2550

HW 5 - Part 1

Solutions

$\boxed{1(a)}$ $\det\begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix} = (3)(4) - (5)(-2) = 22$

Since the determinant is not 0, the matrix $\begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}$ is invertible.

$\boxed{1(b)}$ $\det\begin{pmatrix} 4 & 1 \\ 8 & 2 \end{pmatrix} = (4)(2) - (1)(8) = 0$

Since the determinant is 0, the matrix $\begin{pmatrix} 4 & 1 \\ 8 & 2 \end{pmatrix}$ is not invertible.

$\boxed{1(c)}$ $\det\begin{pmatrix} -5 & 6 \\ -7 & -2 \end{pmatrix} = (5)(-2) - (6)(-7) = 52$

Since the determinant is not 0, the matrix $\begin{pmatrix} -5 & 6 \\ -7 & -2 \end{pmatrix}$ is invertible.

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① (d)

$$\begin{vmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{vmatrix} = +(-2)\begin{vmatrix} 1 & -2 \\ 8 & 4 \end{vmatrix} - 5\begin{vmatrix} 7 & 6 \\ 8 & 4 \end{vmatrix} + 3\begin{vmatrix} 7 & 6 \\ 1 & -2 \end{vmatrix}$$

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

$$\begin{vmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{vmatrix} \quad \begin{vmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{vmatrix} \quad \begin{vmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{vmatrix}$$

$$= (-2)(4+16) - 5(28-48) + 3(-14-6)$$

$$= (-2)(20) - 5(-20) + 3(-20)$$

$$= -40 + 100 - 60 = 0$$

Since the determinant is 0,

$\begin{pmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{pmatrix}$ is not invertible.

$\boxed{1(e)}$ Let's expand on the 3rd row.

$$\det\begin{pmatrix}-2 & 1 & 4\\ 3 & 5 & -7\\ 1 & 6 & 2\end{pmatrix} \qquad \begin{pmatrix}+ & - & +\\ - & + & -\\ + & - & +\end{pmatrix}$$

$$= (1)\underbrace{\begin{vmatrix}1 & 4\\ 5 & -7\end{vmatrix}}_{\begin{vmatrix}-2 & 1 & 4\\ 3 & 5 & -7\\ 1 & 6 & 2\end{vmatrix}} - 6\underbrace{\begin{vmatrix}-2 & 4\\ 3 & -7\end{vmatrix}}_{\begin{vmatrix}-2 & 1 & 4\\ 3 & 5 & -7\\ 1 & 6 & 2\end{vmatrix}} + 2\underbrace{\begin{vmatrix}-2 & 1\\ 3 & 5\end{vmatrix}}_{\begin{vmatrix}-2 & 1 & 4\\ 3 & 5 & -7\\ 1 & 6 & 2\end{vmatrix}}$$

$$= (1)\Big[(1)(-7)-(4)(5)\Big] - 6\Big[(-2)(-7)-(4)(3)\Big]$$

$$+2\Big[(-2)(5)-(1)(3)\Big] = -65$$

Since the determinant is not zero, the matrix is invertible.

$\boxed{1(f)}$ Let's expand on the first row. since it has a zero and two 1's.

$$\det\begin{pmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix} \qquad \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

$$= (1)\cdot\underbrace{\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}}_{\begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \\ 4 & 1 & 3 \end{vmatrix}} - 0\cdot\underbrace{\begin{vmatrix} 3 & 2 \\ 4 & 3 \end{vmatrix}}_{\begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \\ 4 & 1 & 3 \end{vmatrix}} + 1\cdot\underbrace{\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix}}_{\begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \\ 4 & 1 & 3 \end{vmatrix}}$$

$$= (1)\left[(1)(3)-(2)(1)\right] - 0 + 1\cdot\left[(3)(1)-(1)(4)\right]$$

$$= 0$$

Since the determinant is 0, the matrix does not have an inverse.

$\boxed{2(a)}$ Let's expand on the middle column since it has two ~~zeros~~ zeros in it.

$$\det\begin{pmatrix}-3 & 0 & 7\\ 2 & 5 & 1\\ -1 & 0 & 5\end{pmatrix} \qquad \begin{pmatrix}+ & - & +\\ - & + & -\\ + & - & +\end{pmatrix}$$

$$= -0\underbrace{\begin{vmatrix}2 & 1\\ -1 & 5\end{vmatrix}}_{\begin{pmatrix}-3 & 0 & 7\\ 2 & 5 & 1\\ -1 & 0 & 5\end{pmatrix}} + 5\cdot\underbrace{\begin{vmatrix}-3 & 7\\ -1 & 5\end{vmatrix}}_{\begin{pmatrix}-3 & 0 & 7\\ 2 & 5 & 1\\ -1 & 0 & 5\end{pmatrix}} - 0\cdot\underbrace{\begin{vmatrix}-3 & 7\\ 2 & 1\end{vmatrix}}_{\begin{pmatrix}-3 & 0 & 7\\ 2 & 5 & 1\\ -1 & 0 & 5\end{pmatrix}}$$

$$= 0 + 5\left[(-3)(5) - (7)(-1)\right] + 0$$

$$= 5\left[-15 + 7\right] = -40$$

Since the determinant is not zero, the matrix is invertible.

$\boxed{2(b)}$ Expand on row 2 because of the zero.

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

$$\det\begin{pmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{pmatrix}$$

$$= -(1)\cdot\underbrace{\begin{vmatrix} 3 & 1 \\ -3 & 5 \end{vmatrix}}_{\begin{pmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{pmatrix}} + 0\cdot\underbrace{\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}}_{\begin{pmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{pmatrix}} - (-4)\underbrace{\begin{vmatrix} 3 & 3 \\ 1 & -3 \end{vmatrix}}_{\begin{pmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{pmatrix}}$$

$$= -1\cdot\left[(3)(5) - (1)(-3)\right] + 0 + 4\cdot\left[(3)(-3) - (3)(1)\right]$$

$$= -66$$

Since the determinant is not zero, the matrix is investible.

2(c)

Expand on column 2 because of the two zeros

$$\begin{vmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{vmatrix} \qquad \begin{pmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{pmatrix}$$

$$= +0 \underbrace{\begin{vmatrix} 2 & 2 & -2 \\ 4 & 1 & 0 \\ 2 & 10 & 2 \end{vmatrix}}_{\begin{vmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{vmatrix}} - 0 \underbrace{\begin{vmatrix} 3 & 3 & 5 \\ 4 & 1 & 0 \\ 2 & 10 & 2 \end{vmatrix}}_{\begin{vmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{vmatrix}}$$

$$+(-3) \underbrace{\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix}}_{\begin{vmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{vmatrix}} - (3) \underbrace{\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix}}_{\begin{vmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{vmatrix}}$$

$$= (-3)\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} - (3)\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix} \qquad \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

$$= (-3)\left[+3 \underbrace{\begin{vmatrix} 2 & -2 \\ 10 & 2 \end{vmatrix}}_{\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix}} - 2 \underbrace{\begin{vmatrix} 3 & 5 \\ 10 & 2 \end{vmatrix}}_{\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix}} + 2 \underbrace{\begin{vmatrix} 3 & 5 \\ 2 & -2 \end{vmatrix}}_{\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix}} \right]$$

$$-3\left[+4 \underbrace{\begin{vmatrix} 3 & 5 \\ 2 & -2 \end{vmatrix}}_{\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix}} - 1 \cdot \underbrace{\begin{vmatrix} 3 & 5 \\ 2 & -2 \end{vmatrix}}_{\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix}} + 0 \underbrace{\begin{vmatrix} 3 & 3 \\ 2 & 2 \end{vmatrix}}_{\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix}} \right]$$

$$= (-3)\left[+3 \begin{vmatrix} 2 & -2 \\ 10 & 2 \end{vmatrix} - 2 \begin{vmatrix} 3 & 5 \\ 10 & 2 \end{vmatrix} + 2 \begin{vmatrix} 3 & 5 \\ 2 & -2 \end{vmatrix} \right]$$

$$-3 \left[+4 \begin{vmatrix} 3 & 5 \\ 2 & -2 \end{vmatrix} - 1 \cdot \begin{vmatrix} 3 & 5 \\ 2 & -2 \end{vmatrix} + 0 \begin{vmatrix} 3 & 3 \\ 2 & 2 \end{vmatrix} \right]$$

$$= (-3)\left(\underbrace{(3)\overbrace{[4+20]}^{24} - 2\overbrace{[6-50]}^{-44} + 2\overbrace{[-6-10]}^{-16}}_{128} \right)$$

$$-3 \left[\underbrace{4\overbrace{[-6-10]}^{-16} - 1 \cdot \overbrace{[-6-10]}^{-16} + 0}_{-48} \right]$$

$$= -384 + 144 = -240$$

Since the determinant is not 0 the matrix is invertible.

$\boxed{2(d)}$ Expand on the first row because of all the zeros.

$$\begin{vmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{vmatrix} \qquad \begin{pmatrix} + & - & + & - & + \\ - & + & - & + & - \\ + & - & + & - & + \\ - & + & - & + & - \\ + & - & + & - & + \end{pmatrix}$$

$$= 4 \cdot \underbrace{\begin{vmatrix} 3 & 3 & -1 & 0 \\ 2 & 4 & 2 & 3 \\ 4 & 6 & 2 & 3 \\ 2 & 4 & 2 & 3 \end{vmatrix}}_{\begin{vmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{vmatrix}} - 0 \cdot \underbrace{\begin{vmatrix} 3 & 3 & -1 & 0 \\ 1 & 4 & 2 & 3 \\ 9 & 6 & 2 & 3 \\ 2 & 4 & 2 & 3 \end{vmatrix}}_{\begin{vmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{vmatrix}} + 0 \underbrace{\begin{vmatrix} 3 & 3 & -1 & 0 \\ 1 & 2 & 2 & 3 \\ 9 & 4 & 2 & 3 \\ 2 & 2 & 2 & 3 \end{vmatrix}}_{\begin{vmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{vmatrix}}$$

$$-1 \cdot \underbrace{\begin{vmatrix} 3 & 3 & 3 & 0 \\ 1 & 2 & 4 & 3 \\ 9 & 4 & 6 & 3 \\ 2 & 2 & 4 & 3 \end{vmatrix}}_{\begin{vmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{vmatrix}} + 0 \cdot \underbrace{\begin{vmatrix} 3 & 3 & 3 & -1 \\ 1 & 2 & 4 & 2 \\ 9 & 4 & 6 & 2 \\ 2 & 2 & 4 & 2 \end{vmatrix}}_{\begin{vmatrix} 4 & 0 & 0 & 1 & 0 \\ 3 & 3 & 3 & -1 & 0 \\ 1 & 2 & 4 & 2 & 3 \\ 9 & 4 & 6 & 2 & 3 \\ 2 & 2 & 4 & 2 & 3 \end{vmatrix}}$$

NOTE

Normally I wouldn't calculate the three zero terms but I added them to illustrate the process in case you ever had an example without the zeros.

$$= 4 \cdot \begin{vmatrix} 3 & 3 & -1 & 0 \\ 2 & 4 & 2 & 3 \\ 4 & 6 & 2 & 3 \\ 2 & 4 & 2 & 3 \end{vmatrix} - 1 \cdot \begin{vmatrix} 3 & 3 & 3 & 0 \\ 1 & 2 & 4 & 3 \\ 9 & 4 & 6 & 3 \\ 2 & 2 & 4 & 3 \end{vmatrix}$$

Expand both on 4th column

$$\begin{pmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{pmatrix}$$

$$= 4\left[-0 + 3 \begin{vmatrix} 3 & 3 & -1 \\ 4 & 6 & 2 \\ 2 & 4 & 2 \end{vmatrix} - 3 \begin{vmatrix} 3 & 3 & -1 \\ 2 & 4 & 2 \\ 2 & 4 & 2 \end{vmatrix} + 3 \begin{vmatrix} 3 & 3 & -1 \\ 2 & 4 & 2 \\ 4 & 6 & 2 \end{vmatrix} \right]$$

$$- 1 \cdot \left[-0 + 3 \begin{vmatrix} 3 & 3 & 3 \\ 9 & 4 & 6 \\ 2 & 2 & 4 \end{vmatrix} - 3 \begin{vmatrix} 3 & 3 & 3 \\ 1 & 2 & 4 \\ 2 & 2 & 4 \end{vmatrix} + 3 \begin{vmatrix} 3 & 3 & 3 \\ 1 & 2 & 4 \\ 9 & 4 & 6 \end{vmatrix} \right]$$

you can calculate the above if you like, it should all come out to zero

$$= 0$$

Since the determinant is zero, the matrix is invertible.