Test 1 - Mon. Oct 3 / Study guide on website

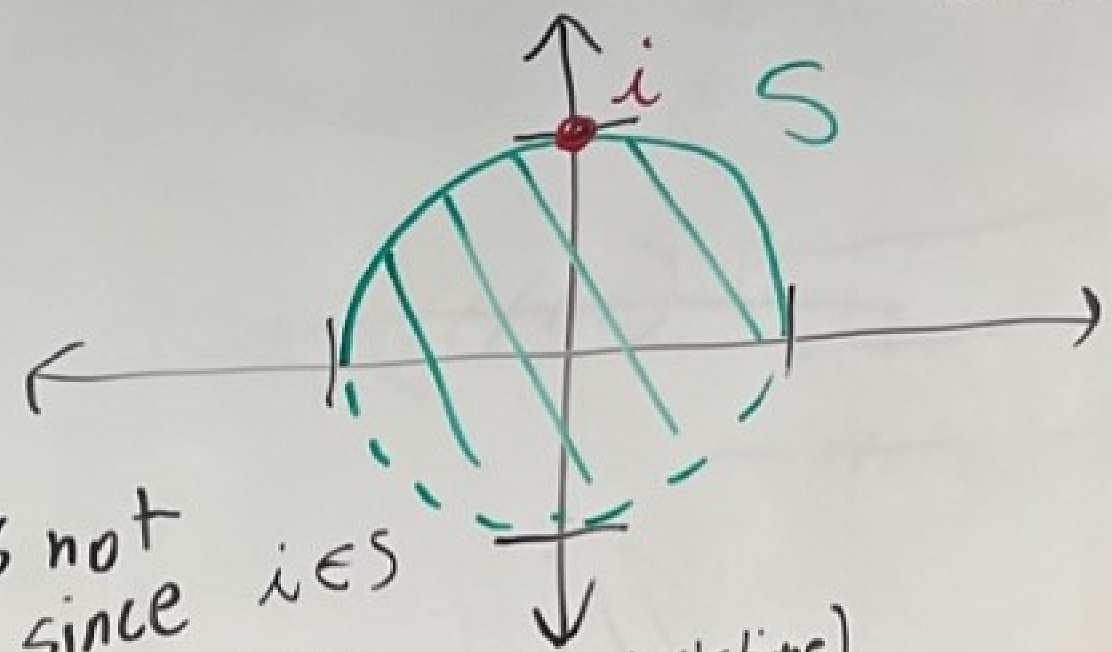
Theorem: Let $A, B \subseteq \mathbb{C}$.

Then:

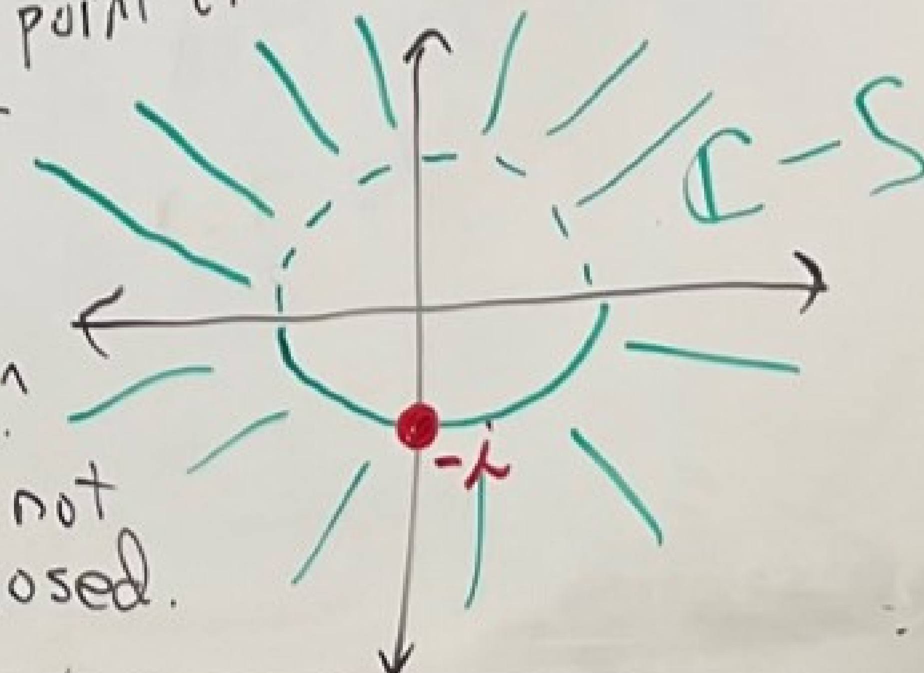
1. $\emptyset$ is open and closed.
2. $\mathbb{C}$ is open and closed.
3. If A and B are both open then $A \cup B$ and $A \cap B$ are both open.
4. If A and B are both closed then $A \cup B$ and $A \cap B$ are both closed.

Proof: HW 3 ☒

Ex: A set S that is not open and not closed.



S is not open since $i \in S$ but i is not an interior point (prove like last time)



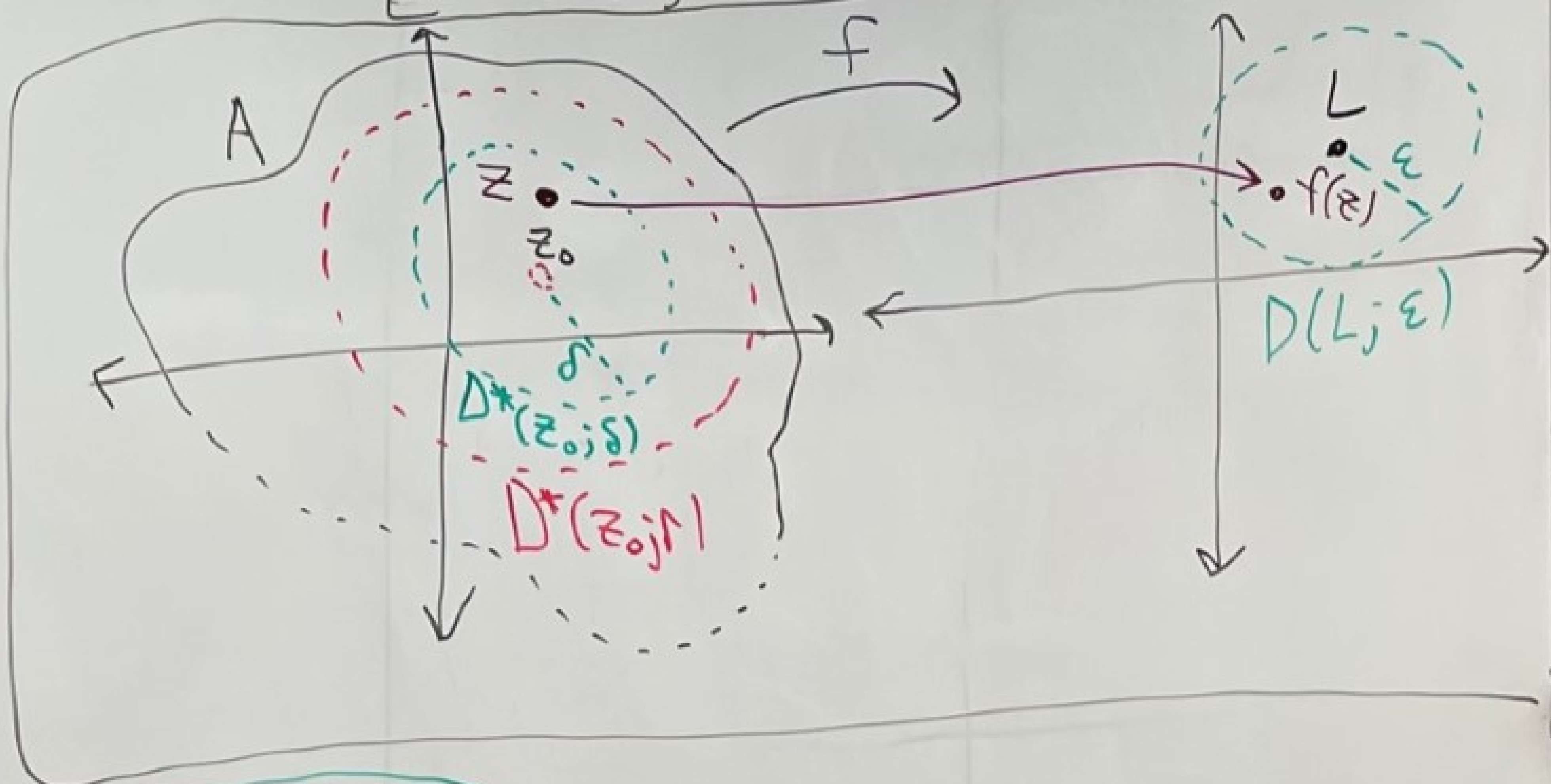
$\mathbb{C} - S$ is not open since $-i$ is in $\mathbb{C} - S$ but not an interior pt.

So, S is not closed.

HW 4 - LIMITS

Def: Let $A \subseteq \mathbb{C}$ and $f: A \to \mathbb{C}$. Let $z_0 \in \mathbb{C}$ where $D^*(z_0; r) \subseteq A$ for some $r > 0$ [So, f is defined on some deleted neighborhood of z_0]

We say that <u>f has limit L as z approaches z_0</u> and write $\lim_{z \to z_0} f(z) = L$ if for every $\varepsilon > 0$ there exists $\delta > 0$ where if $z \in A$ and $\underbrace{0 < |z - z_0| < \delta}_{\text{z is within δ of z_0 but $z \neq z_0$}}$ then $|f(z) - L| < \varepsilon$.



Theorem: If $\lim_{z \to z_0} f(z) = L_1$ and $\lim_{z \to z_0} f(z) = L_2$, then $L_1 = L_2$

proof: HW 4

Note: Let $z = x + iy \in \mathbb{C}$. Can think of z as $(x,y) \in \mathbb{R}^2$.
You can move back and forth between $\mathbb{C}$ and $\mathbb{R}^2$

Then if $f: A \to \mathbb{C}$, where $A \subseteq \mathbb{C}$.

Can write $f(z) = f(x+iy) = f(x,y) = u(x,y) + i\,v(x,y)$ [or $u(x+iy) + i\,v(x+iy)$] where $u, v : A \to \mathbb{R}$ where we think of $A \subseteq \mathbb{R}^2$

Ex: $f(z) = z^2$

$$f(x,y) = f(x+iy) = (x+iy)^2 = (x+iy)(x+iy) = x^2 + ixy + ixy + i^2y^2$$

$$= \underbrace{(x^2 - y^2)}_{u(x,y) = x^2 - y^2} + i\underbrace{(2xy)}_{v(x,y) = 2xy} = u(x,y) + i\,v(x,y)$$

Calc III limits

Let $g : A \to \mathbb{R}$ where $A \subseteq \mathbb{R}^2$.

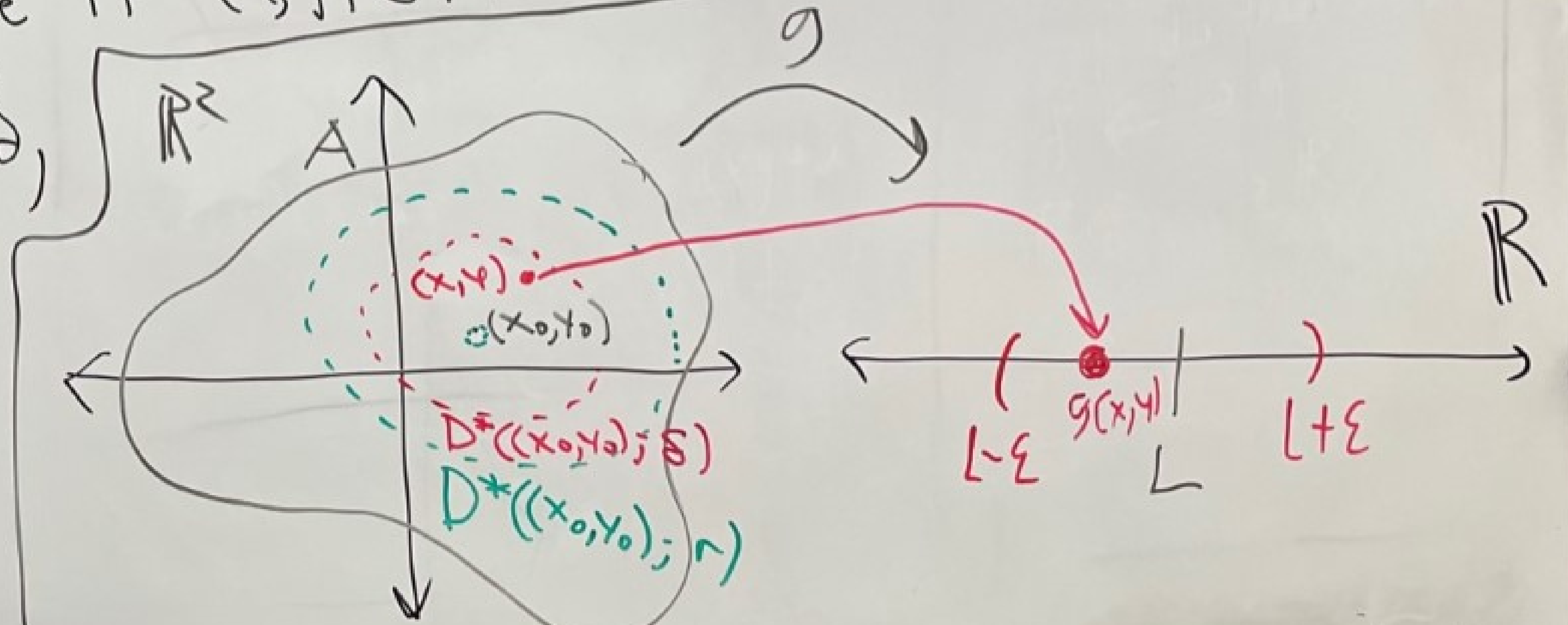
Let $(x_0, y_0) \in \mathbb{R}^2$ where $\underbrace{D^*((x_0,y_0); r) \subseteq A}_{\text{g is defined on some deleted neighborhood of } (x_0,y_0)}$ for some $r > 0$.

We say that $\displaystyle\lim_{(x,y)\to(x_0,y_0)} g(x,y) = L$ if for every $\varepsilon > 0$

there exists $\delta > 0$ where if $(x,y) \in A$

and $0 < \underbrace{\sqrt{(x-x_0)^2 + (y-y_0)^2}}_{\substack{(x,y) \text{ is } \delta \text{ close} \\ \text{to } (x_0,y_0) \text{ but} \\ \text{not equal}}} < \delta$,

then $|g(x,y) - L| < \varepsilon$



Theorem: Suppose $f: A \to \mathbb{C}$ and $z_0 \in \mathbb{C}$ and $D^*(z_0; r) \subseteq A$ for some $r > 0$

Suppose $f(z) = f(x+iy) = u(x,y) + iv(x,y)$.

Let $z_0 = x_0 + iy_0$ and $L = u_0 + iv_0$

Then:

(1) $\lim_{z \to z_0} f(z) = L$ ← $\mathbb{C}$ limit

iff

(2) $\lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$ and $\lim_{(x,y)\to(x_0,y_0)} v(x,y) = v_0$

Calc III limits

Proof: On website under notes for today.

Ex: $f(z) = z^2$

$$\lim_{z\to 1+i} f(z) = \lim_{x+iy \to 1+i} \Big[\underbrace{(x^2-y^2)}_{u} + i\underbrace{(2xy)}_{v}\Big]$$

$$\overset{\text{Thm}}{=} \lim_{(x,y)\to(1,1)} (x^2-y^2) + i \lim_{(x,y)\to(1,1)} (2xy)$$

$$\underset{\substack{\text{Calc limits}\\\text{continuity}}}{=} (1^2-1^2) + i(2(1)(1))$$

$$= 0 + 2i$$

$$= 2i$$

Theorem: Suppose $A \subseteq \mathbb{C}$ and $z_0 \in \mathbb{C}$ with $D^*(z_0; r) \subseteq A$ for some $r > 0$

Suppose $f: A \to \mathbb{C}$ and $g: A \to \mathbb{C}$.

Suppose $\lim_{z \to z_0} f(z) = F$ and $\lim_{z \to z_0} g(z) = G$.

Then:

1. $\lim_{z \to z_0} [f(z) + g(z)] = F + G$
2. $\lim_{z \to z_0} [\alpha f(z)] = \alpha F$ where $\alpha \in \mathbb{C}$
3. $\lim_{z \to z_0} [f(z) g(z)] = F \cdot G$
4. $\lim_{z \to z_0} \frac{f(z)}{g(z)} = \frac{F}{G}$ if $G \neq 0$