previous times we did S.

Suppose we keep repeating S until either A or B occurs. Then the probability that A occurs before B occurs is

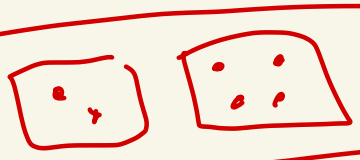
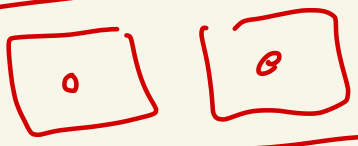
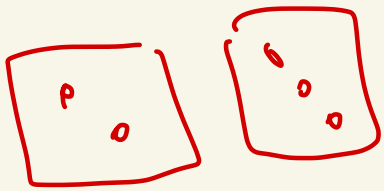
$$\frac{P(A)}{P(A) + P(B)}$$

[Proof: Topic 3 notes online.]

Ex: Suppose we keep rolling two 6-sided dice. We don't stop until either the sum of the dice is 5 or the sum is 7.

What's the probability that sum of 5 will happen before sum of 7 happens?

Example

roll #	dice	Sum
1		6
2		2
3		5

← 5 happened before 7

S = rolling two 6-sided dice

A = sum of dice is 5

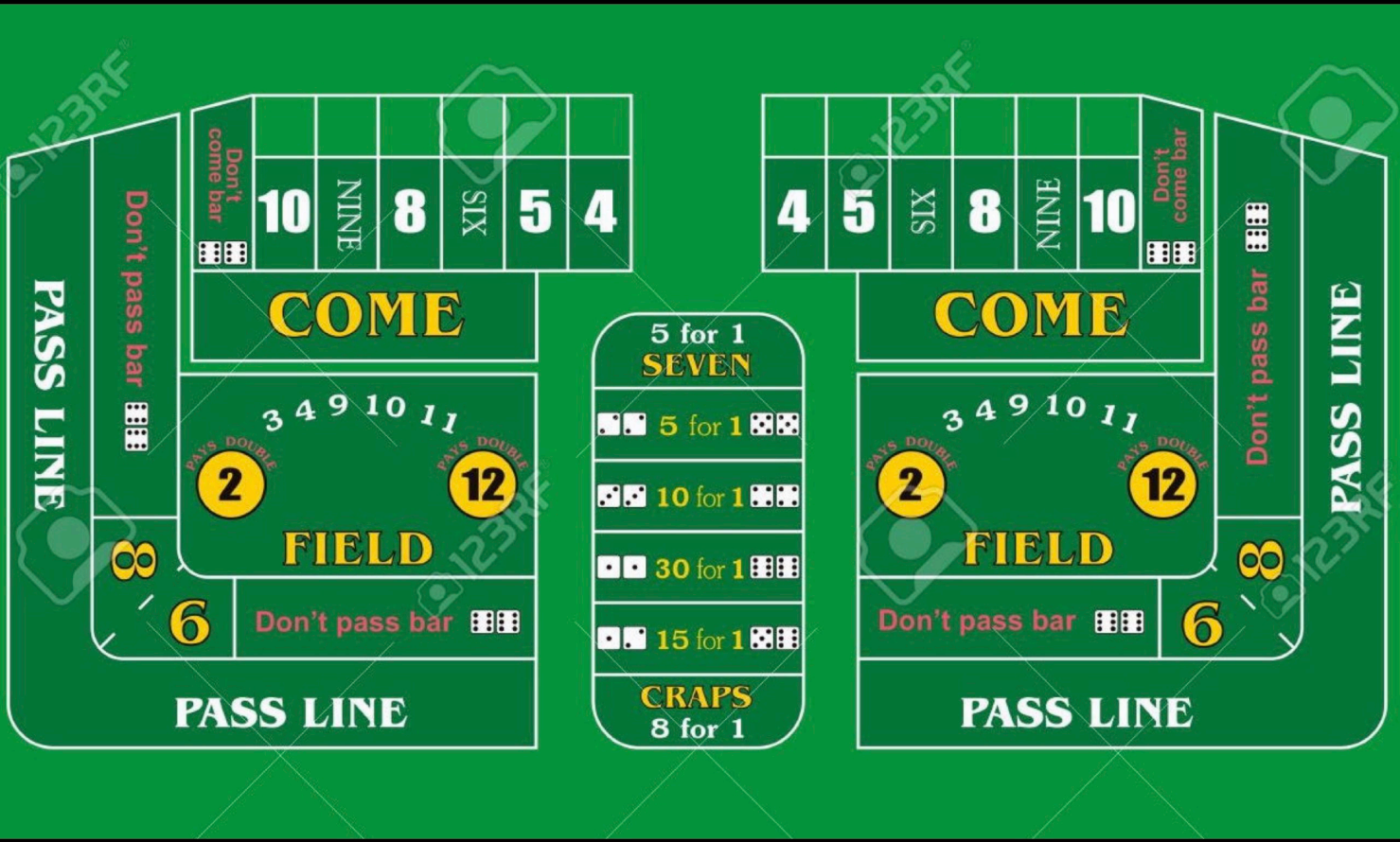
B = sum of dice is 7

$$P(A \text{ before } B) = \frac{P(A)}{P(A) + P(B)}$$

$$= \frac{4/36}{4/36 + 6/36} = \boxed{\frac{4}{10}}$$

$$P(B \text{ before } A) = \frac{P(B)}{P(B) + P(A)}$$

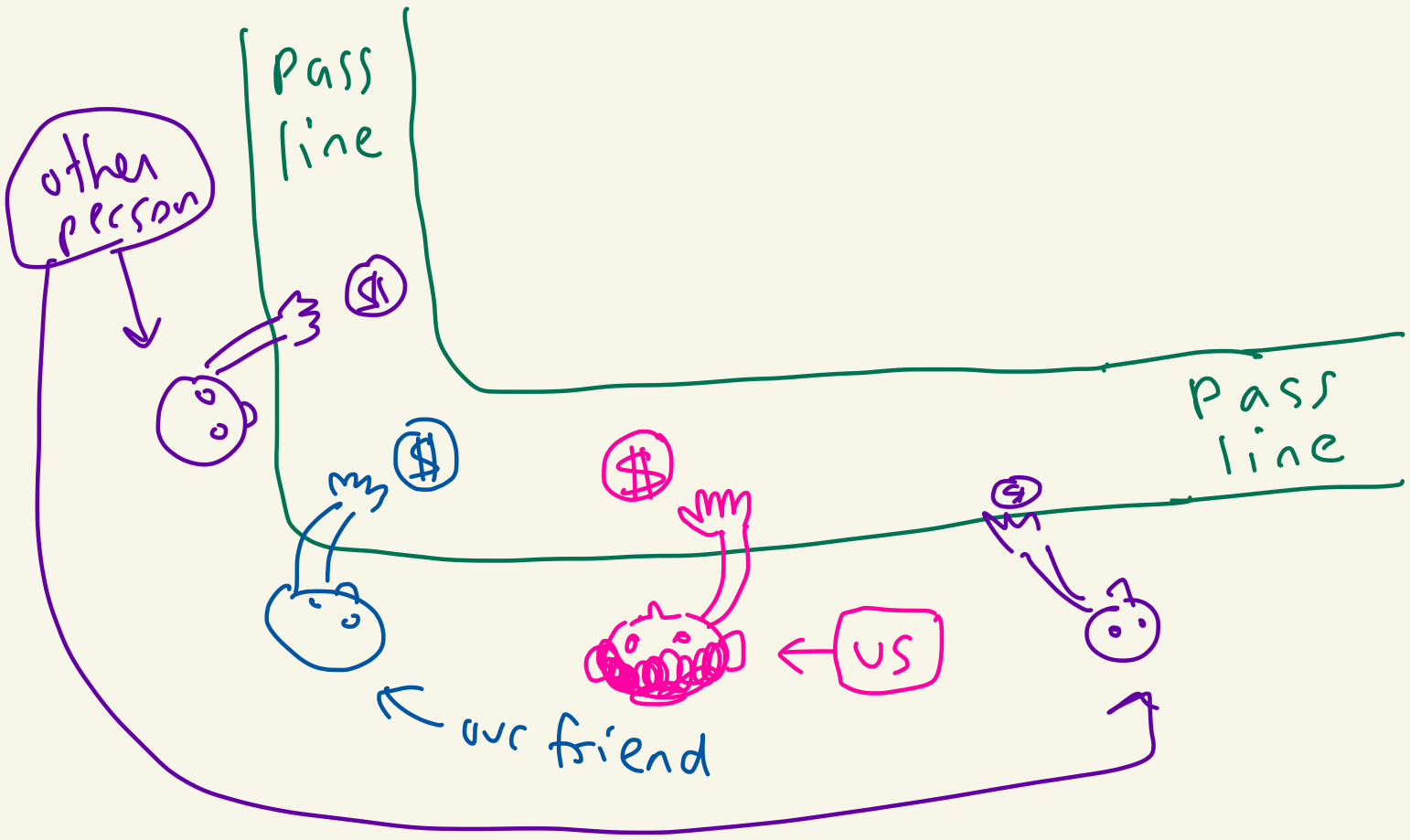
$$= \frac{6/36}{6/36 + 4/36} = \boxed{\frac{6}{10}}$$


PASS LINE
Don't pass bar
Don't come bar
10
NINE
8
SIX
5
4
COME
3 4 9 10 11
PAYS DOUBLE
2
12
FIELD
8
6
Don't pass bar
PASS LINE
5 for 1
SEVEN
5 for 1
10 for 1
30 for 1
15 for 1
CRAPS
8 for 1
4
5
SIX
8
NINE
10
Don't come bar
COME
3 4 9 10 11
PAYS DOUBLE
2
12
FIELD
Don't pass bar
8
6
Don't pass bar
PASS LINE

Craps

The main bet in Craps is called the pass line bet. People place their bets on the pass line on the table and then the game starts.

Suppose we put money on the pass line.



One of the players (called the shooter) rolls the dice.

Two 6-sided dice are rolled.

The first roll is called the "come out roll".

Each time the dice are rolled the sum is measured.

Case 1: If a 7 or 11 is rolled on the come out roll, then we win the pass line bet.

roll called a "natural"

Case 2: If a 2, 3, or 12 is rolled on the come out roll then we lose the

roll called "craps"

pass line bet.

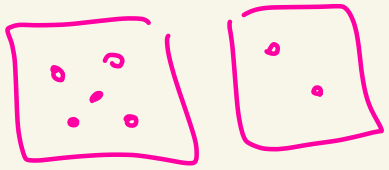
Case 3: If a 4, 5, 6, 8, 9, or 10 is rolled on the come out roll then that number becomes the "point". Now the dice are rolled over and over until either the point or 7 comes up. If the point comes up first, then we win. If 7 comes up first then we lose.

7

- After one of the above cases happens the game is over. It then starts again.
- The casino pays 1:1 on a pass line bet.

EXAMPLE 1

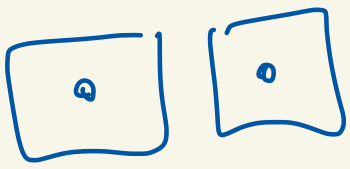
pass line bet = $10

come out roll =  ← Sum is 7

This is case 1, we win.
They pay us $10.
[You keep the $10 bet]

EXAMPLE 2

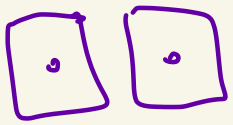
pass line bet = $10

come out roll =  ← sum is 2

This is case 2,
We lose our $10 bet.

EXAMPLE 3

pass line bet = $50

come out roll	roll 2	roll 3	roll 4	
				
5	4	2	5	

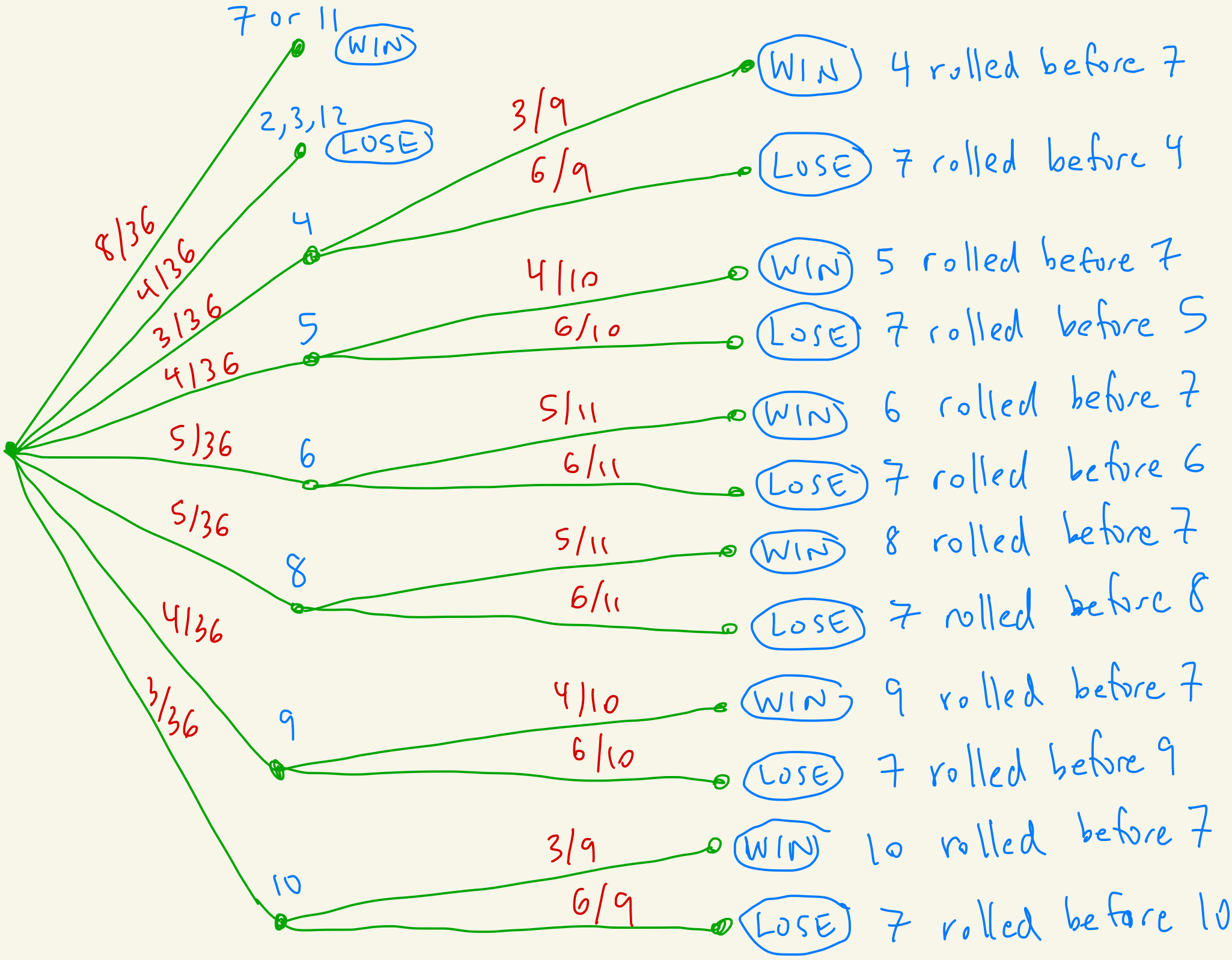
5 is the point

We win $50! 5 happened before 7

If a 7 happened before a 5 we would have lost our $50 bet.

Let's calculate the expected value.
We will need this (case 3):

Point	probability point occuring before 7	probability of 7 occuring before point
4	3/9	6/9
5	4/10	6/10
6	5/11	6/11
8	5/11	6/11
9	4/10	6/10
10	3/9	6/9


7 or 11
WIN
2,3,12
LOSE
4
5
6
8
9
10
8/36
4/36
3/36
4/36
5/36
5/36
4/36
3/36
3/9
6/9
4/10
6/10
5/11
6/11
5/11
6/11
4/10
6/10
3/9
6/9
WIN
LOSE
WIN
LOSE
WIN
LOSE
WIN
LOSE
WIN
LOSE
WIN
LOSE
4 rolled before 7
7 rolled before 4
5 rolled before 7
7 rolled before 5
6 rolled before 7
7 rolled before 6
8 rolled before 7
7 rolled before 8
9 rolled before 7
7 rolled before 9
10 rolled before 7
7 rolled before 10