Math 2550

9/6/23

- Test 1 moved to Weds 10/11

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This gives more time to study Topic 3 since test 1 covers Topics 1, 2, 3

<u>Def:</u> Let A and B be $m \times n$ matrices. [So they have the same size.]

Let

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}$$

Then we define

$$A+B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{pmatrix}$$

$$A-B = \begin{pmatrix} a_{11}-b_{11} & a_{12}-b_{12} & \cdots & a_{1n}-b_{1n} \\ a_{21}-b_{21} & a_{22}-b_{22} & \cdots & a_{2n}-b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1}-b_{m1} & a_{m2}-b_{m2} & \cdots & a_{mn}-b_{mn} \end{pmatrix}$$

If α is a scalar/number then

$$\alpha A = \begin{pmatrix} \alpha a_{11} & \alpha a_{12} & \cdots & \alpha a_{1n} \\ \alpha a_{21} & \alpha a_{22} & \cdots & \alpha a_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha a_{m1} & \alpha a_{m2} & \cdots & \alpha a_{mn} \end{pmatrix}$$

Ex:

$$\underbrace{\begin{pmatrix} 5 & 2 \\ 1 & -\frac{1}{2} \end{pmatrix}}_{2\times 2} + \underbrace{\begin{pmatrix} 1 & 0 \\ -3 & 0 \end{pmatrix}}_{2\times 2}$$

$$= \begin{pmatrix} 5+1 & 2+0 \\ 1-3 & -\frac{1}{2}+0 \end{pmatrix} = \underbrace{\begin{pmatrix} 6 & 2 \\ -2 & -\frac{1}{2} \end{pmatrix}}_{2\times 2}$$

Ex:

$$\underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 2 & 2 \end{pmatrix}}_{3\times 2} + \underbrace{\begin{pmatrix} 3 & 3 \\ 4 & 4 \\ 5 & 5 \end{pmatrix}}_{3\times 2} = \underbrace{\begin{pmatrix} 3 & 3 \\ 5 & 5 \\ 7 & 7 \end{pmatrix}}_{3\times 2}$$

Ex:

$$\underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}}_{2\times 2} + \underbrace{\begin{pmatrix} 5 & 3 \\ 2 & 1 \\ 0 & 7 \end{pmatrix}}_{3\times 2}$$

not the same size

Undefined since not the same size

Ex:

$$10\begin{pmatrix} 0 & 3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 10(0) & 10(3) \\ 10(1) & 10(4) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 30 \\ 10 & 40 \end{pmatrix}$$

Ex:

$$\underbrace{\begin{pmatrix} 1 \\ 5 \end{pmatrix}}_{2\times 1} - \underbrace{\begin{pmatrix} 0 \\ 3 \end{pmatrix}}_{2\times 1} = \begin{pmatrix} 1-0 \\ 5-3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\boxed{\text{Def:}}$ Let A be an $m \times r$ matrix and B be an $r \times n$ matrix. We define the <u>product</u> of A and B, denoted by AB, as the $m \times n$ matrix whose entry in row i and column j is the dot product of row i from A and column j from B.

$$\underbrace{A}_{m \times r} \quad \underbrace{B}_{r \times n}$$

must match

$m \times n$ is size of AB

Ex: Calculate AB, if possible, where

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\underbrace{A}_{2\times 2}\ \underbrace{B}_{2\times 3} = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

same ✓

answer is 2×3

$$= \begin{pmatrix} \overset{\substack{(\text{row } 1\ A)\cdot \\ (\text{col } 1\ B)}}{(1\ \ 2)\cdot\begin{pmatrix}1\\0\end{pmatrix}} & \overset{\substack{(\text{row } 1\ A)\cdot \\ (\text{col } 2\ B)}}{(1\ \ 2)\cdot\begin{pmatrix}2\\1\end{pmatrix}} & \overset{\substack{(\text{row } 1\ A)\cdot \\ (\text{col } 3\ B)}}{(1\ \ 2)\cdot\begin{pmatrix}-1\\0\end{pmatrix}} \\ \underset{\substack{(\text{row } 2\ A)\cdot \\ (\text{col } 1\ B)}}{(-1\ \ 0)\cdot\begin{pmatrix}1\\0\end{pmatrix}} & \underset{\substack{(\text{row } 2\ A)\cdot \\ (\text{col } 2\ B)}}{(-1\ \ 0)\cdot\begin{pmatrix}2\\1\end{pmatrix}} & \underset{\substack{(\text{row } 2\ A)\cdot \\ (\text{col } 3\ B)}}{(-1\ \ 0)\cdot\begin{pmatrix}-1\\0\end{pmatrix}} \end{pmatrix}$$

$$= \begin{pmatrix} (1)(1)+(2)(0) & (1)(2)+(2)(1) & (1)(-1)+(2)(0) \\ (-1)(1)+(0)(0) & (-1)(2)+(0)(1) & (-1)(-1)+(0)(0) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & -1 \\ -1 & -2 & 1 \end{pmatrix}$$

<u>Ex:</u> Calculate AB if possible where

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{pmatrix}$$

pretend we tried

$$\underbrace{A}_{2\times 2}\ \underbrace{B}_{3\times 2} = \begin{pmatrix} \underbrace{\begin{pmatrix}1 & 0\end{pmatrix}\cdot\begin{pmatrix}1\\2\\3\end{pmatrix}}_{\substack{(\text{row } 1\ A)\cdot\\(\text{col } 1\ B)}} & \_\_ & \_\_ \end{pmatrix}$$

$2 \neq 3$

(not possible
undefined)

can't do it

AB not defined.

Ex: Let

$$A = \underbrace{\begin{pmatrix}1\\2\\-1\end{pmatrix}}_{3\times 1} \quad \text{and} \quad B = \underbrace{\begin{pmatrix}0 & 1 & -3\end{pmatrix}}_{1\times 3}$$

Calculate AB if possible.

$$\underbrace{A}_{3\times1}\ \underbrace{B}_{1\times3}$$

answer is 3×3

$$AB = \begin{pmatrix}
\text{(row 1 A)}\cdot\text{(col 1 B)} & \text{(row 1 A)}\cdot\text{(col 2 B)} & \text{(row 1 A)}\cdot\text{(col 3 B)} \\
(1)\cdot(0) & (1)(1) & (1)(-3) \\
\text{(row 2 A)}\cdot\text{(col 1 B)} & \text{(row 2 A)}\cdot\text{(col 2 B)} & \text{(row 2 A)}\cdot\text{(col 3 B)} \\
(2)(0) & (2)(1) & (2)(-3) \\
\text{(row 3 A)}\cdot\text{(col 1 B)} & \text{(row 3 A)}\cdot\text{(col 2 B)} & \text{(row 3 A)}\cdot\text{(col 3 B)} \\
(-1)(0) & (-1)(1) & (-1)(-3)
\end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & -3 \\ 0 & 2 & -6 \\ 0 & -1 & 3 \end{pmatrix} = AB$$

<u>Ex:</u> Let $A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & -3 \end{pmatrix}$ as before. Can we calculate

BA?

$$\underbrace{\underbrace{B}_{1\times 3}\underbrace{A}_{3\times 1}}_{1\times 1 \text{ is answer}} = \begin{pmatrix} 0 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$= \big((0)(1) + (1)(2) + (-3)(-1) \big)$$

(row 1 B)·
(col 1 A)

$$= (5)$$

Note: We saw above that $AB \neq BA$ with matrices.

Def: Let A be an $m \times n$ matrix. The **transpose** of A, denoted by A^T, is defined to be the $n \times m$ matrix that results from interchanging the rows and columns of A.

That is, the i-th column of A^T is the i-th row of A.

[Or, the j-th row of A^T is the j-th column of A].

Ex:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix}$$

3x4

$$A^T = \begin{pmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{pmatrix}$$

4x3

You could have instead turned the columns of A into the rows of A^T

$\boxed{\text{Def:}}$ The $m \times n$ <u>zero matrix</u> is the $m \times n$ matrix whose entries are all zeros. We denote it by $0_{m\times n}$ or just by 0 if you don't want to mention the size.

<u>Ex:</u> $0_{2\times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$0_{3\times 1} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$0_{5\times 3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\boxed{\text{Ex:}}$ Let $A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$

Then,

$$A + O_{2\times 2} = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} = A$$

Similarly, $O_{2\times 2} + A = A$.

__Def:__ The $n \times n$ __identity matrix__, denoted by I_n, or just I, is the $n \times n$ matrix with 1's along the main diagonal and 0's everywhere else.

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$