

2550

HW 3

Solutions



①(a)

This is already a 1

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array} \right) \xrightarrow{R_1 + R_2 \rightarrow R_2} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array} \right)$$

make these zeros

$$\xrightarrow{-3R_1 + R_3 \rightarrow R_3}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array} \right)$$

now make this into a 1

$$\xrightarrow{-R_2 \rightarrow R_2} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{array} \right)$$

make this a zero

$$\xrightarrow{10R_2 + R_3 \rightarrow R_3} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{array} \right)$$

make this into a 1

$$\xrightarrow{-\frac{1}{52}R_3 \rightarrow R_3} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

The reduced system is:

$$\begin{aligned}x_1 + x_2 + 2x_3 &= 8 \\x_2 - 5x_3 &= -9 \\x_3 &= 2\end{aligned}$$

leading variables
are x_1, x_2, x_3 .

No free variables.

$$\begin{aligned}x_1 &= 8 - x_2 - 2x_3 & \textcircled{1} \\x_2 &= -9 + 5x_3 & \textcircled{2} \\x_3 &= 2 & \textcircled{3}\end{aligned}$$



Solve $\textcircled{3}$:

$$x_3 = 2$$

Plug into $\textcircled{2}$:

$$\begin{aligned}x_2 &= -9 + 5x_3 \\&= -9 + 5(2) = 1\end{aligned}$$

Plug into $\textcircled{3}$:

$$\begin{aligned}x_1 &= 8 - x_2 - 2x_3 \\&= 8 - 1 - 2(2) \\&= 3\end{aligned}$$



Thus, the system has
only one solution.

$$x_1 = 3, x_2 = 1, x_3 = 2$$

①(b)

make this into a 1

$$\begin{pmatrix} 2 & 2 & 2 & | & 0 \\ -2 & 5 & 2 & | & 1 \\ 8 & 1 & 4 & | & -1 \end{pmatrix} \xrightarrow{\frac{1}{2}R_1 \rightarrow R_1} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ -2 & 5 & 2 & | & 1 \\ 8 & 1 & 4 & | & -1 \end{pmatrix}$$

make these zeros

$$\xrightarrow{2R_1 + R_2 \rightarrow R_2} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 7 & 4 & | & 1 \\ 8 & 1 & 4 & | & -1 \end{pmatrix}$$

$$\xrightarrow{-8R_1 + R_3 \rightarrow R_3} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 7 & 4 & | & 1 \\ 0 & -7 & -4 & | & -1 \end{pmatrix}$$

make this into a 1

$$\xrightarrow{\frac{1}{7}R_2 \rightarrow R_2} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 1 & 4/7 & | & 1/7 \\ 0 & -7 & -4 & | & -1 \end{pmatrix}$$

make this a zero

$$\xrightarrow{7R_2 + R_3 \rightarrow R_3} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 1 & 4/7 & | & 1/7 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

The reduced system is:

$$x_1 + x_2 + x_3 = 0$$

$$x_2 + \frac{4}{7}x_3 = \frac{1}{7}$$

$$0 = 0$$

leading variables
are x_1, x_2

Free variable
is x_3



$$x_1 = -x_2 - x_3 \quad (1)$$

$$x_2 = \frac{1}{7} - \frac{4}{7}x_3 \quad (2)$$



Set free variables:

$$x_3 = t$$

Solve (2):

$$x_2 = \frac{1}{7} - \frac{4}{7}x_3 = \frac{1}{7} - \frac{4}{7}t$$

Solve (1):

$$x_1 = -x_2 - x_3 = -\left(\frac{1}{7} - \frac{4}{7}t\right) - t = -\frac{1}{7} - \frac{3}{7}t$$



Solution:

$$x_1 = -\frac{1}{7} - \frac{3}{7}t$$

$$x_2 = \frac{1}{7} - \frac{4}{7}t$$

$$x_3 = t$$

where t
is any
real
number

see
next
page



So, there are an infinite number of solutions, one for each t .

For example, if $t = 0$ then

$$x_1 = -\frac{1}{7} - \frac{3}{7}(0) = -\frac{1}{7}$$

$$x_2 = \frac{1}{7} - \frac{4}{7}(0) = \frac{1}{7}$$

$$x_3 = 0$$

is a solution.

Or if $t = 1$, then

$$x_1 = -\frac{1}{7} - \frac{3}{7}(1) = -\frac{4}{7}$$

$$x_2 = \frac{1}{7} - \frac{4}{7}(1) = -\frac{3}{7}$$

$$x_3 = 1$$

is another solution.

And so on.

①(c) we already have a 1 here

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array} \right)$$

make these into zeros

$-2R_1 + R_2 \rightarrow R_2$
 $R_1 + R_3 \rightarrow R_3$
 $-3R_1 + R_4 \rightarrow R_4$

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array} \right)$$

make this a 1

$R_2 \leftrightarrow R_3$

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array} \right)$$

make these into zeros

$$\begin{array}{l} -3R_2 + R_3 \rightarrow R_3 \\ -3R_2 + R_4 \rightarrow R_4 \end{array} \rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

The reduced system is :

$$\begin{array}{rcl} \textcircled{x} - y + 2z - w & = & -1 \\ \textcircled{y} - 2z & = & 0 \\ 0 & = & 0 \\ 0 & = & 0 \end{array}$$

leading variables
are x, y

Free variables
are z, w



$$\begin{array}{l} x = -1 + y - 2z + w \quad \textcircled{1} \\ y = 2z \quad \textcircled{2} \end{array}$$



Set free variables: $z = t, w = s$

Solve $\textcircled{2}$: $y = 2z = 2t$

Solve $\textcircled{1}$: $x = -1 + y - 2z + w = -1 + 2t - 2t + s = -1 + s$

Solution :

$$x = -1 + s$$

$$y = 2t$$

$$z = t$$

$$w = s$$

where t, s
are any
real numbers

There are an infinite number of solutions.

For example, setting $s=0$ and $t=0$
gives $x=0, y=0, z=0, w=0$.

Setting $s=1, t=2$ gives
 $x=0, y=4, z=2, w=1$.

①(d) Need to put a 1 up here

$$\begin{pmatrix} 0 & -2 & 3 & | & 1 \\ 3 & 6 & -3 & | & -2 \\ 6 & 6 & 3 & | & 5 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 3 & 6 & -3 & | & -2 \\ 0 & -2 & 3 & | & 1 \\ 6 & 6 & 3 & | & 5 \end{pmatrix}$$

$$\xrightarrow{\frac{1}{3} R_1 \rightarrow R_1} \begin{pmatrix} 1 & 2 & -1 & | & -2/3 \\ 0 & -2 & 3 & | & 1 \\ 6 & 6 & 3 & | & 5 \end{pmatrix}$$

make those zeros

now make this a 1

$$\xrightarrow{-6R_1 + R_3 \rightarrow R_3} \begin{pmatrix} 1 & 2 & -1 & | & -2/3 \\ 0 & -2 & 3 & | & 1 \\ 0 & -6 & 9 & | & 27/3 \end{pmatrix}$$

$$\xrightarrow{-\frac{1}{2} R_2 \rightarrow R_2} \begin{pmatrix} 1 & 2 & -1 & | & -2/3 \\ 0 & 1 & -3/2 & | & -1/2 \\ 0 & -6 & 9 & | & 27/3 \end{pmatrix}$$

Now make this a zero

$$\xrightarrow{6R_2 + R_3 \rightarrow R_3} \begin{pmatrix} 1 & 2 & -1 & | & -2/3 \\ 0 & 1 & -3/2 & | & -1/2 \\ 0 & 0 & 0 & | & 6 \end{pmatrix}$$

reduced

The system is :

$$\begin{aligned} a + 2b - c &= -2/3 \\ b - \frac{3}{2}c &= -1/2 \\ 0 &= 6 \end{aligned}$$

Answer

Since we have $0 = 6$ there are no solutions the system is inconsistent

② (a)

turn into a 1

$$\begin{pmatrix} 2 & -3 & -2 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

$$\frac{1}{2}R_1 \rightarrow R_1$$

$$\begin{pmatrix} 1 & -3/2 & -1 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

now make these zeros using the 1 in row 1

$$\begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3 \end{array}$$

$$\begin{pmatrix} 1 & -3/2 & -1 \\ 0 & 4 & 3 \\ 0 & 13/2 & 4 \end{pmatrix}$$

now make this a 1

$$\frac{1}{4}R_2 \rightarrow R_2$$

$$\begin{pmatrix} 1 & -3/2 & -1 \\ 0 & 1 & 3/4 \\ 0 & 13/2 & 4 \end{pmatrix}$$

use the 1 in row 2 to make this a zero

$$-\frac{13}{2}R_2 + R_3 \rightarrow R_3$$
$$\begin{pmatrix} 1 & -3/2 & -1 \\ 0 & 1 & 3/4 \\ 0 & 0 & -7/8 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & -3/2 & -1 \\ 0 & 1 & 3/4 \\ 0 & 0 & -7/8 \end{array} \right)$$

Now turn back into a system.

$$\begin{aligned} x_1 - 3/2 x_2 &= -1 \\ x_2 &= 3/4 \\ 0 &= -7/8 \end{aligned}$$

← Since we have
 $0 = -7/8$

this system
has no
solutions

it is
inconsistent.

② (b)

make into a 1

make these zeros

$$\begin{pmatrix} 3 & 2 & -1 & -15 \\ 5 & 3 & 2 & 0 \\ 3 & 1 & 3 & 11 \\ -6 & -4 & 2 & 30 \end{pmatrix} \xrightarrow{\frac{1}{3}R_1 \rightarrow R_1} \begin{pmatrix} 1 & 2/3 & -1/3 & -5 \\ 5 & 3 & 2 & 0 \\ 3 & 1 & 3 & 11 \\ -6 & -4 & 2 & 30 \end{pmatrix}$$

$$\begin{array}{l} -5R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3 \\ 6R_1 + R_4 \rightarrow R_4 \end{array} \rightarrow \begin{pmatrix} 1 & 2/3 & -1/3 & -5 \\ 0 & -1/3 & 11/3 & 25 \\ 0 & -1 & 4 & 26 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

make this into a 1

$$\xrightarrow{-3R_2 \rightarrow R_2} \begin{pmatrix} 1 & 2/3 & -1/3 & -5 \\ 0 & 1 & -11 & -75 \\ 0 & -1 & 4 & 26 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

make these zeros

$$\xrightarrow{R_2 + R_3 \rightarrow R_3} \begin{pmatrix} 1 & 2/3 & -1/3 & -5 \\ 0 & 1 & -11 & -75 \\ 0 & 0 & -7 & -49 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

make this into a 1

$$-\frac{1}{7}R_3 \rightarrow R_3 \rightarrow \left(\begin{array}{ccc|c} 1 & 2/3 & -1/3 & -5 \\ 0 & 1 & -11 & -75 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Write out the reduced system:

$$\begin{aligned} \textcircled{X_1} + \frac{2}{3}X_2 - \frac{1}{3}X_3 &= -5 \\ \textcircled{X_2} - 11X_3 &= -75 \\ \textcircled{X_3} &= 7 \end{aligned}$$

X_1, X_2, X_3 are leading variables.
There are no free variables.



$$\begin{aligned} X_1 &= -5 - \frac{2}{3}X_2 + \frac{1}{3}X_3 & \textcircled{1} \\ X_2 &= -75 + 11X_3 & \textcircled{2} \\ X_3 &= 7 & \textcircled{3} \end{aligned}$$

③ gives $X_3 = 7$.
Now plug into ② to get
 $X_2 = -75 + 11(7)$
 $= 2$
Now plug into ① to get
 $X_1 = -5 - \frac{2}{3}(2) + \frac{1}{3}(7)$
 $= -4$

Answer: $X_1 = -4, X_2 = 2, X_3 = 7$

② (c)

put a 1 here

make these zeros

$$\begin{pmatrix} 4 & -8 & | & 12 \\ 3 & -6 & | & 9 \\ -2 & 4 & | & -6 \end{pmatrix} \xrightarrow{\frac{1}{4}R_1 \rightarrow R_1} \begin{pmatrix} 1 & -2 & | & 3 \\ 3 & -6 & | & 9 \\ -2 & 4 & | & -6 \end{pmatrix}$$

$$\begin{array}{l} -3R_1 + R_2 \rightarrow R_2 \\ 2R_1 + R_3 \rightarrow R_3 \end{array} \rightarrow \begin{pmatrix} 1 & -2 & | & 3 \\ 0 & 0 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

Write down reduced system:

$$\begin{aligned} x_1 - 2x_2 &= 3 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

x_1 is a leading variable
 x_2 is a free variable

$$x_1 = 3 + 2x_2 \quad (1)$$

Set free variables:

$$x_2 = t$$

Solve (1):

$$x_1 = 3 + 2t$$

Answer:

$$x_1 = 3 + 2t$$

$$x_2 = t$$

where t is
any real number

2(d)

make this into a 1

$$\begin{pmatrix} 0 & 10 & -4 & 1 & 1 \\ 1 & 4 & -1 & 1 & 2 \\ 3 & 2 & 1 & 2 & 5 \\ -2 & -8 & 2 & -2 & -4 \\ 1 & -6 & 3 & 0 & 1 \end{pmatrix}$$

make these into zeros

$$R_1 \leftrightarrow R_2$$

$$\begin{pmatrix} 1 & 4 & -1 & 1 & 2 \\ 0 & 10 & -4 & 1 & 1 \\ 3 & 2 & 1 & 2 & 5 \\ -2 & -8 & 2 & -2 & -4 \\ 1 & -6 & 3 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} -3R_1 + R_3 &\rightarrow R_3 \\ 2R_1 + R_4 &\rightarrow R_4 \\ -R_1 + R_5 &\rightarrow R_5 \end{aligned}$$

$$\begin{pmatrix} 1 & 4 & -1 & 1 & 2 \\ 0 & 10 & -4 & 1 & 1 \\ 0 & -10 & 4 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -10 & 4 & -1 & -1 \end{pmatrix}$$

notice that R_3 is $-R_2$ and R_5 is R_2 so an easy way to simplify is as follows

$$\begin{aligned} R_2 + R_3 &\rightarrow R_3 \\ R_2 + R_5 &\rightarrow R_5 \end{aligned}$$

$$\begin{pmatrix} 1 & 4 & -1 & 1 & 2 \\ 0 & 10 & -4 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

now make this a 1

$$\frac{1}{10} R_2 \rightarrow R_2 \rightarrow \left(\begin{array}{cccc|c} 1 & 4 & -1 & 1 & 2 \\ 0 & 1 & -\frac{4}{10} & \frac{1}{10} & \frac{1}{10} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

reduced system is:

$$\begin{aligned} x + 4y - z + w &= 2 \\ y - \frac{4}{10}z + \frac{1}{10}w &= \frac{1}{10} \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

x & y are leading variables
 z & w are free variables



$$\begin{aligned} x &= 2 - 4y + z - w \\ y &= \frac{1}{10} + \frac{4}{10}z - \frac{1}{10}w \end{aligned}$$

① ⇒

②

set free variables:

$$z = t$$

$$w = s$$

plug into ②:

$$y = \frac{1}{10} + \frac{4}{10}t - \frac{1}{10}s$$

plug into ①:

$$\begin{aligned} x &= 2 - 4\left(\frac{1}{10} + \frac{4}{10}t - \frac{1}{10}s\right) \\ &\quad + t - s \\ &= \frac{8}{5} - \frac{3}{5}t - \frac{3}{5}s \end{aligned}$$

Answer:

$$x = \frac{8}{5} - \frac{3}{5}t - \frac{3}{5}s$$

$$y = \frac{1}{10} + \frac{4}{10}t - \frac{1}{10}s$$

$$z = t$$

$$w = s$$

where t & s
can be any real #s



③ (a)

make this a 1

$$\begin{pmatrix} 5 & -2 & 6 & | & 0 \\ -2 & 1 & 3 & | & 1 \end{pmatrix} \xrightarrow{\frac{1}{5}R_1 \rightarrow R_1} \begin{pmatrix} 1 & -\frac{2}{5} & \frac{6}{5} & | & 0 \\ -2 & 1 & 3 & | & 1 \end{pmatrix}$$

make this a zero

$$2R_1 + R_2 \rightarrow R_2 \rightarrow \begin{pmatrix} 1 & -\frac{2}{5} & \frac{6}{5} & | & 0 \\ 0 & \frac{1}{5} & \frac{27}{5} & | & 1 \end{pmatrix}$$

make this a 1

$$5R_2 \rightarrow R_2 \rightarrow \begin{pmatrix} 1 & -\frac{2}{5} & \frac{6}{5} & | & 0 \\ 0 & 1 & 27 & | & 5 \end{pmatrix}$$

reduced system is:

$$\begin{aligned} x_1 - \frac{2}{5}x_2 + \frac{6}{5}x_3 &= 0 \\ x_2 + 27x_3 &= 5 \end{aligned}$$

leading variables are x_1 & x_2
free variables are x_3

$$X_1 = \frac{2}{5}X_2 - \frac{6}{5}X_3 \quad (1)$$

$$X_2 = 5 - 27X_3 \quad (2)$$

Set free variables:

$$X_3 = t$$

plug into (2):

$$X_2 = 5 - 27t$$

plug into (3):

$$X_1 = \frac{2}{5}(5 - 27t) - \frac{6}{5}(t) = 2 - 12t$$

Answer:

$$X_1 = 2 - 12t$$

$$X_2 = 5 - 27t$$

$$X_3 = t$$

where t can be any real #

③(b)

we already have a 1 here

$$\begin{pmatrix} 1 & -2 & 1 & -4 & | & 1 \\ 1 & 3 & 7 & 2 & | & 2 \\ 1 & -12 & -11 & -16 & | & 5 \end{pmatrix}$$

make these into zeros

$$\begin{aligned} -R_1 + R_2 &\rightarrow R_2 \\ -R_1 + R_3 &\rightarrow R_3 \end{aligned}$$

$$\begin{pmatrix} 1 & -2 & 1 & -4 & | & 1 \\ 0 & 5 & 6 & 6 & | & 1 \\ 0 & -10 & -12 & -12 & | & 4 \end{pmatrix}$$

make this a 1

$$\frac{1}{5}R_2 \rightarrow R_2$$

$$\begin{pmatrix} 1 & -2 & 1 & -4 & | & 1 \\ 0 & 1 & 6/5 & 6/5 & | & 1/5 \\ 0 & -10 & -12 & -12 & | & 4 \end{pmatrix}$$

make this a zero

$$10R_2 + R_3 \rightarrow R_3$$

$$\begin{pmatrix} 1 & -2 & 1 & -4 & | & 1 \\ 0 & 1 & 6/5 & 6/5 & | & 1/5 \\ 0 & 0 & 0 & 0 & | & 6 \end{pmatrix}$$

Write down the reduced system:

$$x_1 - 2x_2 + x_3 - 4x_4 = 1$$

$$x_2 + \frac{6}{5}x_3 + \frac{6}{5}x_3 = \frac{1}{5}$$

$$0 = 6$$

Because we have $0 = 6$
there are no solutions to the
system. It is inconsistent.

(4) (a)

put a 1
here

$$\begin{pmatrix} 2 & 1 & 3 & | & 0 \\ 1 & 2 & 0 & | & 0 \\ 0 & 1 & 1 & | & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 2 & 0 & | & 0 \\ 2 & 1 & 3 & | & 0 \\ 0 & 1 & 1 & | & 0 \end{pmatrix}$$

put
zeros here

$$\xrightarrow{-2R_1 + R_2 \rightarrow R_2} \begin{pmatrix} 1 & 2 & 0 & | & 0 \\ 0 & -1 & 3 & | & 0 \\ 0 & 1 & 1 & | & 0 \end{pmatrix}$$

make
this a 1

$$\xrightarrow{-R_2 \rightarrow R_2} \begin{pmatrix} 1 & 2 & 0 & | & 0 \\ 0 & 1 & -3 & | & 0 \\ 0 & 1 & 1 & | & 0 \end{pmatrix}$$

make
this a
0

$$\xrightarrow{-R_2 + R_3 \rightarrow R_3} \begin{pmatrix} 1 & 2 & 0 & | & 0 \\ 0 & 1 & -3 & | & 0 \\ 0 & 0 & 4 & | & 0 \end{pmatrix}$$

make this
a 1

$$\frac{1}{4}R_3 \rightarrow R_3 \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right)$$

Write down the reduced system:

$$\begin{aligned} x_1 + 2x_2 &= 0 \\ x_2 - 3x_3 &= 0 \\ x_3 &= 0 \end{aligned}$$

leading variables are x_1, x_2, x_3
there are no free variables

$$\begin{aligned} x_1 &= -2x_2 & \textcircled{1} \\ x_2 &= 3x_3 & \textcircled{2} \\ x_3 &= 0 & \textcircled{3} \end{aligned}$$

$\textcircled{3}$ gives $x_3 = 0$
plug into $\textcircled{2}$ to get
 $x_2 = 3(0) = 0$
plug into $\textcircled{1}$ to get
 $x_1 = -2(0) = 0.$

Answer:

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \\ x_3 &= 0 \end{aligned}$$

④ (6)

make this a 1

$$\begin{pmatrix} 3 & 1 & 1 & 1 & | & 0 \\ 5 & -1 & 1 & -1 & | & 0 \end{pmatrix}$$

make this a zero

$\frac{1}{3}R_1 \rightarrow R_1$

$$\begin{pmatrix} 1 & 1/3 & 1/3 & 1/3 & | & 0 \\ 5 & -1 & 1 & -1 & | & 0 \end{pmatrix}$$

$-5R_1 + R_2 \rightarrow R_2$

$$\begin{pmatrix} 1 & 1/3 & 1/3 & 1/3 & | & 0 \\ 0 & -8/3 & -2/3 & -8/3 & | & 0 \end{pmatrix}$$

make this a 1

$-\frac{3}{8}R_2 \rightarrow R_2$

$$\begin{pmatrix} 1 & 1/3 & 1/3 & 1/3 & | & 0 \\ 0 & 1 & 1/4 & 1 & | & 0 \end{pmatrix}$$

Write down the reduced system:

$$\begin{aligned} X_1 + \frac{1}{3}X_2 + \frac{1}{3}X_3 + \frac{1}{3}X_4 &= 0 \\ X_2 + \frac{1}{4}X_3 + X_4 &= 0 \end{aligned}$$

leading variables are X_1, X_2

free variables are X_3, X_4



$$X_1 = -\frac{1}{3}X_2 - \frac{1}{3}X_3 - \frac{1}{3}X_4 \quad (1)$$

$$X_2 = -\frac{1}{4}X_3 - X_4 \quad (2)$$



assign free variables:

$$X_3 = s$$

$$X_4 = t$$

plug into (2):

$$X_2 = -\frac{1}{4}s - t$$

plug into (1):

$$\begin{aligned} X_1 &= -\frac{1}{3}\left(-\frac{1}{4}s - t\right) - \frac{1}{3}s - \frac{1}{3}t \\ &= -\frac{5}{12}s \end{aligned}$$



Answer:

$$X_1 = -\frac{5}{12}s$$

$$X_2 = -\frac{1}{4}s - t$$

$$X_3 = s$$

$$X_4 = t$$

where s & t
can be any
real numbers

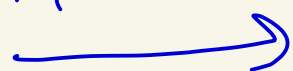
(4)(c)

put a 1 here

$$\begin{pmatrix} 0 & 2 & 2 & 4 & | & 0 \\ 1 & 0 & -1 & -3 & | & 0 \\ 2 & 3 & 1 & 1 & | & 0 \\ -2 & 1 & 3 & -2 & | & 0 \end{pmatrix}$$

make these zeros

$R_1 \leftrightarrow R_2$



$$\begin{pmatrix} 1 & 0 & -1 & -3 & | & 0 \\ 0 & 2 & 2 & 4 & | & 0 \\ 2 & 3 & 1 & 1 & | & 0 \\ -2 & 1 & 3 & -2 & | & 0 \end{pmatrix}$$

make this a 1

$-2R_1 + R_3 \rightarrow R_3$

$2R_1 + R_4 \rightarrow R_4$



$$\begin{pmatrix} 1 & 0 & -1 & -3 & | & 0 \\ 0 & 2 & 2 & 4 & | & 0 \\ 0 & 3 & 3 & 7 & | & 0 \\ 0 & 1 & 1 & -8 & | & 0 \end{pmatrix}$$

$R_2 \leftrightarrow R_4$



$$\begin{pmatrix} 1 & 0 & -1 & -3 & | & 0 \\ 0 & 1 & 1 & -8 & | & 0 \\ 0 & 3 & 3 & 7 & | & 0 \\ 0 & 2 & 2 & 4 & | & 0 \end{pmatrix}$$

make these zeros

$$\begin{aligned} -3R_2 + R_3 &\rightarrow R_3 \\ -2R_2 + R_4 &\rightarrow R_4 \end{aligned} \rightarrow$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & -8 & 0 \\ 0 & 0 & 0 & -17 & 0 \\ 0 & 0 & 0 & 20 & 0 \end{array} \right)$$

make this
a 1

$$-\frac{1}{17}R_3 \rightarrow R_3 \rightarrow$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & -8 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 20 & 0 \end{array} \right)$$

make this
a zero

$$-20R_3 + R_4 \rightarrow R_4 \rightarrow$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & -8 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

reduced system is:

$$\begin{aligned} w - y - 3z &= 0 \\ x + y - 8z &= 0 \\ z &= 0 \end{aligned}$$

leading variables: w, x, z

free variable: y

$$\begin{aligned} w &= y + 3z & (1) \\ x &= -y + 8z & (2) \\ z &= 0 & (3) \end{aligned}$$

Assign free variable:

$$y = t$$

plug into (3):

$$z = 0$$

plug into (2):

$$x = -y + 8z = -t$$

plug into (1):

$$w = y + 3z = t$$



Answer:

$$w = t$$

$$x = -t$$

$$y = t$$

$$z = 0$$

t can be
any real
number