1. See HW 1 – problem 2(c)

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2. $\ell = L_{4,-1} \leftarrow \boxed{L_{m,b}}$

(a) $f : \ell \to \mathbb{R}$

$$f(x, 4x-1) = \sqrt{1+4^2}\, x = \sqrt{17}\, x$$

(b) $f(-1,-5) = -\sqrt{17}$

$$f(2,7) = 2\sqrt{17}$$

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3. $\ell = {}_2L_5 = \{(x,y) \in \mathbb{H} \mid (x-2)^2 + y^2 = 5^2\}$

$P = (2,5)$ and $Q = (-1,4)$

$$d_H(P,Q) = \left| \ln\left(\frac{\frac{2-2+5}{5}}{\frac{-1-2+5}{4}} \right) \right| = \left| \ln\left(\frac{1}{\frac{2}{4}} \right) \right| = \left| \ln(2) \right|$$

$$= \boxed{\ln(2)} \approx \boxed{0.6931\ldots}$$

(4) $\ell = L_{\sqrt{3},0} = \{(x,y) \mid y = \sqrt{3}\,x\}$

$\boxed{m = \sqrt{3}} \rightarrow \boxed{\sqrt{1+m^2} = \sqrt{4} = 2}$

$f : \ell \to \mathbb{R}$

$f(x,y) = 2x$

$f(P) = f(1,\sqrt{3}) = 2 \cdot 1 = 2$

$f(Q) = f(-1,-\sqrt{3}) = -2$

Want $g(P) = 0$ and $g(Q) > 0$.

Set $g(x,y) = -\left[f(x,y) - f(P)\right]$

$= -f(x,y) + 2 = -2x + 2$

So, $g(P) = g(1,\sqrt{3}) = -2(1) + 2 = 0$

$g(Q) = g(-1,-\sqrt{3}) = -2(-1) + 2 = 4 > 0.$

(5)

(A) See HW 1 - problem 8.

(B) See HW 2 - problem 8.

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(6) Suppose $\ell_1 \cap \ell_2$ contain two or more points.

Let $P, Q \in \ell_1 \cap \ell_2$ where $P \neq Q$.

Since we are in an incidence geometry there exists a unique line $\overleftrightarrow{PQ}$ through P and Q.

Since $P, Q \in \ell_1$ and $P, Q \in \overleftrightarrow{PQ}$ we must then have $\ell_1 = \overleftrightarrow{PQ}$.

Since $P, Q \in \ell_2$ and $P, Q \in \overleftrightarrow{PQ}$ we must then have $\ell_2 = \overleftrightarrow{PQ}$.

Thus $\ell_1 = \ell_2 = \overleftrightarrow{PQ}$.