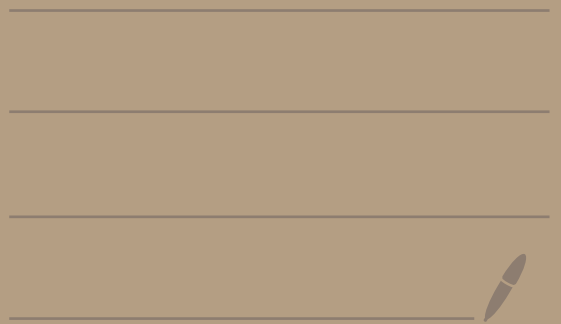


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Ex:

$$\begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 1+2 & 2+3 \\ 0+4 & -3+5 \end{pmatrix} \\ = \begin{pmatrix} 3 & 5 \\ 4 & 2 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \\ -1 & -3 \end{pmatrix} - \begin{pmatrix} 2 & 3 \\ -1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-2 & 0-3 \\ 2-(-1) & 1-(-2) \\ -1-0 & -3-1 \end{pmatrix} \\ = \begin{pmatrix} -1 & -3 \\ 3 & 3 \\ -1 & -4 \end{pmatrix}$$

---

$$(-2) \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} (-2)(1) & (-2)(2) \\ (-2)(3) & (-2)(-4) \end{pmatrix}$$

$$= \begin{pmatrix} -2 & -4 \\ -6 & 8 \end{pmatrix}$$

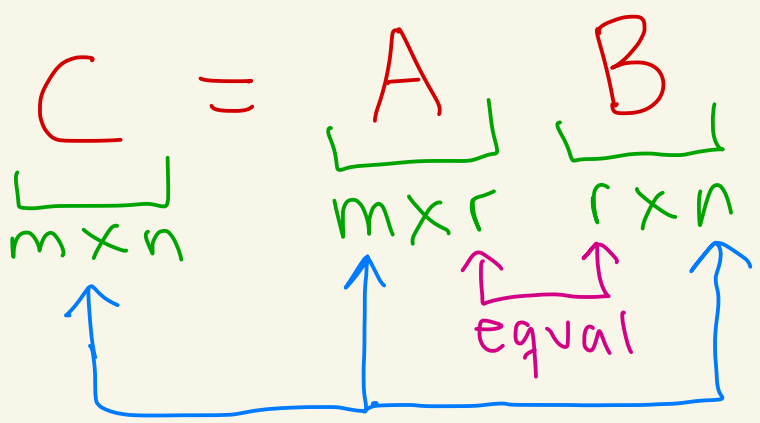
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$$\underbrace{\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}}_{2 \times 2} + \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 5 & -1 \end{pmatrix}}_{3 \times 2}$$

Since the matrices are not the same size we say the addition is undefined

---

Def: Let  $A$  be an  $m \times r$  matrix and  $B$  be an  $r \times n$  matrix. We define the product of  $A$  and  $B$ , denoted by  $AB$ , to be the  $m \times n$  matrix  $C$  whose entry in row  $i$  and column  $j$  equals the dot product of the  $i$ -th row of  $A$  and the  $j$ -th column of  $B$ .

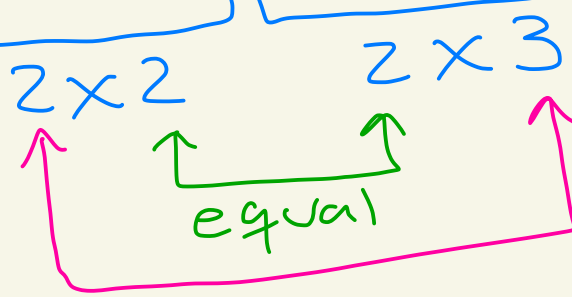


Ex: Calculate AB, if possible,

Where

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$



AB is 2x3

(row 1 A) · (col 1 B)      (row 1 A) · (col 2 B)      (row 1 A) · (col 3 B)

$$\left( \begin{array}{ccc} (1 \ 2) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (1 \ 2) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} & (1 \ 2) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{array} \right)$$

$$\left( \begin{array}{ccc} (-1 \ 0) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (-1 \ 0) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} & (-1 \ 0) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{array} \right)$$

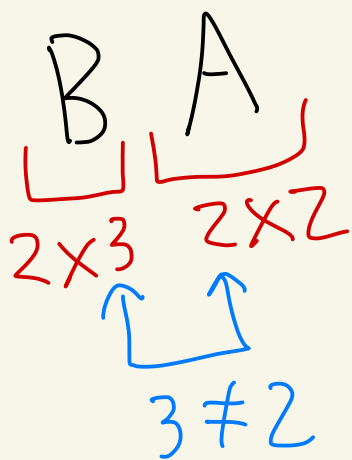
(row 2 A) · (col 1 B)      (row 2 A) · (col 2 B)      (row 2 A) · (col 3 B)

AB =

$$= \begin{pmatrix} (1)(1) + (2)(0) & (1)(2) + (2)(1) & (1)(-1) + (2)(0) \\ (-1)(1) + (0)(0) & (-1)(2) + (0)(1) & (-1)(-1) + (0)(0) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & -1 \\ -1 & -2 & 1 \end{pmatrix}$$

Ex: What if we tried  $BA$   
with the same matrices?



Since  $3 \neq 2$   
 $BA$  is  
undefined.

Let's see why this doesn't work. Let's try to compute BA as above.

$$BA = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$$

(row 1 of B) ·  
(col 1 of A)

$$= \begin{pmatrix} (1 \ 2 \ -1) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} & \text{~~~~~} \\ \text{~~~~~} & \text{~~~~~} \end{pmatrix}$$

not the same size. Can't do this dot product

Ex: Let  $A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$  and  $B = (0 \ 1 \ -3)$

Calculate  $AB$  if possible.

$3 \times 1$     $1 \times 3$

same

$AB$  is  $3 \times 3$

(row 1 A) ·  
(col 1 B)

$$(1) \cdot (0)$$

(row 1 A) ·  
(col 2 B)

$$(1) \cdot (1)$$

(row 1 A) ·  
(col 3 B)

$$(1) \cdot (-3)$$

(row 2 A) ·  
(col 1 B)

$$(2) \cdot (0)$$

(row 2 A) ·  
(col 2 B)

$$(2) \cdot (1)$$

(row 2 A) ·  
(col 3 B)

$$(2) \cdot (-3)$$

(row 3 A) ·  
(col 1 B)

$$(-1) \cdot (0)$$

(row 3 A) ·  
(col 2 B)

$$(-1) \cdot (1)$$

(row 3 A) ·  
(col 3 B)

$$(-1) \cdot (-3)$$

$AB =$



$$= \begin{pmatrix} 0 & 1 & -3 \\ 0 & 2 & -6 \\ 0 & -1 & 3 \end{pmatrix}$$

---

Ex: Can we calculate  $BA$   
Where  $B = (0 \ 1 \ -3)$  and  $A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$   
as above?

$$BA = (0 \ 1 \ -3) \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$1 \times 3$        $3 \times 1$   
same  
 $BA$  is  $1 \times 1$

$$\begin{array}{c} \text{(row 1 B)} \cdot \\ \text{(col 1 A)} \end{array} \left( (0 \ 1 \ -3) \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \right)$$

$$= (0)(1) + (1)(2) + (-3)(-1)$$

$$= (5)$$

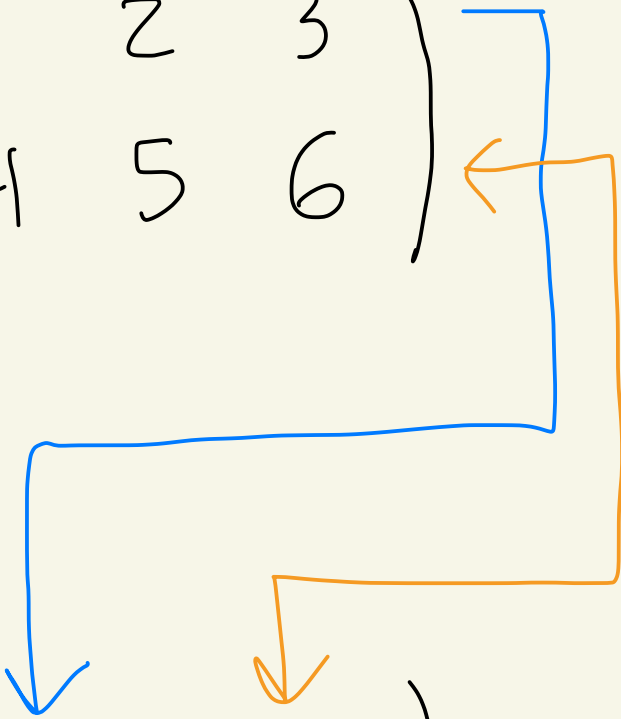
Note: In the above examples we saw that  $AB \neq BA$ .

In general,  $AB = BA$  is not always true for matrices

Def: Let  $A$  be an  $m \times n$  matrix. The transpose of  $A$ , denoted by  $A^T$ , is defined to be the  $n \times m$  matrix that results from interchanging the rows and columns of  $A$ . That is, the  $i$ -th row of  $A^T$  is the  $i$ -th column of  $A$ . Or, the  $j$ -th column of  $A^T$  is the  $j$ -th row of  $A$ .

Some people write  $A^t$  instead of  $A^T$

Ex:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$


A is  
 $2 \times 3$

$$A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

$A^T$  is  
 $3 \times 2$

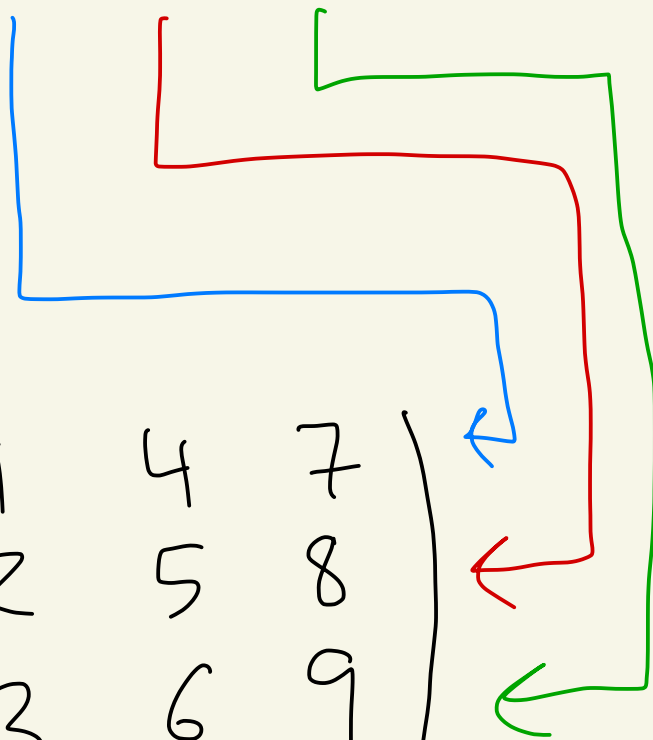
Ex:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

A is  
3x3

$$A^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

$A^T$  is  
3x3



Def: The  $m \times n$  zero matrix

is the  $m \times n$  matrix where every entry is zero.

We denote this matrix

by  $O_{m \times n}$  or just by  $O$

if you don't want to mention the size.

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Ex:  $O_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$O_{3 \times 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$O_{1 \times 6} = (0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

Ex: Let  $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ .

Then

$$A + O_{2 \times 2} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} = A$$

$$O_{2 \times 2} + A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} = A$$

So,  $A + O_{2 \times 2} = A$

and  $O_{2 \times 2} + A = A$ .

} So the zero matrix is like the number zero but in the land of matrices.