Math 2550-03
3/7/24

- Test 1 next Tuesday
- Today is review

HW 1 - Part 1

(5) Compute $\frac{1}{2}\vec{u} - \vec{v}$ where
$\vec{u} = \langle 2, 0, 8, -4, 10 \rangle$ and
$\vec{v} = \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$.

$$\frac{1}{2}\vec{u} - \vec{v} = \frac{1}{2}\langle 2, 0, 8, -4, 10 \rangle - \langle 0, \tfrac{1}{2}, 3, 10, -1 \rangle$$

$$= \langle 1, 0, 4, -2, 5 \rangle - \langle 0, \tfrac{1}{2}, 3, 10, -1 \rangle$$

$$= \langle 1, -\tfrac{1}{2}, 1, -12, 6 \rangle$$

$$\vec{u} \cdot \vec{v} = \langle 2, 0, 8, -4, 10 \rangle \cdot \langle 0, \tfrac{1}{2}, 3, 10, -1 \rangle$$

$$= (2)(0) + (0)\left(\frac{1}{2}\right) + (8)(3) + (-4)(10) + (10)(-1)$$

$$= 0 + 0 + 24 - 40 - 10$$

$$= \boxed{-26}$$

What's the norm/length of $\vec{u} = \langle 2, 0, 8, -4, 10 \rangle$?

$$\|\vec{u}\| = \sqrt{(2)^2 + (0)^2 + (8)^2 + (-4)^2 + (10)^2}$$

$$= \sqrt{4 + 64 + 16 + 100}$$

$$= \boxed{\sqrt{184}} \approx 13.56$$

HW 1 - Part 2

① (d) (modified to $\mathbb{R}^3$)

Let $\vec{u}, \vec{v} \in \mathbb{R}^3$ and $\alpha \in \mathbb{R}$.

Prove that $\alpha(\vec{u} \cdot \vec{v}) = (\alpha \vec{u}) \cdot \vec{v}$

Proof: Let $\vec{u}, \vec{v} \in \mathbb{R}^3$ and $\alpha \in \mathbb{R}$.

Then, $\vec{u} = \langle a, b, c \rangle$ and

$\vec{v} = \langle d, e, f \rangle$ where

$a, b, c, d, e, f \in \mathbb{R}$

We have that

$$\alpha(\vec{u} \cdot \vec{v}) = \alpha(\langle a, b, c \rangle \cdot \langle d, e, f \rangle)$$
$$= \alpha(ad + be + cf)$$

$$= \alpha ad + \alpha be + \alpha cf$$

Also we have that

$$(\alpha \vec{u}) \cdot \vec{v} = (\alpha \langle a, b, c \rangle) \cdot \langle d, e, f \rangle$$

$$= \langle \alpha a, \alpha b, \alpha c \rangle \cdot \langle d, e, f \rangle$$

$$= \alpha ad + \alpha be + \alpha cf$$

SAME

From above we see that

$$\alpha (\vec{u} \cdot \vec{v}) = (\alpha \vec{u}) \cdot \vec{v}$$

$\blacksquare$

HW 2 - Part 1

Compute $2A + BC^T$

where $A = \begin{pmatrix} 1 & 3 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

$$C = \begin{pmatrix} 1 & 2 & 1 & 1 \\ -1 & 0 & -1 & 1 \end{pmatrix}$$

$2A + BC^T$

$$= 2\begin{pmatrix} 1 & 3 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 1 & 1 \\ -1 & 0 & -1 & 1 \end{pmatrix}^T$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \underbrace{\begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}}_{2 \times 4}\underbrace{\begin{pmatrix} 1 & -1 \\ 2 & 0 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}}_{4 \times 2}$$

$\checkmark$

2x2 answer

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} \begin{pmatrix} 1 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1+0+1-1 & -1+0-1-1 \\ 0+0+1+1 & 0+0-1+1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & -3 \\ 2 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix}}$$

HW 2 - Part 2

1 (f)

Let A, B be 2×2 matrices.
Prove that $(A+B)^T = A^T + B^T$

proof: Let A, B be 2×2 matrices.

Then,

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ and } B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}.$$

where a, b, c, d, e, f, g, h are real numbers.

We have that

$$(A+B)^T = \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} \right)^T$$

$$= \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}^T$$

$$= \begin{pmatrix} a+e & c+g \\ b+f & d+h \end{pmatrix}$$

Also,

$$A^T + B^T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^T + \begin{pmatrix} e & f \\ g & h \end{pmatrix}^T$$

$$= \begin{pmatrix} a & c \\ b & d \end{pmatrix} + \begin{pmatrix} e & g \\ f & h \end{pmatrix}$$

SAME

$$= \begin{pmatrix} a+e & c+g \\ b+f & d+h \end{pmatrix}$$

Thus, $(A+B)^T = A^T + B^T$

$\blacksquare$

HW 3 - 1 (c) Solve

$$\boxed{\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\qquad - 3w &= -3 \end{aligned}}$$

already a 1

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right)$$

make these 0

make this 1

$$\xrightarrow[\substack{R_1 + R_3 \to R_3 \\ -3R_1 + R_4 \to R_4}]{-2R_1 + R_2 \to R_2} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right)$$

make these 0

$$\xrightarrow[-3R_2+R_4 \to R_4]{-3R_2+R_3 \to R_3} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Back to equations:

$$\boxed{\begin{aligned} x - y + 2z - w &= -1 \\ y - 2z &= 0 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}}$$

leading x, y

free z, w

Solve for leading variables &

give free variables

new names.

$$\boxed{\begin{aligned} x &= -1 + y - 2z + w \quad &(1)\\ y &= 2z \quad &(2)\\ z &= t \quad &(3)\\ w &= s \quad &(4) \end{aligned}}$$

Back substitute:

(4) $w = s$

(3) $z = t$

(2) $y = 2z = 2t$

(1) $x = -1 + y - 2z + w$

$= -1 + 2t - 2t + s = -1 + s$

Answer:

$x = -1 + s$

$y = 2t$

$z = t$

$w = s$

where s, t can be any real numbers