Math 2550

8-28-23

Def: Let $n \geq 1$ be an integer.

[So n can be $1, 2, 3, 4, 5, 6, \ldots$]

An <u>n-dimensional real vector</u> is a list of n real numbers.

We use brackets $<$ and $>$ to denote vectors and commas to separate the numbers. We use an arrow over a variable to denote that its a vector such as $\vec{v}$.

Ex: Some 2-dimensional vectors

$$\vec{v} = \langle 5, 10 \rangle \qquad \vec{z} = \langle 10, 5 \rangle$$

$$\vec{w} = \langle \tfrac{1}{2}, e \rangle \qquad \vec{v} \neq \vec{z}$$

Ex: Some 3-dimensional vectors.

$$\vec{v} = \langle 1, 0, -1 \rangle$$

$$\vec{w} = \langle 0, 0, 0 \rangle$$

Ex: Some 10-dimensional vectors.

$\vec{v} = \langle 1, -1, \pi, 5, \frac{1}{2}, e, 2, 3, 4, 5 \rangle$

$\vec{w} = \langle 0, 0, 0, 1, 1, 1, -1, -1, -1, 10 \rangle$

$\boxed{\text{Sets } \underline{\text{vs}} \text{ vectors}}$

set $\rightarrow$ $\{5, 4\} = \{4, 5\}$ $\leftarrow$ the same

vectors $\rightarrow$ $\langle 4, 5 \rangle \neq \langle 5, 4 \rangle$ $\leftarrow$ not the same

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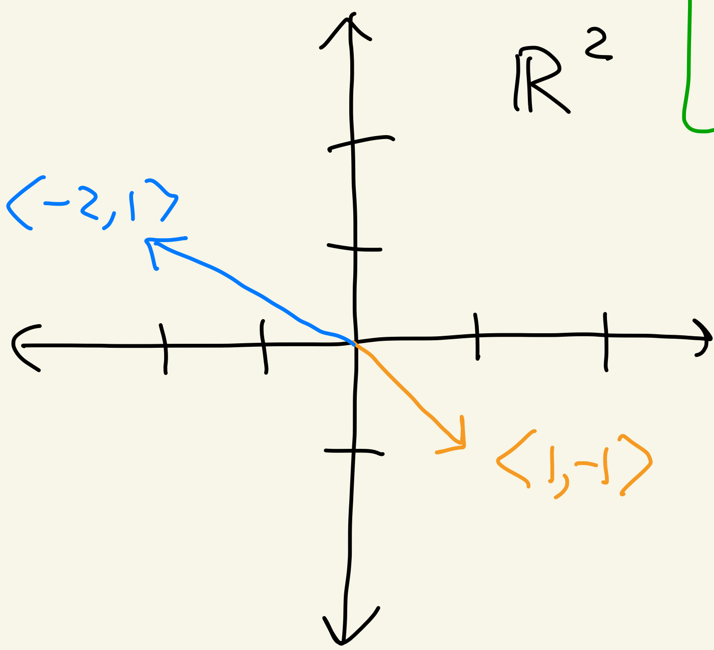
Def: Let $n \geq 1$ be an integer. Define $\mathbb{R}^n$ to be the set of all n-dimensional real vectors. That is,

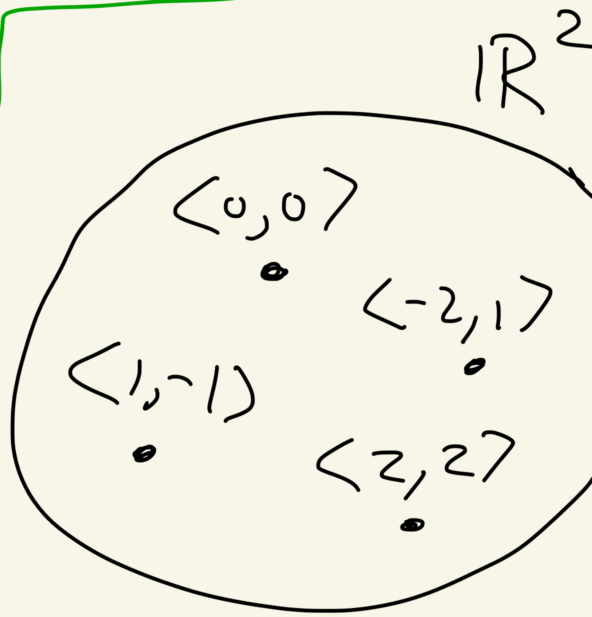
$$\mathbb{R}^n = \{ \langle a_1, a_2, \ldots, a_n \rangle \mid a_1 \in \mathbb{R}, a_2 \in \mathbb{R}, \ldots, a_n \in \mathbb{R} \}$$

Ex: $\mathbb{R}^2 = \{\langle a_1, a_2 \rangle \mid a_1 \in \mathbb{R},\ a_2 \in \mathbb{R}\}$

$$= \{\langle 0,0\rangle, \langle 1,-1\rangle, \langle 2,2\rangle, \langle 10000, e^2\rangle, \ldots\}$$

infinitely many more





Ex: $\mathbb{R}^3 = \{\langle a_1, a_2, a_3 \rangle \mid a_1, a_2, a_3 \in \mathbb{R}\}$

$$= \{\langle 1,1,1\rangle, \langle 0,5,7\rangle, \langle 3,-10,e^2\rangle, \ldots\}$$

infinitely many more

Ex:

$$\mathbb{R}^6 = \{\langle a_1, a_2, a_3, a_4, a_5, a_6 \rangle \mid a_1, a_2, a_3, a_4, a_5, a_6 \in \mathbb{R}\}$$

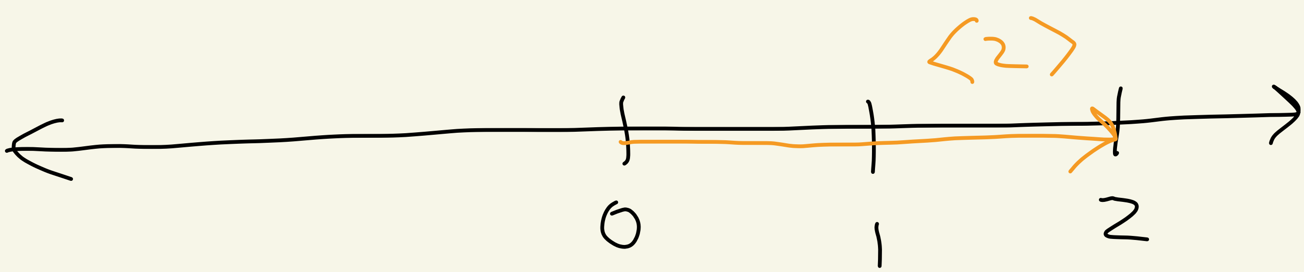
$$= \{\langle 1, 0, 8, 4, \pi, e \rangle, \langle 1, 2, 3, 3, 2, 1 \rangle, \langle 0, 0, 0, 0, 0, 0 \rangle, \ldots\}$$

infinitely many more

$\boxed{\text{Ex:}}$

$$\mathbb{R}^1 = \{\langle a_1 \rangle \mid a_1 \in \mathbb{R}\}$$

$$= \{\langle 1 \rangle, \langle 0 \rangle, \langle -5 \rangle, \ldots\}$$



<u>Def:</u> Let $\vec{v} = \langle a_1, a_2, \ldots, a_n \rangle$ be a vector in $\mathbb{R}^n$.

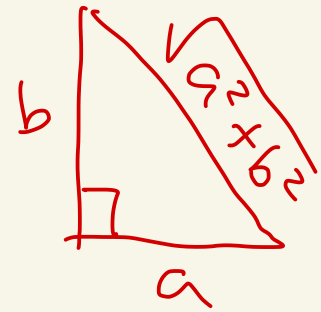
Define the <u>length</u> (or <u>magnitude</u> or <u>norm</u>) of $\vec{v}$ to be

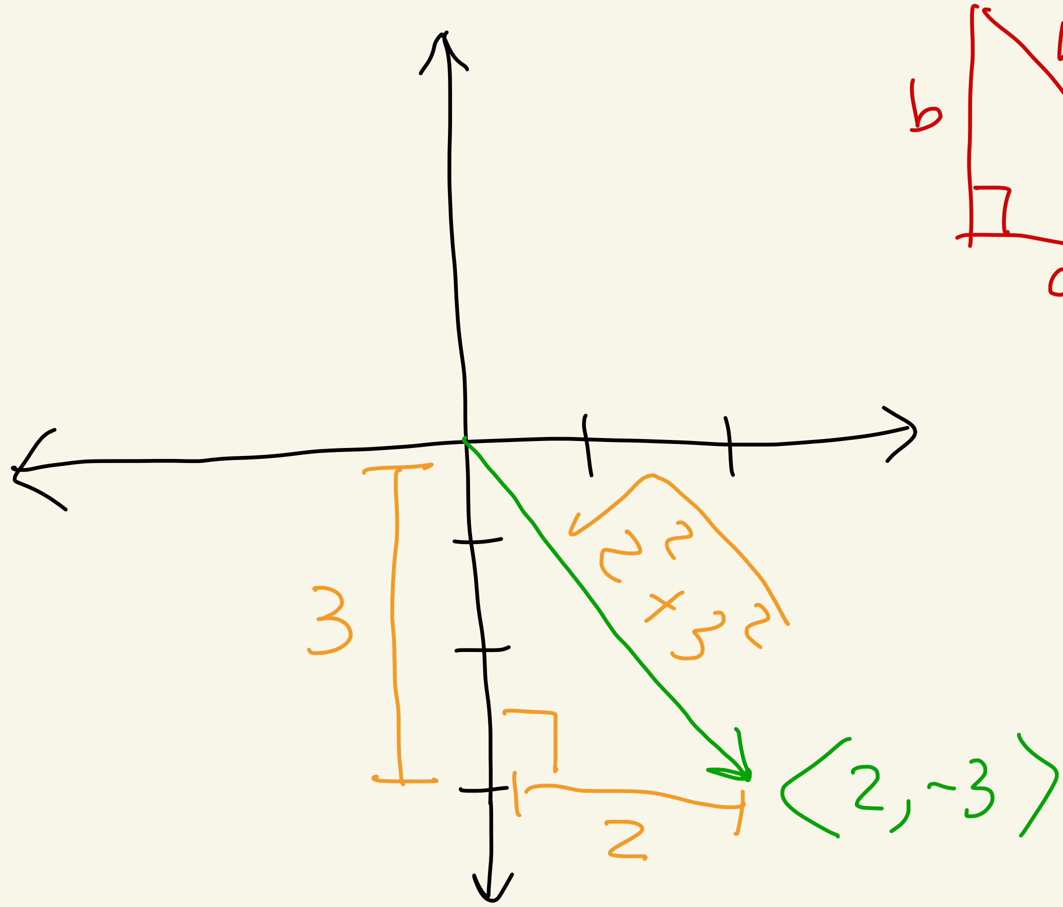
$$\|\vec{v}\| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$$

Some people write $|\vec{v}|$ instead of $\|\vec{v}\|$

Ex: In $\mathbb{R}^2$, let $\vec{v} = \langle 2, -3 \rangle$

$$\|\vec{v}\| = \sqrt{2^2 + (-3)^2} = \sqrt{13} \approx 3.6055$$





Ex: In $\mathbb{R}^5$,

let $\vec{v} = \langle 0, -1, 2, -3, 4 \rangle$

Then,

$$\|\vec{v}\| = \sqrt{0^2 + (-1)^2 + 2^2 + (-3)^2 + 4^2}$$

$$= \sqrt{1 + 4 + 9 + 16}$$

$$= \sqrt{30} \approx 5.4772$$

Operations on vectors

Let $\vec{v}, \vec{w}$ be vectors in $\mathbb{R}^n$ and let α be a $\underbrace{\text{scalar}}_{\text{another name for number}}$ in $\mathbb{R}$

Some Greek letters

α - alpha
β - beta
γ - gamma
δ - delta

Suppose

$$\vec{v} = \langle a_1, a_2, \ldots, a_n \rangle$$

$$\vec{w} = \langle b_1, b_2, \ldots, b_n \rangle$$

Define <u>vector addition</u> as

$$\vec{v} + \vec{w} = \langle a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n \rangle$$

and <u>scalar multiplication</u> as

$$\alpha \vec{v} = \langle \alpha a_1, \alpha a_2, \ldots, \alpha a_n \rangle$$

and <u>vector subtraction</u> as

$$\vec{v} - \vec{w} = \langle a_1 - b_1, a_2 - b_2, \ldots, a_n - b_n \rangle$$

Ex: In $\mathbb{R}^2$, let

$\vec{v} = \langle 2, 4 \rangle \qquad \alpha = 3$

$\vec{w} = \langle -1, 5 \rangle$

Then,

$$\vec{v} + \vec{w} = \langle 2, 4 \rangle + \langle -1, 5 \rangle$$
$$= \langle 2 + (-1), 4 + 5 \rangle$$
$$= \langle 1, 9 \rangle$$

$$\alpha \vec{v} = 3\vec{v} = 3\langle 2, 4 \rangle$$
$$= \langle 3(2), 3(4) \rangle$$
$$= \langle 6, 12 \rangle$$

$$\vec{v} - \vec{w} = \langle 2, 4 \rangle - \langle -1, 5 \rangle$$

$= \langle 2-(-1), 4-5 \rangle$

$= \langle 3, -1 \rangle$

<u>Ex:</u> In $\mathbb{R}^6$, let

$\vec{v} = \langle 1, 2, 3, 4, 5, 6 \rangle$

$\vec{w} = \langle 0, 1, 0, -1, 0, 2 \rangle$

$\vec{v} + \vec{w}$

$= \langle 1+0, 2+1, 3+0, 4+(-1), 5+0, 6+2 \rangle$

$= \langle 1, 3, 3, 3, 5, 8 \rangle$

$-2\vec{v}$

$= \langle -2(1), -2(2), -2(3), -2(4), -2(5), -2(6) \rangle$

$= \langle -2, -4, -6, -8, -10, -12 \rangle$

Notation: In $\mathbb{R}^n$, the zero vector is the vector with all 0's. It's notated by $\vec{0}$.

In $\mathbb{R}^2$, $\vec{0} = \langle 0, 0 \rangle$

In $\mathbb{R}^3$, $\vec{0} = \langle 0, 0, 0 \rangle$

In $\mathbb{R}^4$, $\vec{0} = \langle 0, 0, 0, 0 \rangle$

and so on...

Properties of vectors

Let $\vec{u}, \vec{v}, \vec{w}$ be vectors in $\mathbb{R}^n$

and let α, β be scalars in $\mathbb{R}$.

Then:

① $\vec{u} + \vec{w} = \vec{w} + \vec{u}$ (commutativity)

② $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (associativity)

③ $\alpha(\beta \vec{u}) = (\alpha\beta)\vec{u}$

④ $(\alpha + \beta)\vec{u} = \alpha\vec{u} + \beta\vec{u}$

⑤ $\alpha(\vec{u} + \vec{v}) = \alpha\vec{u} + \alpha\vec{v}$

distributive rules

⑥ $\vec{u} + \vec{0} = \vec{0} + \vec{u} = \vec{u}$

⑦ $\vec{u} + (-\vec{u}) = \vec{0}$

$(-\vec{u}) + \vec{u} = \vec{0}$

zero vector magic