Math 2550

8/23/23

Topic 0 – Sets

Def: A set is a collection of objects/elements.

If S is a set and x is an element of S then we write $x \in S$.

read: "x is in S."

If x is not an element of S, then we write

$x \notin S$.

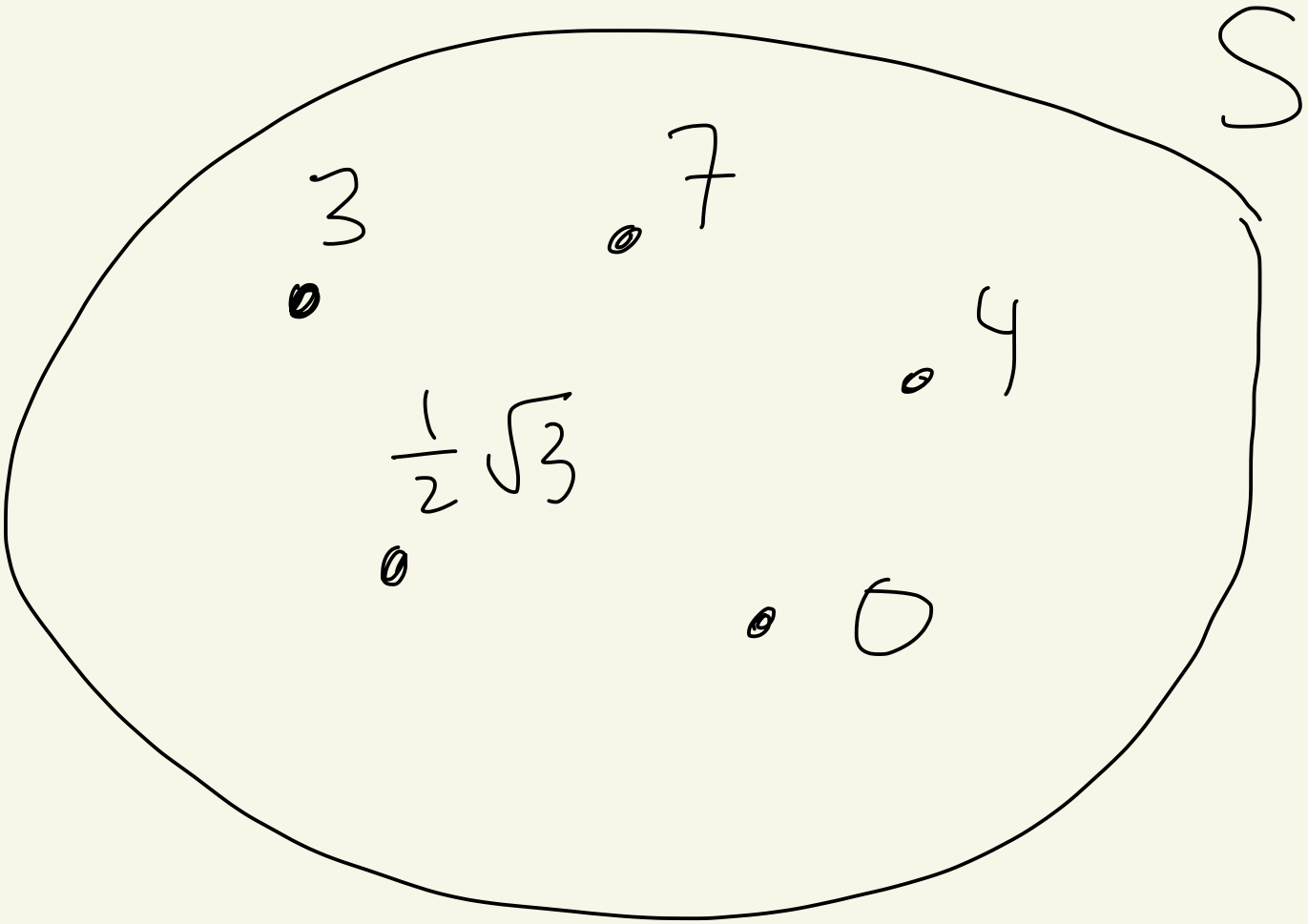
read: "x is not in S."

Ex:

$S = \{3, 7, 4, \frac{1}{2}\sqrt{3}, 0\}$

$3 \in S$ $\quad \frac{1}{2} \notin S$

$0 \in S$



Order doesn't matter in a set. For example,

$$S = \{3, 7, 4, \tfrac{1}{2}\sqrt{3}, 0\}$$
$$= \{4, \tfrac{1}{2}\sqrt{3}, 0, 7, 3\}$$

Set's can't have duplicate elements.

So, $\{1, 1, 2\}$ is not a set.

If someone wrote $\{1, 1, 2\}$ they really mean $\{1, 2\}$.

Ex: $\mathbb{R}$ denotes the set of real numbers. The real numbers are the numbers with decimal expansions.

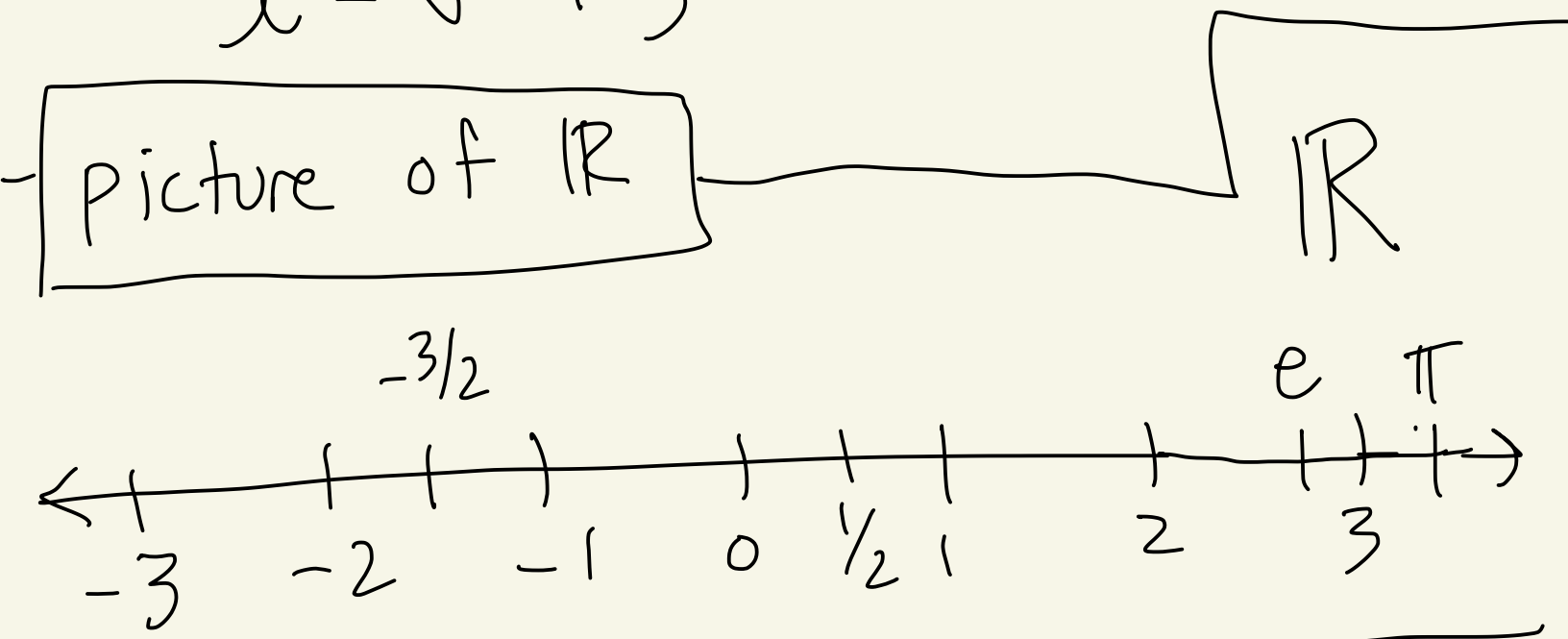
Some real numbers are:

$$-1,\ 0.5 = \frac{1}{2},\ \pi \approx 3.14\ldots$$

Not real numbers:

$$i = \sqrt{-1},\ 1+i$$

picture of $\mathbb{R}$



$\pi \in \mathbb{R}$ $\quad i \notin \mathbb{R}$

$\frac{1}{2} \in \mathbb{R}$

Notation:

$x, y \in S$ is shorthand for $x \in S$ and $y \in S$.

read:
"x is in S
and y is in S"

Ex: $0, 2 \in \mathbb{R}$

means: $0 \in \mathbb{R}$ and $2 \in \mathbb{R}$

Ex: $0, 2, \pi, \frac{1}{2} \in \mathbb{R}$

means: $0 \in \mathbb{R}$ and $2 \in \mathbb{R}$

and $\pi \in \mathbb{R}$ and $\frac{1}{2} \in \mathbb{R}$

General way to define a set

$$\{ \text{description of what the elements look like} \mid \text{conditions that the elements must satisfy to be in the set} \}$$

read $\mid$ as "such that" or "where"

Ex: Let

$$S = \{(x,y) \mid x \in \mathbb{R} \text{ and } y \in \mathbb{R}\}$$

read: S consists of all (x,y) where x is a real # and y is a real #

Some elements from S:

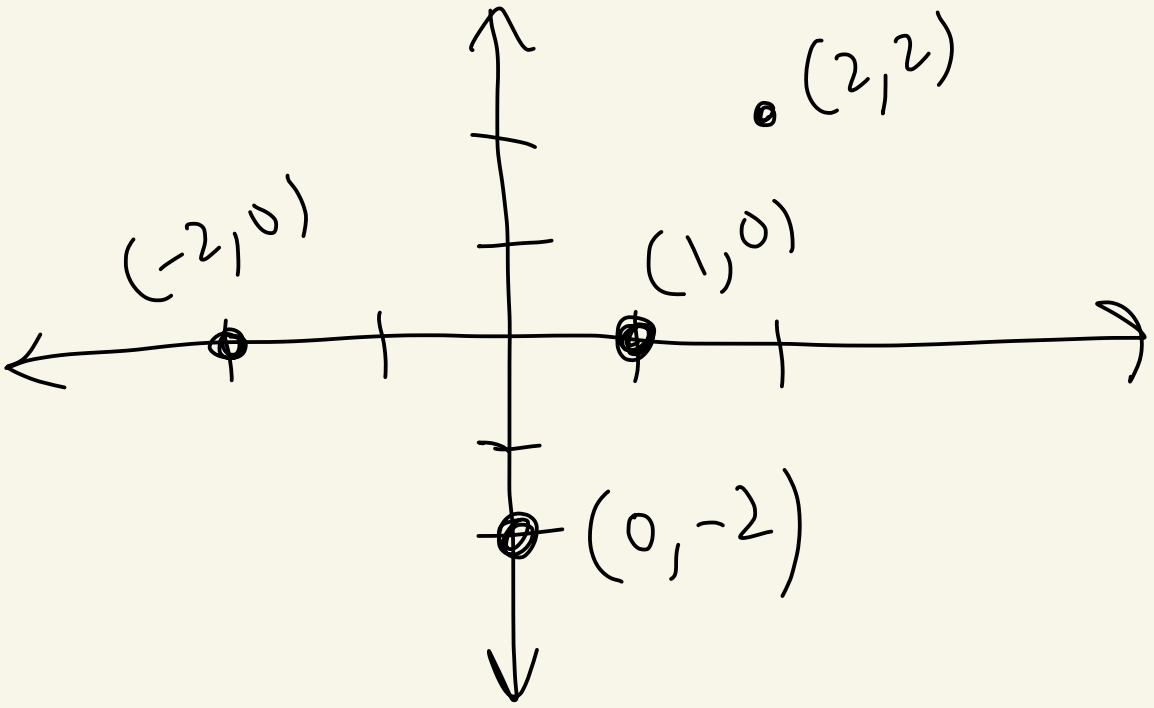
$(2,-1) \in S$

$(0,0) \in S$

$(i,2) \notin S$ because $i = \sqrt{-1}$ is not in $\mathbb{R}$

Use parenthes when order matters $(2,-1) \neq (-1,2)$

S is the xy-plane.



Ex:

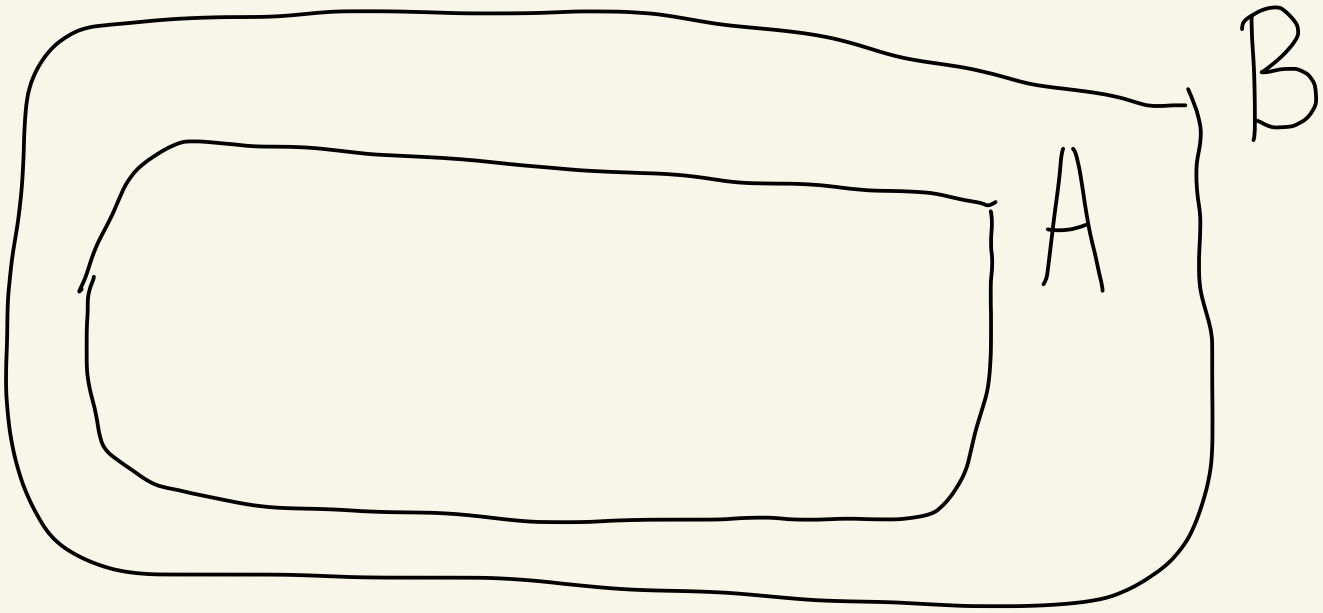
$$S = \{ x \mid x \in \mathbb{R} \text{ and } x^2 - 1 = 0 \}$$

$$= \{ 1, -1 \}$$

S consists of all x where x is a real # and $x^2 - 1 = 0$

Def: The empty set, denoted by ϕ or $\{\}$, is defined to be the set with no elements.

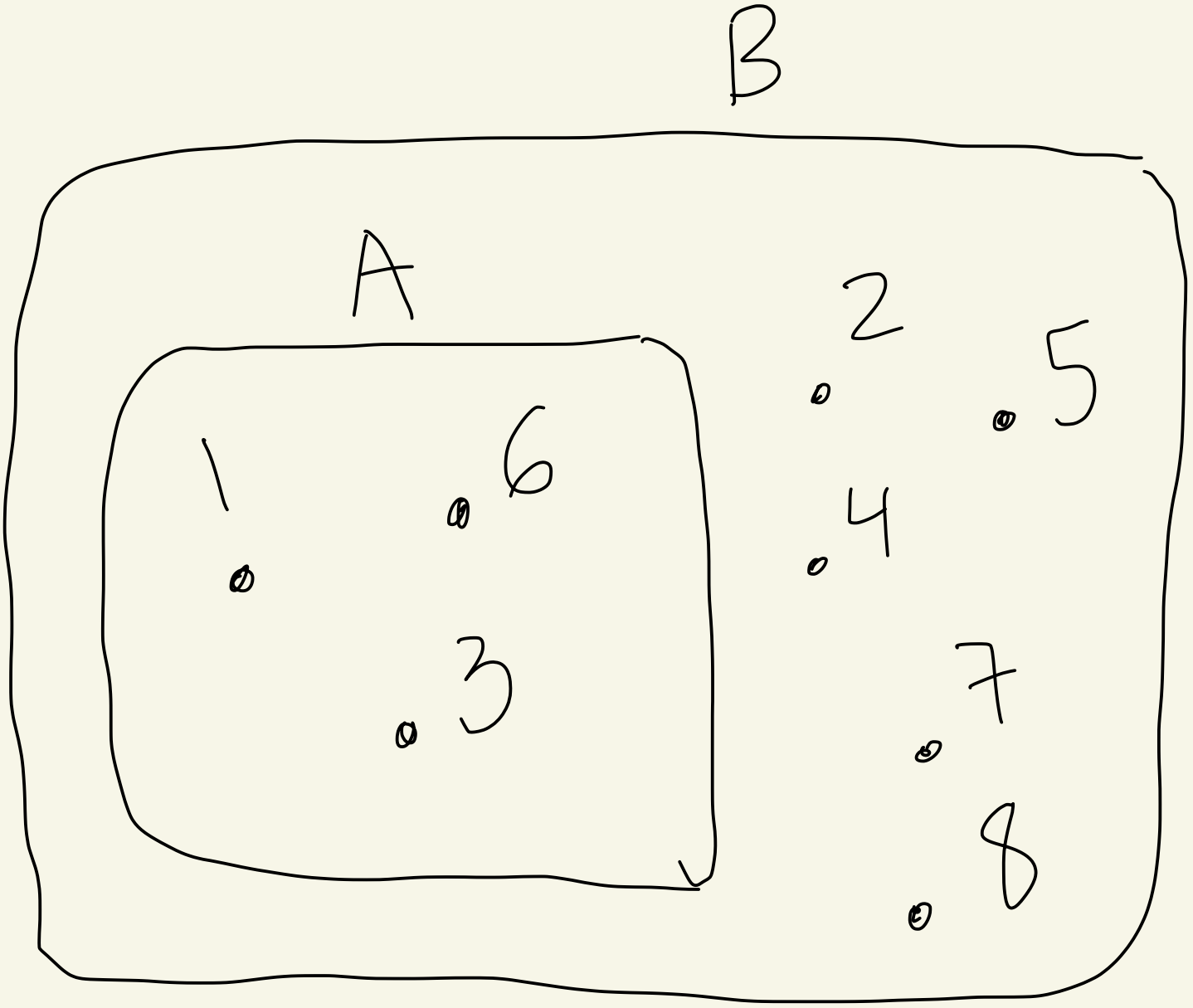
Def: Let A and B be sets. We say that A is a subset of B, and write $A \subseteq B$, if every element of A is also an element of B.


B
A

Ex: Let

$B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A = \{1, 6, 3\}$



Here we have $A \subseteq B$.