Math 2550-03

2/6/24

Ex:

$$\begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 1+2 & 2+3 \\ 0+4 & -3+5 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 5 \\ 4 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \\ -1 & -3 \end{pmatrix} - \begin{pmatrix} 2 & 3 \\ -1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-2 & 0-3 \\ 2-(-1) & 1-(-2) \\ -1-0 & -3-1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -3 \\ 3 & 3 \\ -1 & -4 \end{pmatrix}$$

$$(-2)\begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} (-2)(1) & (-2)(2) \\ (-2)(3) & (-2)(-4) \end{pmatrix}$$

$$= \begin{pmatrix} -2 & -4 \\ -6 & 8 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}}_{2\times 2} + \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 5 & -1 \end{pmatrix}}_{3\times 2}$$

Since the matrices are not the same size we say the addition is undefined

---

Def: Let A be an $m \times r$ matrix and B be an $r \times n$ matrix. We define the **product** of A and B, denoted by AB, to be the $m \times n$ matrix C whose entry in row i and column j equals the dot product of the i-th row of A and the j-th column of B.

$$\underbrace{C}_{m \times n} = \underbrace{A}_{m \times r}\ \underbrace{B}_{r \times n}$$

equal

Ex: Calculate AB, if possible, where

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$AB = \underbrace{\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}}_{2\times 2}\underbrace{\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}}_{2\times 3}$$

equal

AB is 2×3

$$AB = \begin{pmatrix} \overbrace{\begin{pmatrix} 1 & 2 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix}}^{(\text{row } 1\ A)\cdot(\text{col } 1\ B)} & \overbrace{\begin{pmatrix} 1 & 2 \end{pmatrix}\cdot\begin{pmatrix} 2 \\ 1 \end{pmatrix}}^{(\text{row } 1\ A)\cdot(\text{col } 2\ B)} & \overbrace{\begin{pmatrix} 1 & 2 \end{pmatrix}\cdot\begin{pmatrix} -1 \\ 0 \end{pmatrix}}^{(\text{row } 1\ A)\cdot(\text{col } 3\ B)} \\ \\ \underbrace{\begin{pmatrix} -1 & 0 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{(\text{row } 2\ A)\cdot(\text{col } 1\ B)} & \underbrace{\begin{pmatrix} -1 & 0 \end{pmatrix}\cdot\begin{pmatrix} 2 \\ 1 \end{pmatrix}}_{(\text{row } 2\ A)\cdot(\text{col } 2\ B)} & \underbrace{\begin{pmatrix} -1 & 0 \end{pmatrix}\cdot\begin{pmatrix} -1 \\ 0 \end{pmatrix}}_{(\text{row } 2\ A)\cdot(\text{col } 3\ B)} \end{pmatrix}$$

$$= \begin{pmatrix} (1)(1)+(2)(0) & (1)(2)+(2)(1) & (1)(-1)+(2)(0) \\ (-1)(1)+(0)(0) & (-1)(2)+(0)(1) & (-1)(-1)+(0)(0) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & -1 \\ -1 & -2 & 1 \end{pmatrix}$$

<u>Ex:</u> What if we tried BA with the same matrices?

$$\underbrace{B}_{2\times 3}\ \underbrace{A}_{2\times 2}$$

$$3 \neq 2$$

Since $3 \neq 2$, BA is undefined.

Let's see why this doesn't work. Let's try to compute BA as above.

$$BA = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \overbrace{(1\ 2\ -1)\cdot\begin{pmatrix} 1 \\ -1 \end{pmatrix}}^{\substack{(\text{row } 1 \text{ of } B)\cdot \\ (\text{col } 1 \text{ of } A)}} & \\ & \end{pmatrix}$$

not the same size. Can't do this dot product

<u>Ex:</u> Let $A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & -3 \end{pmatrix}$

Calculate $\underbrace{A}_{3\times 1}\ \underbrace{B}_{1\times 3}$ if possible.

same

AB is 3×3

$$AB = \begin{pmatrix} \overbrace{(1)\cdot(0)}^{(\text{row } 1\ A)\cdot(\text{col } 1\ B)} & \overbrace{(1)\cdot(1)}^{(\text{row } 1\ A)\cdot(\text{col } 2\ B)} & \overbrace{(1)\cdot(-3)}^{(\text{row } 1\ A)\cdot(\text{col } 3\ B)} \\ \overbrace{(2)\cdot(0)}^{(\text{row } 2\ A)\cdot(\text{col } 1\ B)} & \overbrace{(2)\cdot(1)}^{(\text{row } 2\ A)\cdot(\text{col } 2\ B)} & \overbrace{(2)\cdot(-3)}^{(\text{row } 2\ A)\cdot(\text{col } 3\ B)} \\ \overbrace{(-1)\cdot(0)}^{(\text{row } 3\ A)\cdot(\text{col } 1\ B)} & \overbrace{(-1)\cdot(1)}^{(\text{row } 3\ A)\cdot(\text{col } 2\ B)} & \overbrace{(-1)\cdot(-3)}^{(\text{row } 3\ A)\cdot(\text{col } 2\ B)} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & -3 \\ 0 & 2 & -6 \\ 0 & -1 & 3 \end{pmatrix}$$

<u>Ex:</u> Can we calculate BA where $B = \begin{pmatrix} 0 & 1 & -3 \end{pmatrix}$ and $A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ as above?

$$BA = \underbrace{\begin{pmatrix} 0 & 1 & -3 \end{pmatrix}}_{1 \times 3} \underbrace{\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}}_{3 \times 1}$$

same

BA is 1×1

$$= \left(\overbrace{\begin{pmatrix}0 & 1 & -3\end{pmatrix} \cdot \begin{pmatrix}1\\2\\-1\end{pmatrix}}^{(\text{row } 1\ B)\cdot(\text{col } 1\ A)} \right)$$

$$= \left((0)(1)+(1)(2)+(-3)(-1) \right)$$

$$= (5)$$

Note: In the above examples we saw that $AB \neq BA$.

In general, $AB = BA$ is <u>not</u> always true for matrices

Def: Let A be an $m \times n$ matrix. The **transpose** of A, denoted by A^T, is defined to be the $n \times m$ matrix that results from interchanging the rows and columns of A. That is, the i-th row of A^T is the i-th column of A. Or, the j-th column of A^T is the j-th row of A.

Some people write A^t instead of A^T

Ex:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

A is 2x3

$$A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

A^T is 3x2

Ex:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

$\boxed{A \text{ is } 3 \times 3}$

$$A^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

$\boxed{A^T \text{ is } 3 \times 3}$

Def: The $m\times n$ **zero matrix** is the $m\times n$ matrix where every entry is zero. We denote this matrix by $0_{m\times n}$ or just by 0 if you don't want to mention the size.

Ex: $0_{2\times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$0_{3\times 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$0_{1\times 6} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Ex: Let $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$.

Then

$$A + 0_{2\times 2} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} = A$$

$$0_{2\times 2} + A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} = A$$

So, $A + 0_{2\times 2} = A$

and $0_{2\times 2} + A = A$.

So the zero matrix is like the number zero but in the land of matrices.