Math 4300 Homework 1 Solutions

(1) (a)
$$P = (-1,2), Q = (3,2)$$

They don't lie on a vertical line, so
plug them into $y = mx+b$ to get:
 $Z = m(-1) + b$
 $Z = m(3) + b$
or
 $-m+b=2$ (2)
 $3m+b=2$ (2)
(1) - (2) gives - 4m = 0 or m = 0.
Thus, $b = 2$.
Thus, P, Q lie on $L_{m,b} = L_{0,2}$.
 $y=2$ Q
 $y=2$ Q
 $L_{0,2}$



(2) (a)
$$P = (1,2), Q = (3,4)$$

P,Q don't lie on a vertical line so they
must lie on some Lr .
Plugging them into $(x-c)^2 + y^2 = r^2$ gives:
 $(1-c)^2 + 2^2 = r^2$
 $(3-c)^2 + 4^2 = r^2$

which becomes

$$C^{2} - 2C + 5 = r^{2}$$

$$C^{2} - 6C + 25 = r^{2}$$
(1)

$$D-Q$$
 gives $4C-20=0$.
So, $c=5$.
Then, D gives $r^2 = 5^2 - 2(5) + 5 = 20$.
So, $r = \sqrt{20} \approx 4.47$.
Thus, $P = (1,2)$ and $Q = (3,4)$ lie on $5^{-}\sqrt{20}$.
Picture 4



(2)(c)
$$P = (2,1), Q = (4,3)$$

These points don't lie on a vertical line.
These points don't lie on a vertical line.
So, they must lie on a ler for some c,r .
So, they must lie on a ler for some c,r .
Plug P, Q into $(x-c)^2 + y^2 = r^2$ to get;

$$(2-c)^{2}+l^{2}=r^{2}$$

 $(4-c)^{2}+3^{2}=r^{2}$

This gives

$$c^{2}-4c+5=r^{2}$$

$$c^{2}-8c+2S=r^{2}$$

(1) - (2) gives
$$4c - 20 = 0$$
.
So, $c = 5$.
(1) then gives $r^2 = 5^2 - 4(5) + 5 = 10$.
So, $r = \sqrt{10} \approx 3.16$



this would imply that
$$l = L_3$$
.
But $R = (1, -1)$ does not satisfy $x = 3$.
So, there is no unique line that P, Q, R
all lie on.
Thus, P, Q, R are noncollinear.

(3)(b) Let
$$P = (2, 1)$$
, $Q = (4, 3)$, $R = (6, 5)$.
Are these points collinear?
Are these points collinear?
Suppose they all lie on some line Q .
Suppose they all lie on some line Q .
Suppose they all lie on some line Q .
 Q can't be vertical since P, Q, R have
 Q can't be vertical since P, Q, R have
 Q can't be of the form $Y = mx + b$.
So Q must be of the form $Y = mx + b$.
So Q must be of the form $Y = mx + b$.
 $Q = (4,3)$ into
 $P = (2,11)$ and $Q = (4,3)$ into
 $Q = mx + b$ gives
 $Q = mx + b$ gives
 $Q = m(4) + b$
 $Q = (4,3)$ into
 $Q = (4,3)$ into

Thus,
$$m=1$$
.
And by $\bigcirc b = 1-2m = 1-2=-1$.
So, P and $\bigcirc both$ lie on $\lim_{m,b} = \lim_{m,b} -1$.
Since the line through P
and $\bigcirc is$ vnique we
have $l = \lim_{m \to -1} \cdot$
Does $R = (6,5)$ lie on this line also?
We have $\underbrace{5=6-1}_{\substack{y=5}}$.
We have $\underbrace{5=6-1}_{\substack{y=5}}$.
So, R also lies on $\lim_{m \to -1} \cdot$
Thus, P, Q, R are collinear.

3(c)
$$P = (0,1), Q = (0,3), R = (0,-S), S = (0,10)$$

one collinear since they all P
lie on $X = D$, ie Lo.

$$\begin{array}{c} (4) (a) \\ (3) \\ (4) \\ (4) \\ (4) \\ (5) \\ ($$

(4) (b)
$$P = (0,1)$$
, $Q = (1,2)$, $R = (4,1)$
Are these points collinear?
Suppose they are.
Then they lie on a variage line l.
Then they have different x coundinates the
Since they have different x coundinates the
line is not vertical, and is of
the form $l = cLr = (x-c)^2 + y^2 = r^2$
the form Q into $(x-c)^2 + y^2 = r^2$ and solving
Plugging P and Q into $(x-c)^2 + y^2 = r^2$ and solving
 $(0-c)^2 + l^2 = r^2$
 $(1-c)^2 + 2^2 = r^2$

$$c^{2} + 1 = r^{2}$$

$$c^{2} + 1 = 2^{2} + 1 = 5$$

$$r^{2} = c^{2} + 1 = 2^{2} + 1 = 5$$

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$$r^{2} = c^{2} + 1 = 2^{2} + 1 = 5$$

$$r^{2} = c^{2} + 1 =$$

(+)(c)
$$A = (1,1), B = (3,1), C = (2,3)$$

Are these points collinear?
Suppose they are.
Then they all lie on some unique line 1.
Then they have different x-coordinates
Since they have different x-coordinates
Some C,Γ .
Plugging $A = (1,1), B = (3,1)$ into $(x-c)+y=r^2$
 $(1-c)^2+1^2 = r^2$
 $(3-c)^2+1^2 = r^2$
 $(3-c)^2+1^2 = r^2$
 $(3-c)^2+1^2 = r^2$
 $(2-6c+10 = r^2)$
 $(3-c)^2 = 2^2 - 2(c) + 2 = 2$.
Then, (1) gives $r^2 = 2^2 - 2(c) + 2 = 2$.

Thus,
$$A = (1,1), B = (3,1)$$
 lie on $L_r = 2L_2$.
Does $(= (2,3) \text{ also}$ lie
on this line?
Plugging $X = 2, Y = 3$ in we get $(x-2)^2 + y^2 = 4$
 $(x-2)^2 + y^2 = (2-2)^2 + 3^2 = 9 \neq 4$
So, C does not lie on $2L_2$.
Thus, there is no
Unique line that
passes through
 A_1B_1C and
these points
are honcollinear. $(-1)^{1/2} + 4^{1/2} = -2$

(a)
$$L_1 = L_1$$
 so they are parallel
(b) $L_{-3} \neq L_1$ and $L_{-3} \cap L_1 = \phi$.
So, L_{-3} and L_1
are parallel.
(L-3) L_1
(L-3) L_1
are parallel.
(L-3) L_1
(L-3) L_1

$$(5)(d) L_{-1/2} \neq L_{1/1}$$

$$D_{0} \text{ they intersect}.$$

$$Take y = -x+2 \text{ and}$$

$$plug \text{ it into } y = x+1$$

$$to get -x+2 = x+1$$

$$Then, x = \frac{1}{3}.$$

$$Plug \text{ this into } y = -x+2$$

$$to get y = -\frac{1}{3}+2$$

$$= \frac{5}{3}.$$

$$Thus, L_{-1/2} \cap L_{1/1} = \frac{2(\frac{1}{3}, \frac{5}{3})}{2} \neq \phi$$
and so the lines are not parallel.
(1/2), 1/2 (1/2),

$$\begin{aligned} & (a) \quad oL_{1} \neq sL_{2} \\ & box{ have two equations:} \\ & (x-o)^{2}+y^{2}=1 \\ & (x-5)^{2}+y^{2}=2 \\ & (x-5)^{2}+y^{2}+y^{2}=2 \\ & (x-5)^{2}+y^{2}+y^{2}=2 \\ & (x-5)^{2}+y^{$$

$$\begin{array}{c} \hline (6)(b) \\ \circ L_{1} \neq 2L_{2} \\ D_{0} \text{ they intusect?} \\ We have \\ \hline (x-0)^{2} + y^{2} = 1^{2} \\ (x-2)^{2} + y^{2} = 2^{2} \\ \hline (x-2)^{2} + y^{2} = 4 \\ \hline (x-2)^{2} \\ \hline (x-2)^{2} + y^{2} = 4 \\ \hline (x-2)^{2} \\ \hline (x-2)^{2} \\ \hline (x-2)^{2} + y^{2} = 4 \\ \hline (x-2)^{2} \\ \hline (x-2)^{2} + y^{2} \\ \hline (x-2)^{2} \\ \hline (x-2)^{2} + y^{2} \\ \hline (x-2)^{2} \\ \hline (x-2)^{2} + y^{2} \\ \hline (x-2)^{2} \\ \hline (x-2)^{2}$$

$$\begin{split} & (G)(C) \circ L_{10} \neq sL_{2} \\ & D_{0} \quad \text{they intersect?} \\ & We have the equations \\ & (X-0)^{2}+y^{2}=10^{2} \\ & (X-5)^{2}+y^{2}=2^{2} \\ & C \\ & (X-5)^{2}+y^{2}=100 \\ & (X-5)^{2}+y^{2}=2^{2} \\ & (X-5)^{2}+y^{2}=100 \\ & (X-5)^{2}+y^{2}=100$$



$$\begin{split} \widehat{(e)} & (L_{10} \neq -sL) \\ D_{0} & \text{they intersect ?} \\ We have these equations: \\ & (x-1)^{2} + y^{2} = 10^{2} \\ & x = -5 \\ \hline (x-1)^{2} + y^{2} = 10^{2} \\ & x = -5 \\ \hline (x-1)^{2} + y^{2} = 10^{2} \\ & z = -s \\ \hline (-s-1)^{2} + y^{2} = 100 \\ & y^{2} = 64 \\ & y = \sqrt{64} = 8 \\ \hline \\ Thus, \quad |L_{10} \cap -sL = \frac{5}{2}(-5,8)^{2} \neq \% \\ S_{0}, \quad |L_{10} \quad and \quad -sL \quad are not puraledl. \\ \end{split}$$

$$\widehat{\mathbb{G}}(\widehat{f}) : L_{1} \neq 2L_{2}$$
Do they intered?
We have

$$[(x-1)^{2} + y^{2} = 1^{2} \quad (z + 1) \quad (x-2)^{2} + y^{2} = 2^{2} \quad (z + 2) \quad (z + 2)^{2} \quad (x-2)^{2} + y^{2} = 2^{2} \quad (z + 2)^{2} \quad (z$$

(7) Let (P,X) be an incidence geometry. Let P,Q,R be distinct points from P that are collinear. Then there exists a line I from Z where P, Q, and R all lie on l. We must show l is unique. Since (P, 2) is an incidence geometry there is a vnique line through any two distinct points. That is the unly line through P and Q is PQ. Thus, l=pà and it is unique. $\langle \Delta \rangle$

that P=A, B, and C are
collinear.
Thus, there must exist a line 1
that P does not lie on.
Case 2: Suppose P#A, P#B, and P#C.
If P does not lie on any of
AB, AC, or BC then we
are done.
Thus the only case we
have left th consider
is if P lies on all
three of AB, AC, and BC.
If so, then A and P lie on AB and AC.
Since there is a unique line through any two
points this gives
$$\overrightarrow{AB} = \overrightarrow{AC}$$
.
Also, we would then have P and B lie on AB and BC
Since there is a unique line through any two
points this gives $\overrightarrow{AB} = \overrightarrow{BC}$.

But then
$$\overrightarrow{AB} = \overrightarrow{AC} = \overrightarrow{BC}$$
.
But then $\overrightarrow{A},\overrightarrow{B},\overrightarrow{C}$ would be collinear.
Contradiction.
Thus in care 2 we also have that there
must exist a line that P does
hot lie on.

Similarly since A,P,C are collinean p
We have that
$$\overrightarrow{AP} = \overrightarrow{AC} = \overrightarrow{PC} \overrightarrow{O} \overrightarrow{A}$$

(i) (a)
(i) (a)
(a) Is there
a line
$$l = L_a$$
 through
(b) 1) that is paralell
to L6? The only line
La through (o) 1) is Lo.
In this case,
Lo $\Pi L_6 = \{(0, y) \mid y \in IR\} \cap \{(6, y) \mid y \in IR\}$
 $= \phi.$
So, Lo is paralell to La
Case (ii) Suppose $l = L_{m, L}$ goes through (0,1).
Then, $l = L_{m, 1} = \{(x, y) \mid y = mx + 1\}$.
Can l be paralell to L_6 ?
No.
This is because the
point (6, 6m+1) $\in L_{m, 1} \cap L_6$.

So,
$$L_{m,1}$$
, $\Lambda L_6 \neq \phi$ and the two lines
are not paralell.

II (b)
Let
$$l$$
 be a line and P a point not on l .
We must find a vnique line m such
When $P \in m$ and m is paralell to l .
That $P \in m$ and m is paralell to l .
 $\frac{case 1:}{Suppose} l = L_a$ and $P = (e, f)$
where $P \notin L_a$.
Since $P \notin L_a$.
 $P = (e, f)$
 L_e
 L_e
 L_a



Case 2: Suppose
$$l = L_{m,b}$$
 and $P=(e,f)$
where $P \notin L_{m,b}$.
Since $P \notin L_{m,b}$ we know $f \neq me+b$.
Consider the line
 $L_{m,b'} = \{(x,y) | y=mx+b'\}$
where $b'=f-me$.
Then, $f=me+b'$ and so
 $P = (e,f) \in L_{m,b'}$.
Since $b \neq f-me$ and $b'=f-me$ we have
that $b \neq b'$.
Thus, $L_{m,b} \cap L_{m,b'} = \oint$ because if
 $(x,y) \in L_{m,b} \cap L_{m,b'}$
Se, $L_{m,b}$ and $L_{m,b'}$ are puralell.

Thus,
$$P \in L_{m,b'}$$
 we $L_{m,b'}$ is paralell to $L_{m,b}$.
Can there be any other such lines?
Any vertical line L_a must intersect $L_{m,b}$
because $(a, ma+b) \in L_a \cap L_{m,b}$.
What about a non-vertical line?
Suppose $P \in L_{n,q} \neq L_{m,b'}$.
 P_{oints} that
 P_{oints} then P_{oints} that
 P_{oints}

We then have
$$(\hat{x}, \hat{y}) = (\frac{b-q}{n-m}, n(\frac{b-q}{n-m}) + q)$$

lies on both $L_{n,q}$ and $L_{m,b'}$.
This shows $L_{n,q}$ and $L_{m,b'}$ are not purallel.
Why does (\hat{x}, \hat{y}) lie on both lines?
Plug it in!
We have
 $\hat{y} - (n\hat{x}+q) = n(\frac{b-q}{n-m}) + q - (n(\frac{b-q}{n-m}) + q) = 0$
So, $\hat{y} = n\hat{x} + q$.
So, $(\hat{x}, \hat{y}) \in L_{n,q}$.

$$\begin{aligned} A|\varsigma_{0}, \\ \widehat{\gamma} - (m\widehat{x}+b) &= n(\frac{b-q}{n-m})+q - (m(\frac{b-q}{n-m})+b) \\ &= \frac{n(b-q)-m(b-q)}{n-m} + q - b \\ &= \frac{(n-m)(b-q)}{n-m} + (q-b) \\ &= (b-q)+(q-b) = 0 \end{aligned}$$

 $S_{o}, (\hat{x}, \hat{y}) \in L_{m,b}$ Thus, Ln, I Lm, + & and they men't panallel. Therefore, the only line through P that is paralell to Lm, b is Lm, b'.

We need
$$c < 6$$
 and $c + r \leq 6$
so that $c r \cap_6 L = \phi$
For example, pick any c with
 $0 < c < 1$.
Then for such c , set $r = \sqrt{c^2 + 1^2}$
Then, $c^2 + 1^2 = r^2$ (So, $P = (o,1) \in c - r$)
and $0 < c < 1 < 6$ (Co)
and $c + r = c + \sqrt{1 + c^2}$
 $< 1 + \sqrt{1 + 1^2}$
 $= 1 + \sqrt{3}$
 $\approx 2.73 \leq 6$
So, if $0 < c < 1$ and $r = \sqrt{c^2 + 1}$,
then $P \in c - r$ and $r = \sqrt{c^2 + 1}$,
then $P \in c - r$ and $c - r = \phi$

12(b) Problem 12(a) shows that
the following statement is not true
by using
$$P = (0, 1)$$
 and $l = _6L$.

"Consider the hyperbolic plane
$$ff = (HI, Z_H)$$
.
Let L be a line in ZH and PEHI.
Then there exists a unique line m
Where PEM and m is purdell to L".

B
Let
$$P = (x_{i}, y_{i})$$
 and $Q = (x_{2}, y_{2})$
Where $x_{i} \neq x_{2}$.
Let $C = \frac{y_{2}^{2} - y_{1}^{2} + x_{2}^{2} - x_{1}^{2}}{Z(x_{2} - x_{1})}$
and $r = \sqrt{(x_{1} - c)^{2} + y_{1}^{2}}$
We will show that P and Q
both lie on cL_{r} .
Since $r = \sqrt{(x_{r} - c)^{2} + y_{1}^{2}} = r^{2}$ and thus
that $(x_{1} - c)^{2} + y_{1}^{2} = r^{2}$ and thus
 $P = (x_{1}, y_{1})$ lies on cL_{r} .
Expanding out $(x_{1} - c)^{2} + y_{1}^{2} = r^{2}$ which
becomes $x_{1}^{2} - 2x_{1}c + c^{2} + y_{1}^{2} = r^{2} - c^{2}$ (*)

Since
$$c = \frac{y_2^2 - y_1^2 + x_2^2 - x_1^2}{2(x_2 - x_1)}$$

We know

$$2x_2c - 2x_1c = y_2^2 - y_1^2 + x_2^2 - x_1^2$$
.
And so,
 $x_1^2 - 2x_1c + y_1^2 = x_2^2 - 2x_2c + y_2^2$ (**)
Now sub (*) into (**) to get
 $r^2 - c^2 = x_2^2 - 2x_2c + y_2^2$
This gives
 $x_2^2 - 2x_2c + c^2 + y_2^2 = r^2$
So,
 $(x_2 - c)^2 + y_2^2 = r^2$
Thus,
 $Q = (x_2, y_2)$ also lies on chr.

P and Q both lie On. We will show that in all cases we have Q=m and this will show there must be a unique line through P and Q.

Suppose there exist two lines I and m that

case 1: Suppose I and m are both
vertical lines. That is suppose

$$l = {}_{a}L$$
 and $m = {}_{b}L$.
Since $P = (x_{1},y_{1})$ and $Q = (x_{2},y_{2})$
both lie on ${}_{a}L$ $P = (x_{1},y_{2})$
we know that $Q = (x_{2},y_{2})$
 $x_{1} = a = x_{2}$.
Since P and Q both lie on ${}_{b}L$
we know that $x_{1} = b = x_{2}$.
Thus, $a = b$.
So, $l = {}_{a}L = {}_{b}L = M$.
Case 2: Suppose Q is a vertical line
and m is a non-vertical line.
Then, $l = {}_{a}L$ and $m = {}_{c}Lr$.
Remember, we are assuming that

$$P = (x_{1}, y_{1}) \text{ and } Q = (x_{2}, y_{2})$$
both lie on Q and m m.
Stace P_Q both lie on Q = L we know

$$P = (x_{1}, y_{1}) = (a, y_{1}) \qquad p = (a_{1}y_{2}) \qquad q = (a_{1}y_{2})$$
and $Q = (x_{2}, y_{2}) = (a_{1}y_{2}) \qquad Q = (a_{1}y_{2}) \qquad q = (a_{1}y_{2})$
Since P and Q both
lie un M = c - r (x - c)^{2} + y^{2} = r^{2}
$$(a - c)^{2} + y^{2}_{1} = r^{2} \qquad P \in c - r$$
and $(a - c)^{2} + y^{2}_{2} = r^{2} \qquad Q \in c - r$
Subtracting gives $y^{2}_{1} - y^{2}_{2} = 0$.
So, $(y_{1} + y_{2})(y_{1} - y_{2}) = 0$
Thus either $y_{1} + y_{2} = 0$ or $y_{1} - y_{2} = 0$

Suppose $y_1 + y_2 = 0$. Then, $y_1 = -y_2 < 0$. But $y_1 > 0$. We can't have both $y_1 < 0$ and $y_1 > 0$. Thus, we can't have $y_1 + y_2 = 0$

Suppose $y_1 - y_2 = D$ Then y= y2. But then $P = (a, y_1) = (a, y_2) = Q$. But P and Q were distinct. Thus, we can't have $y_1 - y_2 = 0$. Both $y_1 + y_2 = 0$ and $y_1 - y_2 = 0$ can't happen So we now know that case 2 where l is vertical and mis non-vertical can't happen and we are done with this case.

Case 3:
$$l = {}_{c_1} {}_{c_1}$$
 and $m = {}_{c_2} {}_{c_2}$
we both non-vertical.
Since P and Q both lie on ${}_{c_1} {}_{c_1}$,
 $(x_1-c_1)^2 + y_1^2 = {}_{c_1}^2$ (D)
 $(x_2-c_1)^2 + y_2^2 = {}_{c_1}^2$ (E)
If $x_1 = x_2$ then this leads to $y_1^2 - y_2^2 = 0$
If $x_1 = x_2$ then this leads to $y_1^2 - y_2^2 = 0$
 $x_1 = x_2$ then this leads x using the
by subtracting and using the
by subtracting and using the
 $x_1 = y_2$ and p and Q
then $get y_1 = y_2$ and p and Q
then $get y_1 = y_2$ and p and Q
then $get y_1 = y_2$ and p and Q
then $y_1 + z_2$.
The above becomes
 $x_1^2 - 2c_1x_1 + c_1^2 + y_2^2 = c_1^2$ (D)
 $x_2^2 - 2c_1x_2 + c_1^2 + y_2^2 = c_1^2$ (D)

Then
$$(i) - (2)$$
 gives
 $\chi_1^2 - \chi_2^2 - 2c_1(\chi_1 - \chi_2) + y_1^2 - y_2^2 = 0$

Then,

$$c_{1} = -\frac{y_{1}^{2} + y_{2}^{2} - \chi_{1}^{2} + \chi_{2}^{2}}{-2(\chi_{1}^{-}\chi_{2})}$$

And

$$r_1 = \int (x_1 - c_1)^2 + y_1^2$$
.
Since P and Q both lie on c_2 r_2
We can do the same thing as above to
 $we can do = \frac{2}{3}$

$$\left(\chi_{1} - c_{1} \right)^{2} + y_{1}^{2} = \Gamma_{1}^{2}$$

$$\left(\chi_{2} - c_{1} \right)^{2} + y_{2}^{2} = \Gamma_{1}^{2}$$

to get that

$$c_{2} = \frac{-y_{1}^{2} + y_{2}^{2} - \chi_{1}^{2} + \chi_{2}^{2}}{-2(\chi_{1}^{2} - \chi_{2})}$$

and

$$r_{z} = \int (\chi_{1} - c_{2})^{2} + \gamma_{1}^{2}$$

4

Thus,
$$l = l = c_1 = c_2 = m$$
.

