

Math 2550-03

3/7/24



- Test 1 next Tuesday
- Today is review

HW 1 - Part 1

⑤ Compute $\frac{1}{2}\vec{u} - \vec{v}$ where

$$\vec{u} = \langle 2, 0, 8, -4, 10 \rangle \text{ and}$$

$$\vec{v} = \langle 0, \frac{1}{2}, 3, 10, -1 \rangle.$$

$$\frac{1}{2}\vec{u} - \vec{v} = \frac{1}{2} \langle 2, 0, 8, -4, 10 \rangle$$

$$- \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$$

$$= \langle 1, 0, 4, -2, 5 \rangle$$

$$- \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$$

$$= \langle 1, -\frac{1}{2}, 1, -12, 6 \rangle$$

$$\vec{u} \cdot \vec{v} = \langle 2, 0, 8, -4, 10 \rangle \cdot \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$$

$$\begin{aligned}
 &= (2)(0) + (0)\left(\frac{1}{2}\right) + (8)(3) \\
 &\quad + (-4)(10) + (10)(-1)
 \end{aligned}$$

$$= 0 + 0 + 24 - 40 - 10$$

$$= \boxed{-26}$$

What's the norm/length of

$$\vec{u} = \langle 2, 0, 8, -4, 10 \rangle ?$$

$$\|\vec{u}\| = \sqrt{(2)^2 + (0)^2 + (8)^2 + (-4)^2 + (10)^2}$$

$$= \sqrt{4 + 64 + 16 + 100}$$

$$= \boxed{\sqrt{184}} \approx 13.56$$

HW 1 - Part 2

①(d) (modified to \mathbb{R}^3)

Let $\vec{u}, \vec{v} \in \mathbb{R}^3$ and $\alpha \in \mathbb{R}$.
Prove that $\alpha(\vec{u} \cdot \vec{v}) = (\alpha\vec{u}) \cdot \vec{v}$

Proof: Let $\vec{u}, \vec{v} \in \mathbb{R}^3$ and $\alpha \in \mathbb{R}$.
Then, $\vec{u} = \langle a, b, c \rangle$ and
 $\vec{v} = \langle d, e, f \rangle$ where
 $a, b, c, d, e, f \in \mathbb{R}$

We have that

$$\begin{aligned}\alpha(\vec{u} \cdot \vec{v}) &= \alpha(\langle a, b, c \rangle \cdot \langle d, e, f \rangle) \\ &= \alpha(ad + be + cf)\end{aligned}$$

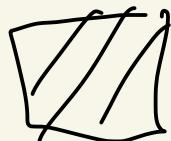
$$= \alpha ad + \alpha be + \alpha cf \quad \text{SAMEN}$$

Also we have that

$$\begin{aligned} (\alpha \vec{u}) \cdot \vec{v} &= (\alpha \langle a, b, c \rangle) \cdot \langle d, e, f \rangle \\ &= \langle \alpha a, \alpha b, \alpha c \rangle \cdot \langle d, e, f \rangle \\ &= \alpha ad + \alpha be + \alpha cf \end{aligned}$$

From above we see that

$$\alpha (\vec{u} \cdot \vec{v}) = (\alpha \vec{u}) \cdot \vec{v}$$



HW 2 - Part 1

Compute $2A + BC^T$

where $A = \begin{pmatrix} 1 & 3 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

$$C = \begin{pmatrix} 1 & 2 & 1 & 1 \\ -1 & 0 & -1 & 1 \end{pmatrix}$$

$$2A + BC^T$$

$$= 2 \begin{pmatrix} 1 & 3 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 & 1 \\ -1 & 0 & -1 & 1 \end{pmatrix}^T$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 0 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$$

2 × 4
4 × 2

✓

2x2 answer

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} (1 & 0 & 1 & -1) \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} & (1 & 0 & 1 & -1) \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \\ (0 & 0 & 1 & 1) \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} & (0 & 0 & 1 & 1) \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1+0+1-1 & -1+0-1-1 \\ 0+0+1+1 & 0+0-1+1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & -3 \\ 2 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix}}$$

HW 2 - Part 2

I (f)

Let A, B be 2×2 matrices.
Prove that $(A+B)^T = A^T + B^T$

Proof: Let A, B be 2×2 matrices.

Then,

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ and } B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}.$$

where a, b, c, d, e, f, g, h are
real numbers.

We have that

$$(A+B)^T = \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} \right)^T$$

$$= \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}^T$$

$$= \begin{pmatrix} a+e & c+g \\ b+f & d+h \end{pmatrix}$$


Also,

$$A^T + B^T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^T + \begin{pmatrix} e & f \\ g & h \end{pmatrix}^T$$

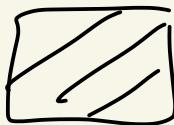
$$= \begin{pmatrix} a & c \\ b & d \end{pmatrix} + \begin{pmatrix} e & g \\ f & h \end{pmatrix}$$

S
A
M
E

$$= \begin{pmatrix} a+e & c+g \\ b+f & d+h \end{pmatrix}$$



Thus, $(A+B)^T = A^T + B^T$



HW 3 - ①(c) Solve

$$\begin{array}{l}
 x - y + 2z - w = -1 \\
 2x + y - 2z - 2w = -2 \\
 -x + 2y - 4z + w = 1 \\
 \hline
 3x \qquad \qquad \qquad -3w = -3
 \end{array}$$

already a 1

$$\left(\begin{array}{cccc|c}
 1 & -1 & 2 & -1 & -1 \\
 2 & 1 & -2 & -2 & -2 \\
 -1 & 2 & -4 & 1 & 1 \\
 3 & 0 & 0 & -3 & -3
 \end{array} \right)$$

make these 0

make this 1

$$-2R_1 + R_2 \rightarrow R_2$$

$$R_1 + R_3 \rightarrow R_3$$

$$-3R_1 + R_4 \rightarrow R_4$$

$$\left(\begin{array}{cccc|c}
 1 & -1 & 2 & -1 & -1 \\
 0 & 3 & -6 & 0 & 0 \\
 0 & 1 & -2 & 0 & 0 \\
 0 & 3 & -6 & 0 & 0
 \end{array} \right)$$

$$R_2 \leftrightarrow R_3 \rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array} \right)$$

make these 0

$$\begin{array}{l} -3R_2 + R_3 \rightarrow R_3 \\ -3R_2 + R_4 \rightarrow R_4 \end{array} \rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Back to equations:

$$\begin{aligned} X - y + 2z - w &= -1 \\ y - 2z &= 0 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

leading
x, y

free
z, w

Solve for leading variables &
give free variables

New names -

$$x = -l + y - 2z + w \quad (1)$$

$$y = 2z \quad (2)$$

$$z = t \quad (3)$$

$$w = s \quad (4)$$

Back substitute:

$$(4) w = s$$

$$(3) z = t$$

$$(2) y = 2z = 2t$$

$$(1) x = -l + y - 2z + w$$
$$= -l + 2t - 2t + s = -l + s$$

Answer:

$$x = -1 + s$$

$$y = 2t$$

$$z = t$$

$$w = s$$

where
 s, t
can be
any real
numbers