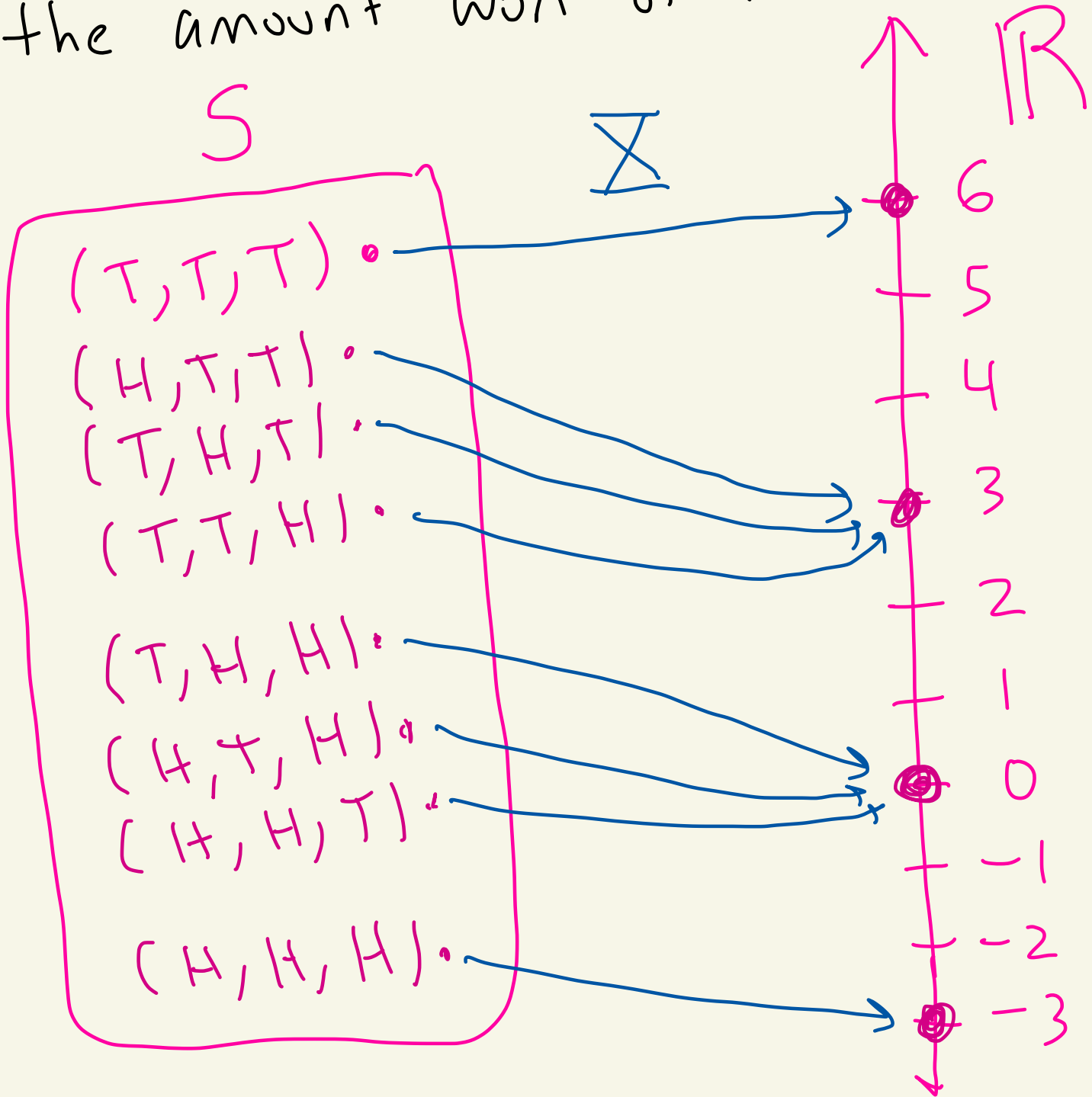
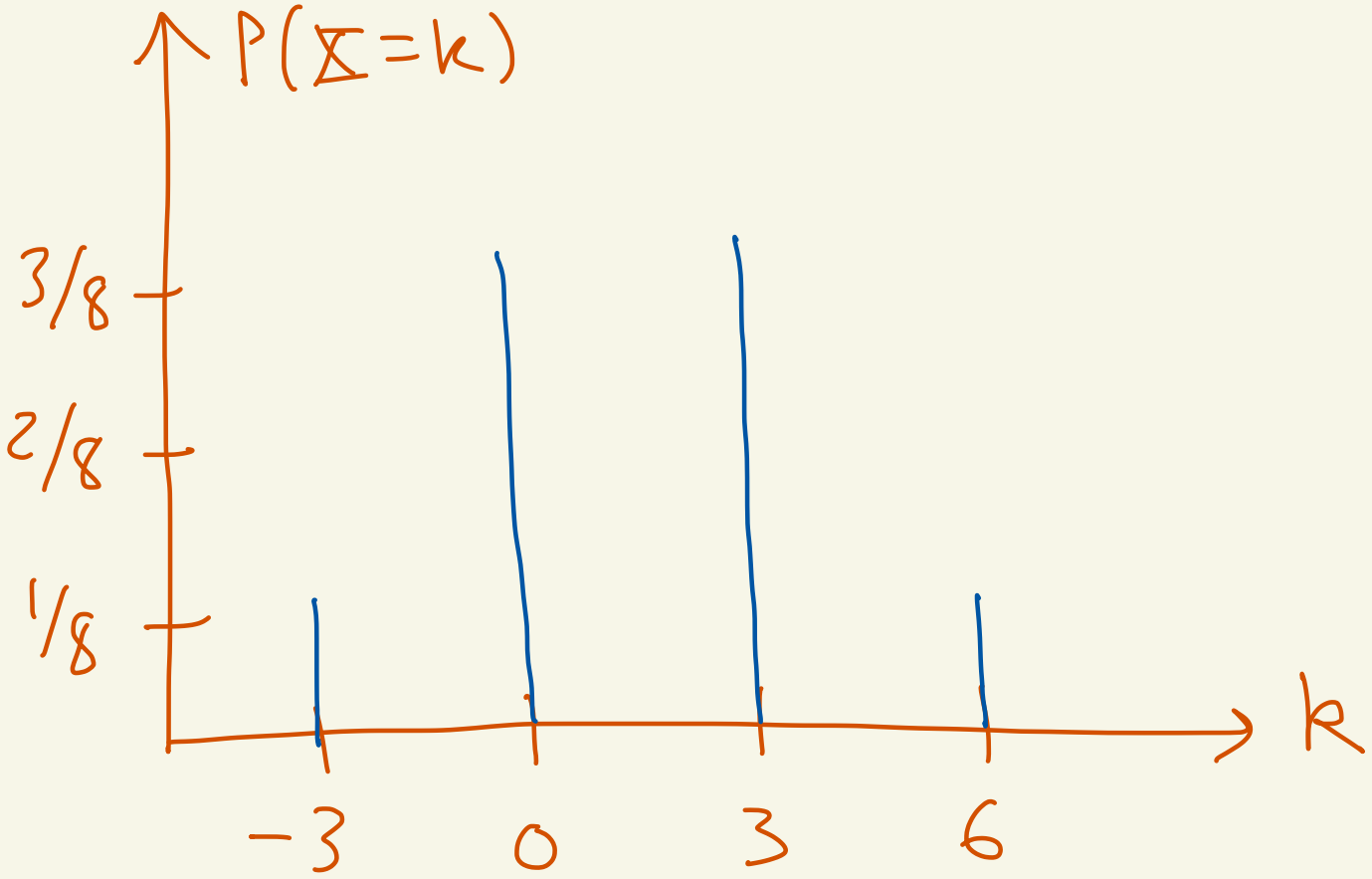
Math 4740

3/10/24

Ex: Suppose you flip a coin three times. For every head you lose $1. For every tail you win $2. Let X be the amount won or lost.



probability function $p(k) = P(X = k)$



The expected value is:

$$E[X] = (\$6)\left(\frac{1}{8}\right) + (\$3)\left(\frac{3}{8}\right) + (\$0)\left(\frac{3}{8}\right) + (-\$3)\left(\frac{1}{8}\right)$$

$$= \$\frac{12}{8} = \boxed{\$1.5}$$

If you play this game many times

on average you expect to win $1.50 per game.

What's the probability you win something when you play?

$$P(X>0) = P(X=3) + P(X=6)$$

$$= \frac{3}{8} + \frac{1}{8} = \frac{1}{2} = \boxed{0.5}$$

If you played this game 1,000,000 times you'd expect to win around

$$(1{,}000{,}000)\underbrace{(\$1.5)}_{E[X]} = \$1{,}500{,}000$$

Odds Let E be an event

Define

$$\text{"odds against } E\text{"} = \frac{P(\bar{E})}{P(E)} = \frac{1-P(E)}{P(E)}$$

This is what casinos quote when they talk about odds payouts

Ex: Suppose you roll a 4-sided die. Let E be the event that a 4 is rolled.

$$\left(\begin{matrix}\text{odds against} \\ E\end{matrix}\right) = \frac{P(\bar{E})}{P(E)} = \frac{3/4}{1/4} = \boxed{\frac{3}{1}}$$

This is written 3:1

Convert back to probability

odds against E

$c : d$

$\longrightarrow$

$$P(E) = \frac{d}{c+d}$$

Ex: If odds against E is $3:1$ then

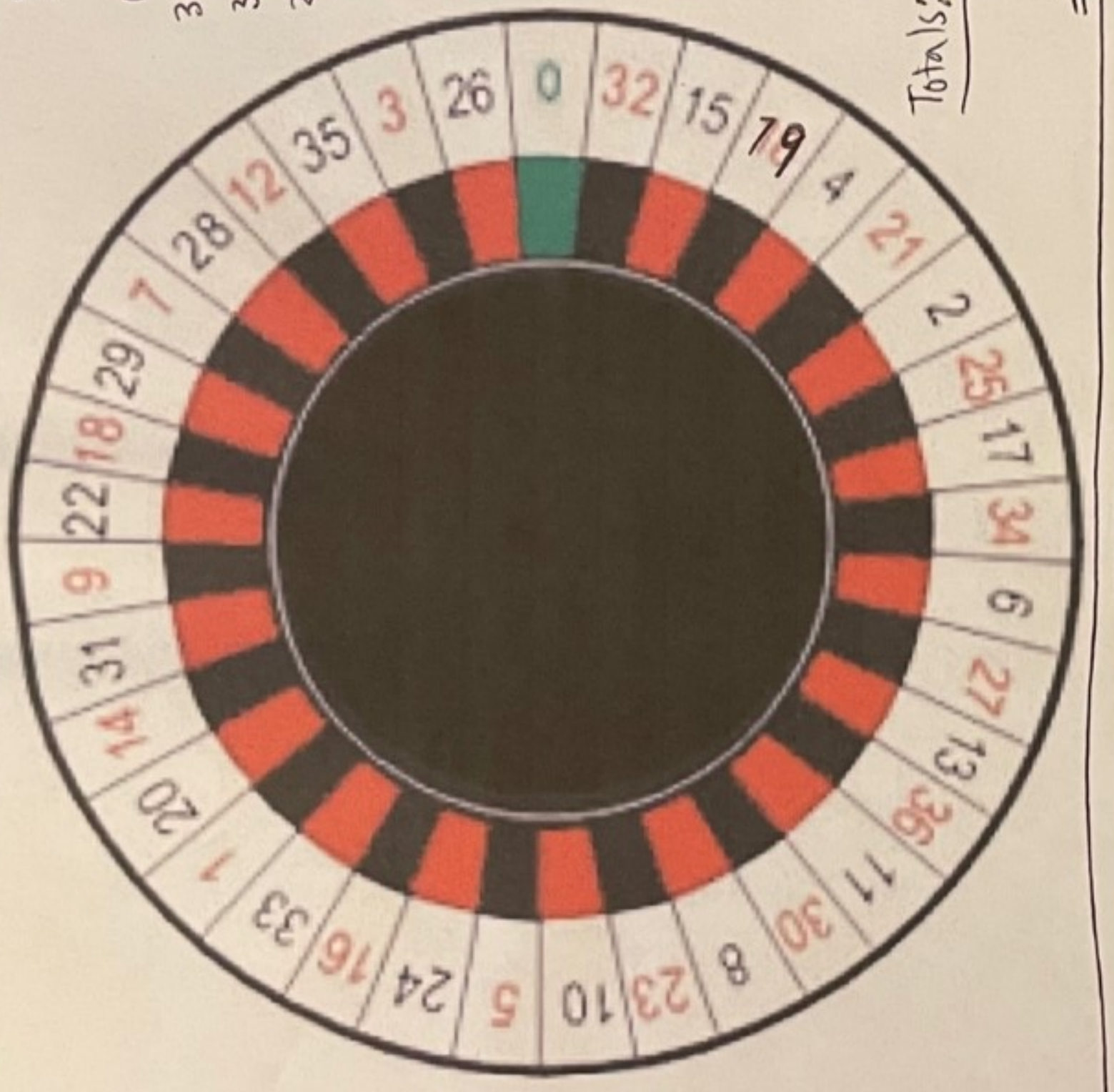
$$P(E) = \frac{1}{3+1} = \frac{1}{4}$$

green
0

red
32, 19, 21, 25,
34, 27, 36, 30,
23, 5, 16, 1,
14, 9, 18, 7
12, 3

black
15, 4, 2, 17
6, 13, 11, 8
10, 24, 33, 20
31, 22, 29, 28
35, 26

Totals: 1 green
18 red
18 black
=37 total



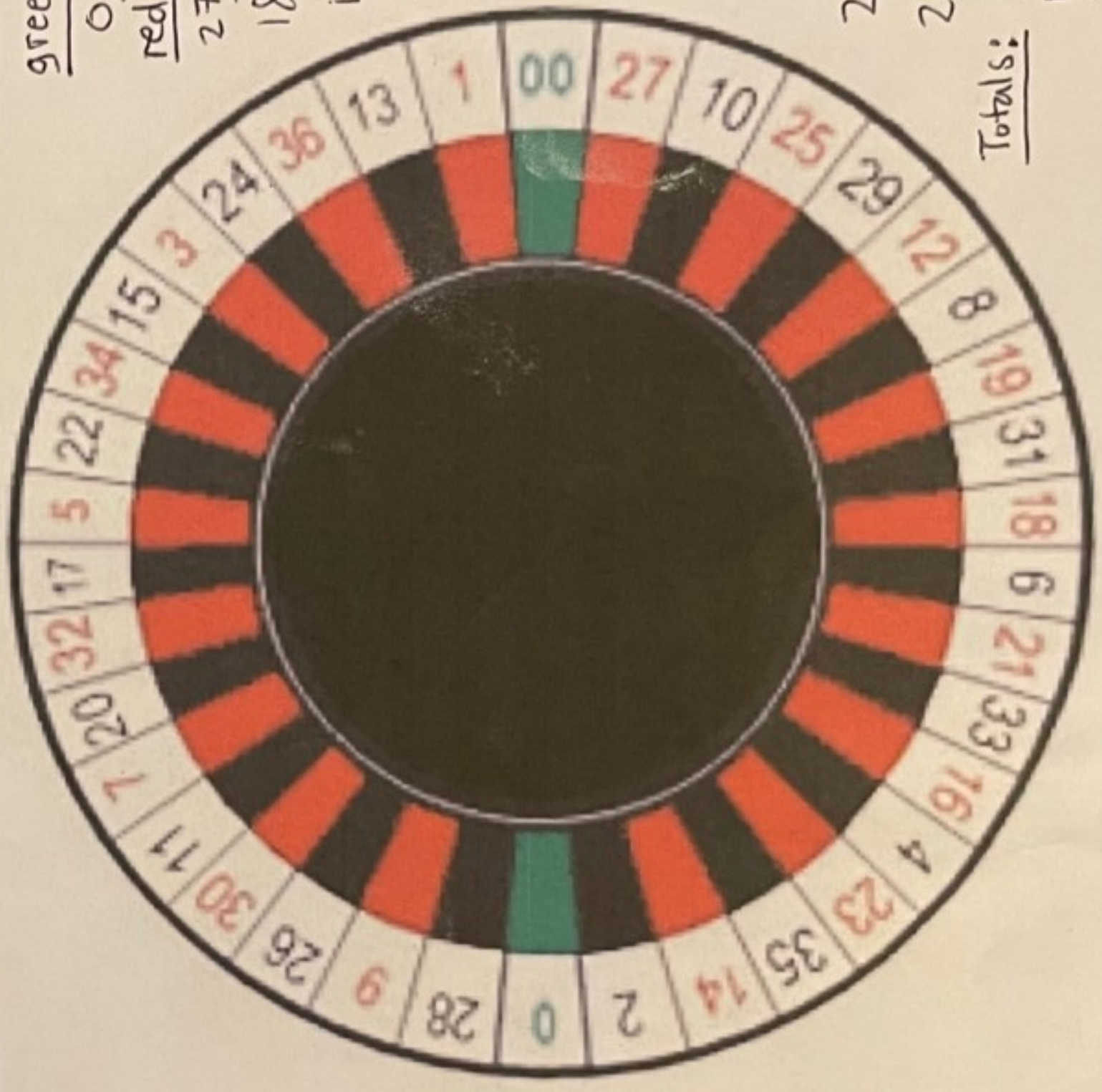
EUROPEAN

green
0, 00

red
27, 25, 12, 19
18, 21, 16, 23
14, 9, 30, 7
32, 5, 34, 3
36, 1

black
10, 29, 8, 31
6, 33, 4, 35
2, 28, 26, 11
20, 17, 22, 15
24, 13

Totals: 2 green
18 red
18 black
= 38 total



AMERICAN

/ Handout

American version

Casino payouts

Type of Bets And [illegible]

Inside bets

Bet Name	Ex.	Numbers to bet on	Payout	Chances	True odds
Straight up	A	30	35:1	38:1	37:1
Split Bet	B	11 or 14	17:1	18:1	36:2
Street Bet	C	19, 20, 21	11:1	12:1	35:3
Corner	D	25, 26, 28, 29	8:1	9:1	34:4
Five Numbers	E	0, 00, 1, 2, 3	6:1	6:1	
Line Bet	F	4, 5, 6, 7, 8, 9	5:1	6:1	32:6

Outside Bets

Bet Name	Ex	Numbers to bet on	Payout	Chances	True odds
Column	G	Set of column numbers	2:1	38:12	26:12
Dozen	H	25 through 36	2:1	38:12	26:12
Red or Black	I	Red numbers	1:1	38:18	20:18
Even or Odd	J	Odd numbers	1:1	38:18	20:18
Low or High	K	19 through 36	1:1	38:18	20:18

Sample space of Roulette

$S = \{ 0, 00, 1, 2, 3, 4, 5, 6,$
$7, 8, 9, 10, 11, 12, 13,$
$14, 15, 16, 17, 18, 19, 20,$
$21, 22, 23, 24, 25, 26,$
$27, 28, 29, 30, 31, 32,$
$33, 34, 35, 36 \}$

green = 2 of them
red = 18 of them
black = 18 of them

Each # equally likely
with probability $1/38$

Straight up bet (35:1 payout)

Let's say we bet $1 on (7)
What's the expected value
of this bet?
Let X be the amount won/lost

$$X(w) = \begin{cases} \$35 & \text{if } w = 7 \\ -\$1 & \text{if } w \neq 7 \end{cases}$$

w is outcome from wheel

$$E[X] = (\$35)\left(\frac{1}{38}\right) + (-\$1)\left(\frac{37}{38}\right)$$

$$= -\$\frac{2}{38} \approx \boxed{-\$0.0526} \approx \boxed{-5.26¢}$$

So on average with many of these bets you'd lose -5.26¢ per spin.

You'd lose 5.26% of your bet each time on average

The casino pays less than the "true odds" or "odds against". What's the true odds here?

Let $E = \{7\}$

$$\text{odds against } E = \frac{P(\bar{E})}{P(E)} = \frac{37/38}{1/38} = \frac{37}{1}$$

37:1

What would the expected value be if they paid 37:1 if you win? It would be:

$$(\$37)\left(\frac{1}{38}\right) + (-\$1)\left(\frac{37}{38}\right) = \$0$$

On average you'd break even over the long run.

win/lose $0

Red bet (payout is 1:1)

Suppose you bet $1 on red. What's the expected value?

$$(\$1)\underbrace{\left(\frac{18}{38}\right)}_{\text{win}} + (-\$1)\underbrace{\left(\frac{20}{38}\right)}_{\text{lose}}$$

$= -\$\frac{2}{38} \approx \boxed{-\$0.0526}$

$\approx \boxed{-5.26¢}$

On the side:

$P(\text{win}) = 18/38 \approx 0.47$

$P(\text{lose}) = 20/38 \approx 0.53$

What are the true odds?

E = red # occuring

$$\text{odds against } E = \frac{P(\bar{E})}{P(E)} = \frac{20/38}{18/38} = \frac{20}{18} = \frac{10}{9}$$

If they paid you 10:9 on your win what would the expected value be?

$$\left(\$\frac{10}{9}\right)\left(\frac{18}{38}\right) + \left(-\$1\right)\left(\frac{20}{38}\right) = \$0$$

win

lose

This would make the game "fair"