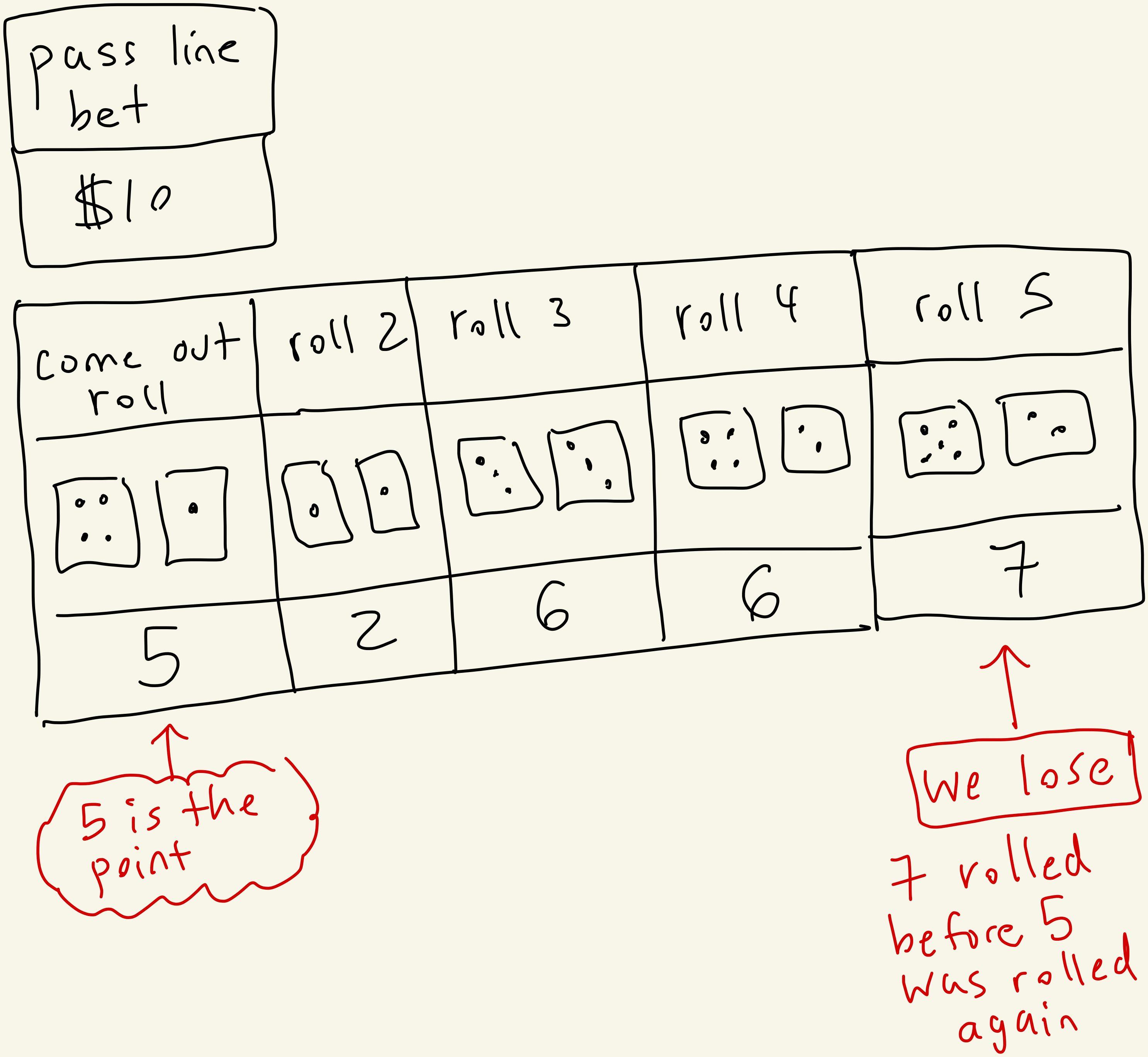
Math 4740

10/25/23



Let's calculate the expected value of betting on the pass line.

Sum of dice	# ways to roll
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

probabilites for come out roll

roll	probability
7 or 11	8/36
2,3,or 12	4/36
4	3/36
5	4/36
6	5/36
8	5/36
9	4/36
10	3/36

WIN

LOSE

point is made with probability 24/36

Let's calculate the probabilities of winning or losing once a point is made

Ex: Suppose the come out roll sums to 8.
So, 8 is the point.

Now we keep rolling until either an 8 or a 7 comes up.

On an individual roll, let A be the event that the sum is 8 and B be the event that the sum is 7.

If we keep rolling, then...

- the probability that A occurs before B (ie an 8 is rolled before a 7) is

$$\frac{P(A)}{P(A)+P(B)} = \frac{5/36}{5/36 + 6/36} = \frac{5}{11}$$

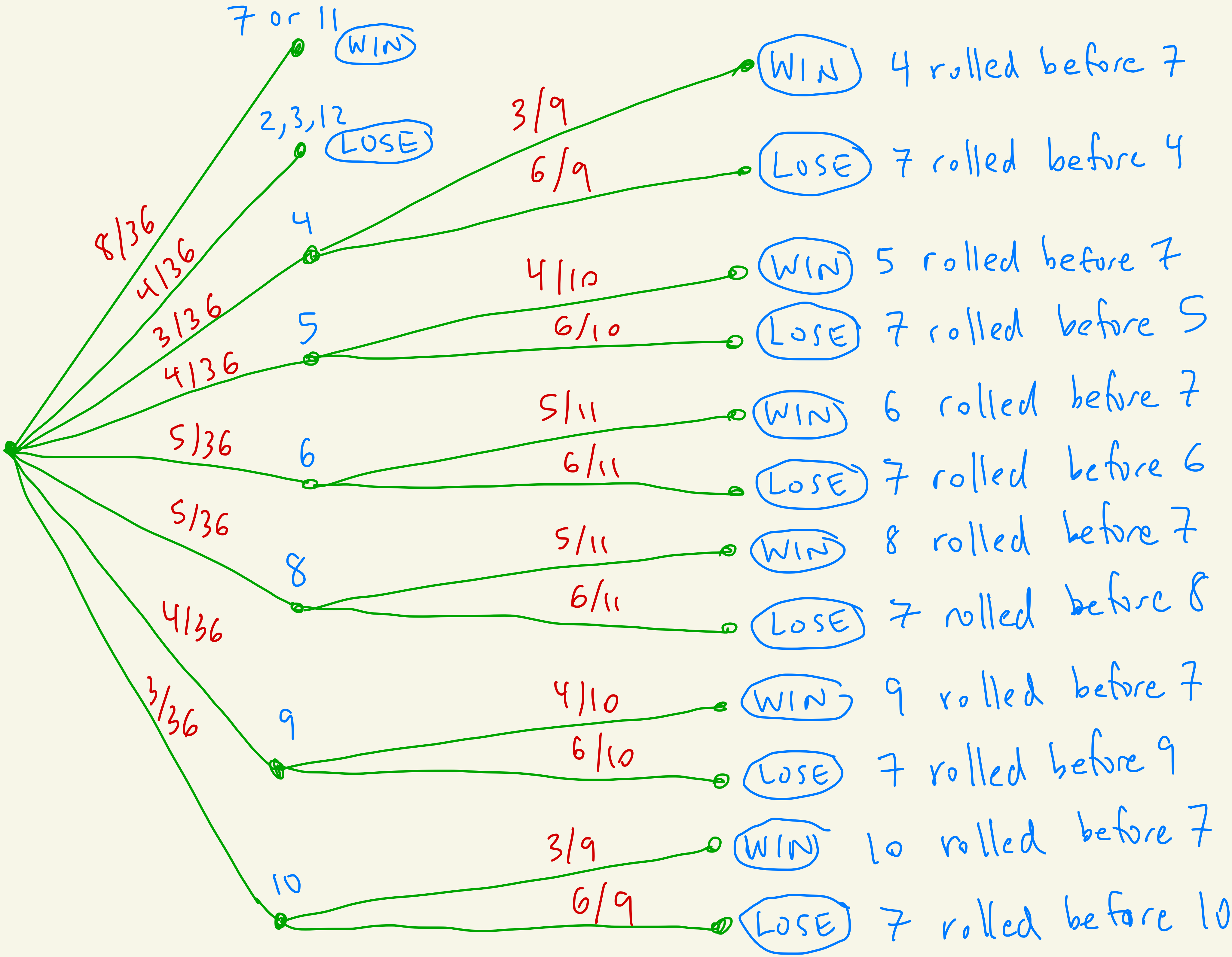
- the probability that B occurs before A (ie a 7 before an 8)

$$\frac{P(B)}{P(B)+P(A)} = \frac{6/36}{6/36 + 5/36} = \frac{6}{11}$$

Here's the table for all possible points.

point	probability of point being rolled before 7	probability of 7 rolled before point
4	3/9	6/9
5	4/10	6/10
6	5/11	6/11
8	5/11	6/11
9	4/10	6/10
10	3/9	6/9

Let's make the tree of all possibilities.


7 or 11
WIN
2,3,12
LOSE
8/36
4/36
3/36
4/36
5/36
5/36
4/36
3/36
4
5
6
8
9
10
3/9
6/9
WIN
4 rolled before 7
LOSE
7 rolled before 4
4/10
6/10
WIN
5 rolled before 7
LOSE
7 rolled before 5
5/11
6/11
WIN
6 rolled before 7
LOSE
7 rolled before 6
5/11
6/11
WIN
8 rolled before 7
LOSE
7 rolled before 8
4/10
6/10
WIN
9 rolled before 7
LOSE
7 rolled before 9
3/9
6/9
WIN
10 rolled before 7
LOSE
7 rolled before 10

The probability of winning a pass line bet is

$$\frac{8}{36} + \frac{3}{36}\cdot\frac{3}{9} + \frac{4}{36}\cdot\frac{4}{10} + \frac{5}{36}\cdot\frac{5}{11}$$

$$+ \frac{5}{36}\cdot\frac{5}{11} + \frac{4}{36}\cdot\frac{4}{10} + \frac{3}{36}\cdot\frac{3}{9}$$

$$= \boxed{\frac{244}{495}} \approx \boxed{0.4929}$$

The probability of losing a pass line bet is

$$1 - \frac{244}{495} = \boxed{\frac{251}{495}} \approx \boxed{0.5071}$$

Expected value

(Pass line bet is paid 1:1)

Suppose you bet \$1 on the pass line. Let X be the amount won or lost.

$$E[X] = \underbrace{(\$1)\left(\frac{244}{495}\right)}_{\text{WIN}} + \underbrace{(-\$1)\left(\frac{251}{495}\right)}_{\text{LOSE}}$$

$$= \boxed{-\$\frac{7}{495}} \approx \boxed{-\$0.01414\ldots}$$

The 1:1 payout is less than the true odds, ie the odds against winning.

$$\text{True odds} = \frac{p(\text{losing})}{p(\text{winning})}$$

$$= \frac{251/495}{244/495} = \frac{251}{244}.$$

If the casino paid you $\frac{251}{244}:1$ then the expected value would be

$$\underbrace{\left(\$\frac{251}{244}\right)\left(\frac{244}{495}\right)}_{\text{WIN}} + \underbrace{(-\$1)\left(\frac{251}{495}\right)}_{\text{LOSE}} = \$0$$

But then its break even for the casino in the long run.

However, the casino does allow an extra "free odds" bet if a point is made. The free odds bets are paid off at their true odds making them a "fair bet"

[fair bet means expected value 0, ie casino has no edge]

point	true odds
4	2:1
5	3:2
6	6:5
8	6:5
9	3:2
10	2:1

point is 4

$p(\text{win}) = 3/9$

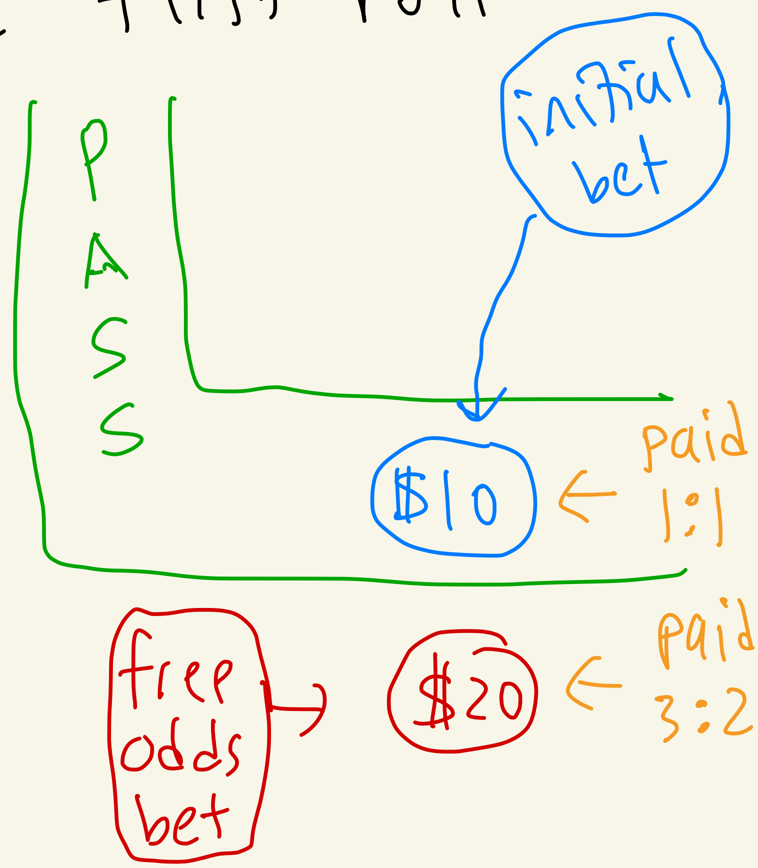
$p(\text{lose}) = 6/9$

$$\text{odds against} = \text{true odds} = \frac{p(\text{lose})}{p(\text{win})}$$

$$= \frac{6/9}{3/9} = \frac{6}{3} = \frac{2}{1}$$

Ex: Suppose you bet \$10 on the pass line. The first roll gives point 5.

Now that the point is made you can make a "free odds" bet. Let's say we bet \$20 more as a free odds bet



come out roll	roll 2	roll 3	roll 4
5	3	4	5

WIN

We would win

$$\underbrace{(\$10)}_{1:1} + \underbrace{\left(\frac{3}{2}\right)(\$20)}_{\text{free odds}} = \boxed{\$40}$$

If we would have lost we would have lost $\boxed{\$30}$