Math 2550-03
4/18/24

HW 7 - Part 1

② (b) Is $\langle 3,1,5\rangle$ in the span of $\vec{u}=\langle 0,-2,2\rangle$ and $\vec{v}=\langle 1,3,-1\rangle$ If so, write it as a linear combo of $\vec{u}$ and $\vec{v}$.

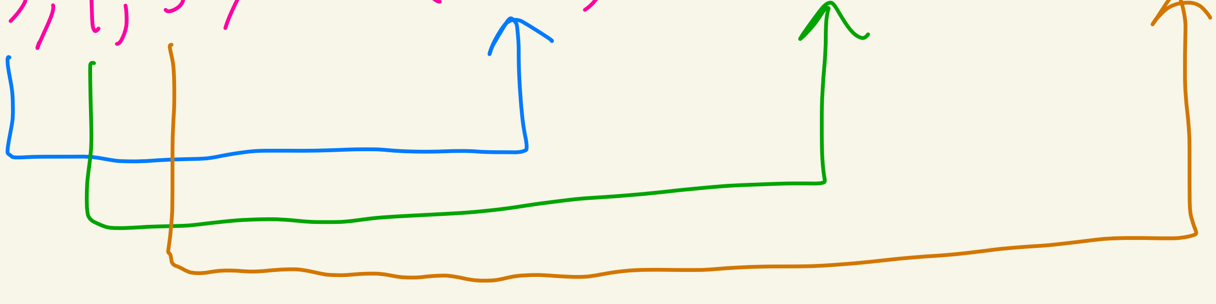
We want to solve

$$\langle 3,1,5\rangle = \underbrace{c_1\langle 0,-2,2\rangle + c_2\langle 1,3,-1\rangle}_{c_1\vec{u}+c_2\vec{v}}$$

This gives

$$\langle 3,1,5\rangle = \langle 0,-2c_1,2c_1\rangle + \langle c_2,3c_2,-c_2\rangle$$

$$\langle 3,1,5\rangle = \langle c_2,-2c_1+3c_2,2c_1-c_2\rangle$$



Need to solve

$$\boxed{\begin{array}{r} c_2 = 3 \\ -2c_1 + 3c_2 = 1 \\ 2c_1 - c_2 = 5 \end{array}} \quad \begin{array}{l} \leftarrow \boxed{c_2 = 3} \\ \leftarrow \boxed{c_1 = 4,\ c_2 = 3} \\ \leftarrow \boxed{c_1 = 4,\ c_2 = 3} \end{array}$$

Thus,

$$\langle 3,1,5\rangle = \underbrace{4\langle 0,-2,2\rangle + 3\langle 1,3,-1\rangle}_{4\vec{u}+3\vec{v}}$$

So, $\langle 3,1,5\rangle$ is in the span of $\vec{u}$ and $\vec{v}$ and $\langle 3,1,5\rangle = 4\vec{u}+3\vec{v}$.

HW 7 - Part 1

4(d) In $V = P_2$ are $\vec{p}_1 = 3 - 2x + x^2$, $\vec{p}_2 = 1 + x + x^2$, $\vec{p}_3 = 6 - 4x + 2x^2$ linearly independent or linearly dependent?

We need to solve

$$c_1 \vec{p}_1 + c_2 \vec{p}_2 + c_3 \vec{p}_3 = \vec{0}$$

If the only solution is $c_1 = 0, c_2 = 0, c_3 = 0$ then $\vec{p}_1, \vec{p}_2, \vec{p}_3$ are linearly independent.

If there are more solutions, then the vectors are linearly dependent.

We need to solve

$$c_1(3-2x+x^2)+c_2(1+x+x^2)+c_3(6-4x+2x^2)$$
$$=0+0x+0x^2$$

This gives

$$3c_1-2c_1x+c_1x^2+c_2+c_2x+c_2x^2+6c_3-4c_3x+2c_3x^2$$
$$=0+0x+0x^2$$

So,

$$(3c_1+c_2+6c_3)+(-2c_1+c_2-4c_3)x+(c_1+c_2+2c_3)x^2$$
$$=0+0x+0x^2$$

Need to solve

$$\boxed{\begin{aligned} 3c_1+c_2+6c_3&=0\\ -2c_1+c_2-4c_3&=0\\ c_1+c_2+2c_3&=0 \end{aligned}}$$

$R_1 \leftrightarrow R_3$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ -2 & 1 & -4 & 0 \\ 3 & 1 & 6 & 0 \end{array}\right)$$

$$\xrightarrow[-3R_1+R_3\to R_3]{2R_1+R_2\to R_2} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{\frac{1}{3}R_2\to R_2} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{2R_2+R_3\to R_3} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

which gives

$$\boxed{\begin{array}{rcl} c_1 + c_2 + 2c_3 &=& 0 \\ c_2 &=& 0 \\ 0 &=& 0 \end{array}} \begin{array}{l} \textcircled{1} \\ \textcircled{2} \\ \end{array}$$

leading c_1, c_2

free c_3

Solution:

$$\boxed{\begin{aligned} &c_3 = t \\ &c_2 = 0 \\ &c_1 = -c_2 - 2c_3 = -(0) - 2t = -2t \end{aligned}}$$

There's infinitly many sols. For example, $t = 1$ gives

$c_1 = -2$, $c_2 = 0$, $c_3 = 1$.

Thus,

$$\boxed{-2\vec{p}_1 + 0\vec{p}_2 + 1\vec{p}_3 = \vec{0}}$$

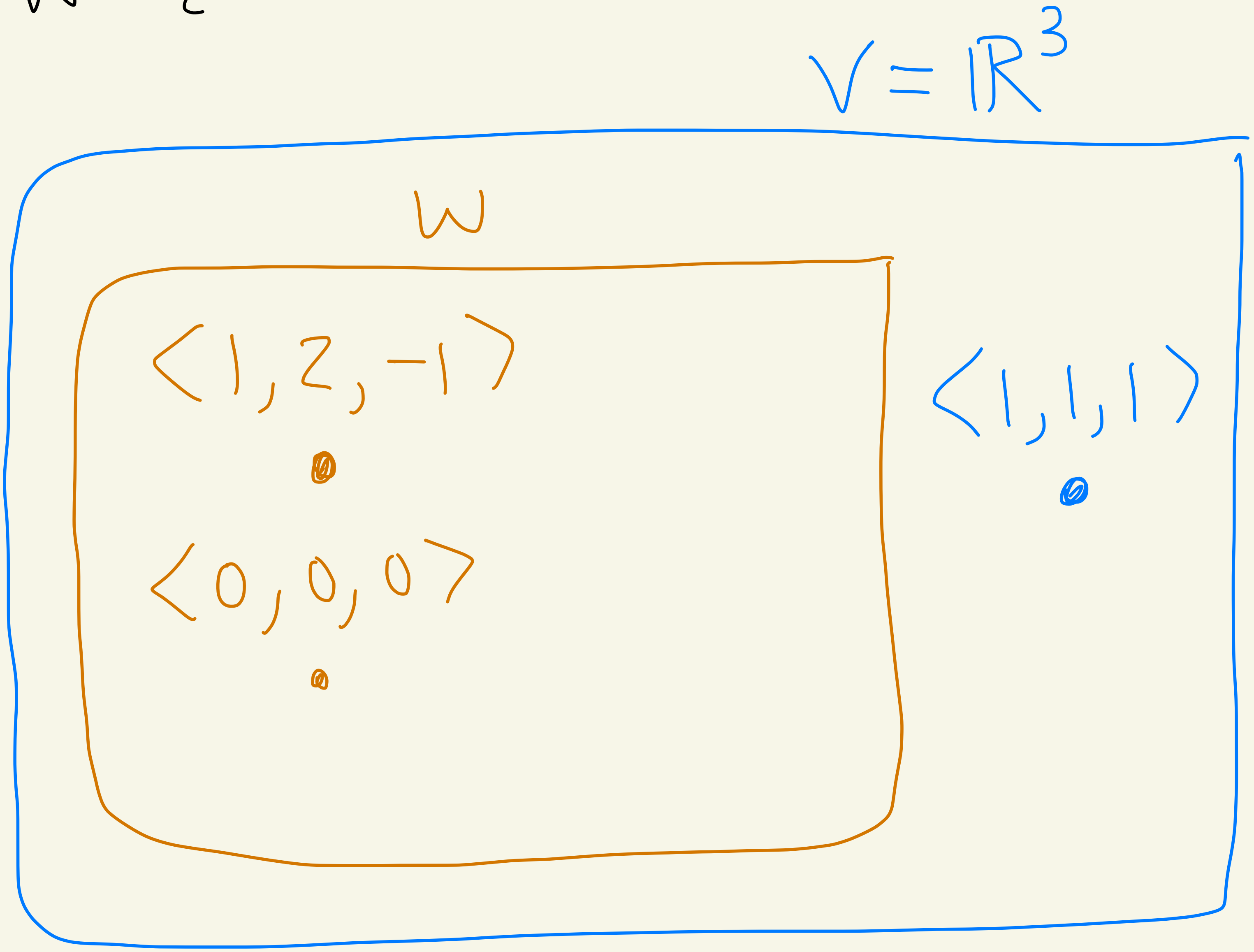
Thus, $\vec{p}_1, \vec{p}_2, \vec{p}_3$ are lin. dep.

HW 6 - Part 2

Let $V = \mathbb{R}^3$, $F = \mathbb{R}$.

Let

$$W = \{\langle a,b,c\rangle \mid \boxed{4a-b+2c=0},\ a,b,c \in \mathbb{R}\}$$



$\langle 1,2,-1\rangle$ is in W since $4(1)-(2)+2(-1)=0$

$\langle 0,0,0\rangle$ is in W since $4(0)-(0)+2(0)=0$

$\langle 1,1,1\rangle$ is not in W since $4(1)-(1)+2(1) \neq 0$

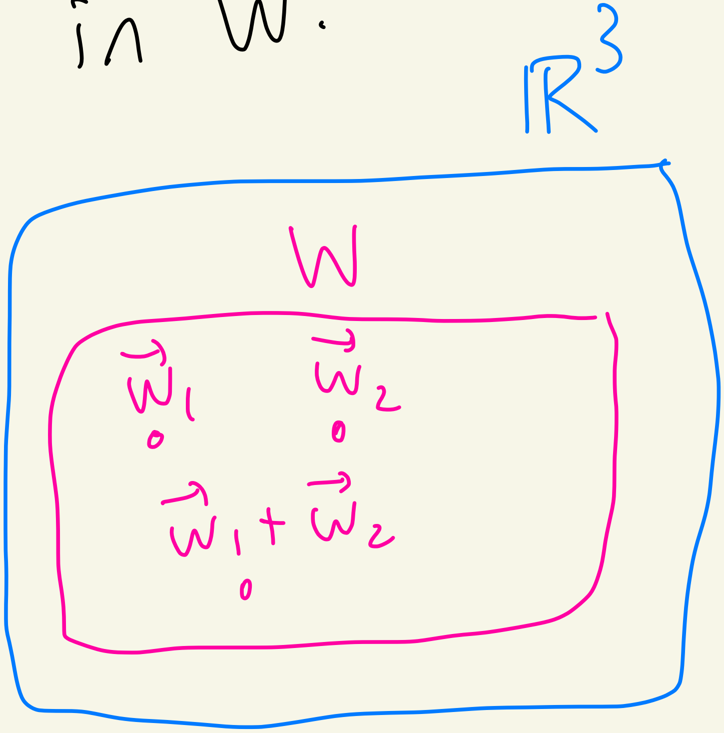
Prove W is a subspace of $\mathbb{R}^3$

$$W = \{\langle a,b,c\rangle \mid \boxed{b=4a+2c},\ a,b,c \in \mathbb{R}\}$$

① (zero vector)

Setting $a=0, b=0, c=0$, then $b=4a+2c$

So, $\vec{0} = \langle 0,0,0\rangle$ is in W.



② (closed under +)

Let $\vec{w}_1 = \langle a_1, b_1, c_1\rangle$

and $\vec{w}_2 = \langle a_2, b_2, c_2)$

be in W.

Then, $\boxed{b_1 = 4a_1 + 2c_1 \text{ and } b_2 = 4a_2 + 2c_2.}$

Then,

$$\vec{w}_1 + \vec{w}_2 = \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$$

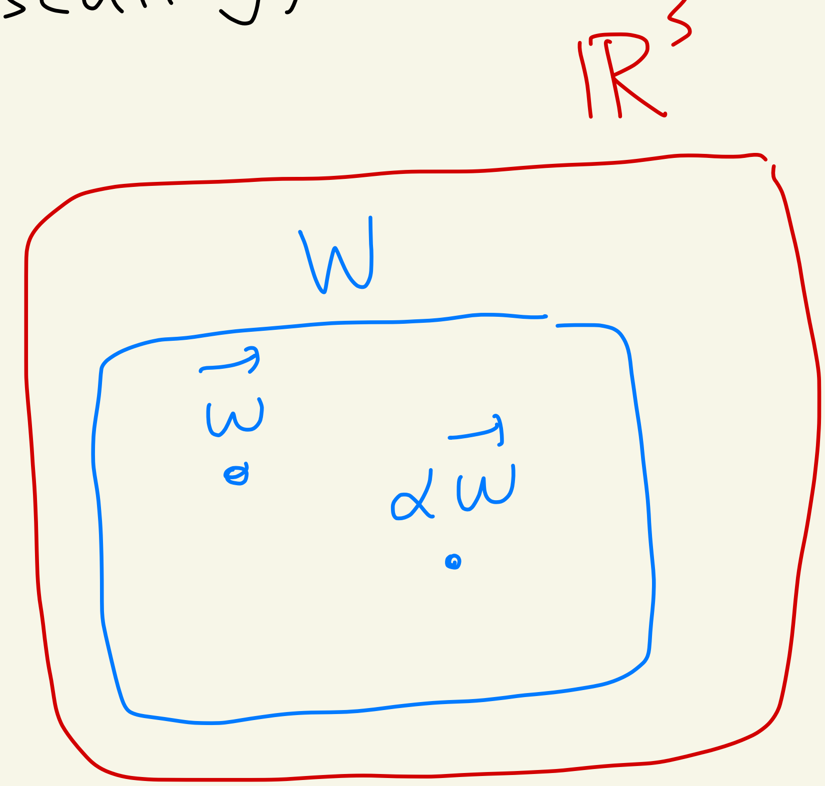
and

$$b_1 + b_2 = 4a_1 + 2c_1 + 4a_2 + 2c_2$$
$$= 4(a_1 + a_2) + 2(c_1 + c_2)$$

So, $\vec{w}_1 + \vec{w}_2$ is in W.

③ (closed under scaling)

Let $\vec{w} = \langle a, b, c \rangle$ be in W and α be a scalar/number.



We know $\boxed{b = 4a + 2c}$ since $\vec{w} = \langle a, b, c \rangle$ is in W.

Then,

$$\alpha\vec{w} = \langle \alpha a, \alpha b, \alpha c \rangle$$

and

$$\alpha b = \alpha(4a+2c) = 4(\alpha a)+2(\alpha c)$$

So, $\alpha\vec{w}$ is in W.

By ①, ②, ③ W is a subspace.

▨

Hw 4 - Part 1

Find A^{-1} if it exists where

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

$$\left(\underbrace{\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{matrix}}_{A} \middle| \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I_3}\right)$$

$$\xrightarrow{-R_1+R_3\to R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{-R_2+R_3\to R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{-\frac{1}{2}R_3 \to R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{array}\right)$$

$$\xrightarrow[-R_3+R_2 \to R_2]{-R_3+R_1 \to R_1} \left(\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I_3} \left| \underbrace{\begin{matrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{matrix}}_{A^{-1}} \right. \right)$$

So,

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

HW 4 - Part 1

Ex: Solve

$$\boxed{\begin{aligned} x \quad\quad &+ z = 1 \\ y &+ z = -2 \\ x + y \quad\quad &= 4 \end{aligned}}$$

by inverting the coefficient matrix

This system is the same as

$$\underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$$

Multiply by A^{-1} on both sides:

$$\underbrace{\cancel{\begin{pmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\end{pmatrix}}}_{A^{-1}}\underbrace{\cancel{\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 0\end{pmatrix}}}_{A}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \underbrace{\begin{pmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\end{pmatrix}}_{A^{-1}}\begin{pmatrix}1\\ -2\\ 4\end{pmatrix}$$

So,

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\ -\frac{1}{2} & 1/2 & 1/2\\ 1/2 & 1/2 & -1/2\end{pmatrix}\begin{pmatrix}1\\ -2\\ 4\end{pmatrix}$$

So,

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}(\frac{1}{2})(1) + (-\frac{1}{2})(-2) + (\frac{1}{2})(4)\\ (-\frac{1}{2})(1) + (\frac{1}{2})(-2) + (\frac{1}{2})(4)\\ (\frac{1}{2})(1) + (\frac{1}{2})(-2) + (-\frac{1}{2})(4)\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}3.5\\ 0.5\\ -2.5\end{pmatrix}$$

Answer: $x = 3,5$, $y = 0,5$, $z = -2,5$