Math 2550

9/11/23

I will use your calstatela email to email the class announcements.

Ex: Let $A = \begin{pmatrix} 5 & 3 \\ 8 & -4 \end{pmatrix}$ ← $\boxed{2\times 2}$

$$\underbrace{A}_{2\times 2}\ \underbrace{I_2}_{2\times 2} = \begin{pmatrix} 5 & 3 \\ 8 & -4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

answer is 2×2

$$= \begin{pmatrix} (5\ \ 3)\cdot\begin{pmatrix}1\\0\end{pmatrix} & (5\ \ 3)\cdot\begin{pmatrix}0\\1\end{pmatrix} \\ (8\ \ -4)\cdot\begin{pmatrix}1\\0\end{pmatrix} & (8\ \ -4)\cdot\begin{pmatrix}0\\1\end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} (5)(1)+(3)(0) & (5)(0)+(3)(1) \\ (8)(1)+(-4)(0) & (8)(0)+(-4)(1) \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 3 \\ 8 & -4 \end{pmatrix} = A$$

You can check that $I_2 A = A$.

Ex: Let $A = \underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}}_{2\times 3}$

Then,

$$\underset{2\times 2}{I_2}\ \underbrace{A}_{2\times 3} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

answer is 2×3

$$= \begin{pmatrix} (1\ 0)\cdot\begin{pmatrix}1\\4\end{pmatrix} & (1\ 0)\cdot\begin{pmatrix}2\\5\end{pmatrix} & (1\ 0)\cdot\begin{pmatrix}3\\6\end{pmatrix} \\ (0\ 1)\cdot\begin{pmatrix}1\\4\end{pmatrix} & (0\ 1)\cdot\begin{pmatrix}2\\5\end{pmatrix} & (0\ 1)\cdot\begin{pmatrix}3\\6\end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 2+0 & 3+0 \\ 0+4 & 0+5 & 0+6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = A$$

What about an I on the right?

$$\underbrace{A}_{2\times 3}\,\underbrace{I_3}_{3\times 3} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= A$$

you can check this

Theorem: Let A, B, C be matrices and α, β be

alpha beta

real numbers. Then the following are true where we will assume that the sizes of the matrices are such that the operations are defined:

① $A+B=B+A$

② $A+(B+C)=(A+B)+C$

③ $A(BC)=(AB)C$

④ $A(B+C)=AB+AC$

⑤ $(B+C)A=BA+CA$

6. $A(B-C) = AB - AC$

7. $(B-C)A = BA - CA$

8. $\alpha(B+C) = \alpha B + \alpha C$

9. $\alpha(B-C) = \alpha B - \alpha C$

10. $(\alpha+\beta)A = \alpha A + \beta A$

11. $(\alpha-\beta)A = \alpha A - \beta A$

12. $\alpha(\beta A) = (\alpha\beta)A$

13. $\alpha(AB) = (\alpha A)B = A(\alpha B)$

14. $(A^T)^T = A$

15. $(A+B)^T = A^T + B^T$

16. $(A-B)^T = A^T - B^T$

(17) $(\alpha A)^T = \alpha A^T$

(18) $(AB)^T = B^T A^T$

note the reversal of the order

(19) If A is $m \times n$, then

$$\underbrace{A}_{m \times n} \underbrace{I_n}_{n \times n} = A$$

(20) If A is $m \times n$, then

$$\underbrace{I_m}_{m \times m} \underbrace{A}_{m \times n} = A$$

(21) If A is $m \times n$, then

$$A - A = O_{m \times n}$$

22 If A is $m \times n$, then

$$A + O_{m \times n} = A$$

$$O_{m \times n} + A = A$$

END OF THEOREM ■

Let's prove 5 when A, B, C are 2×2 matrices.

<u>proof:</u> We must show that

$$(B+C)A = BA + CA$$

when A, B, C are 2×2 matrices.

Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

and $C = \begin{pmatrix} i & j \\ k & l \end{pmatrix}$

where $a, b, c, d, e, f, g, h, i, j, k, l$ are real numbers.

Then,

$$(B+C)A = \left[\begin{pmatrix} e & f \\ g & h \end{pmatrix} + \begin{pmatrix} i & j \\ k & l \end{pmatrix}\right]\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} e+i & f+j \\ g+k & h+l \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} (e+i \quad f+j)\begin{pmatrix} a \\ c \end{pmatrix} & (e+i \quad f+j)\begin{pmatrix} b \\ d \end{pmatrix} \\ (g+k \quad h+l)\begin{pmatrix} a \\ c \end{pmatrix} & (g+k \quad h+l)\begin{pmatrix} b \\ d \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} ea+ia+fc+jc & eb+ib+fd+jd \\ ga+ka+hc+lc & gb+kb+hd+ld \end{pmatrix}$$

Also we have

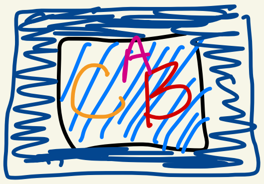
$$BA+CA = \begin{pmatrix} e & f \\ g & h \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} i & j \\ k & l \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} ea+fc & eb+fd \\ ga+hc & gb+hd \end{pmatrix}$$

$$+ \begin{pmatrix} ia+jc & ib+jd \\ ka+lc & kb+ld \end{pmatrix}$$

$$= \begin{pmatrix} ea+fc+ia+jc & eb+fd+ib+jd \\ ga+hc+ka+lc & gb+hd+kb+ld \end{pmatrix}$$

Comparing the above we see that $(B+C)A = BA+CA$



HW 1 – Part 1

(9) List 3 elements from the set

$$S = \{ c_1 \langle 1,1 \rangle + c_2 \langle 0,-1 \rangle + c_3 \langle -2,1 \rangle \mid c_1, c_2, c_3 \in \mathbb{R} \}$$

Some elements from S are:

$$\underbrace{1}_{c_1 = 1} \cdot \langle 1,1 \rangle + \underbrace{0}_{c_2 = 0} \cdot \langle 0,-1 \rangle + \underbrace{(-3)}_{c_3 = -3} \langle -2,1 \rangle$$

$$= \langle 1,1 \rangle + \langle 0,0 \rangle + \langle 6,-3 \rangle$$

$$= \boxed{\langle 7,-2 \rangle} \leftarrow \boxed{\text{in } S}$$

first element $\langle 7,-2 \rangle$

$$0 \cdot \langle 1,1 \rangle + 0 \cdot \langle 0,-1 \rangle + 0 \cdot \langle -2,1 \rangle =$$

$$= \langle 0,0 \rangle + \langle 0,0 \rangle + \langle 0,0 \rangle$$

$$= \boxed{\langle 0,0 \rangle} \leftarrow \boxed{\text{in } S}$$

second element $\langle 0,0 \rangle$

$$1\cdot\langle 1,1\rangle + 1\cdot\langle 0,-1\rangle + 1\cdot\langle -2,1\rangle$$

$$= \langle 1,1\rangle + \langle 0,-1\rangle + \langle -2,1\rangle$$

$$= \boxed{\langle -1,1\rangle} \leftarrow \boxed{\text{in } S}$$

third element $\langle -1,1\rangle$

$$8\cdot\langle 1,1\rangle + 8\cdot\langle 0,-1\rangle + 4\langle -2,1\rangle$$

$$= \langle 8,8\rangle + \langle 0,-8\rangle + \langle -8,4\rangle$$

$$= \boxed{\langle 0,4\rangle} \leftarrow \boxed{\text{in } S}$$

4th element $\langle 0,4\rangle$