

two Test 2 is in 4123. Weeks. Tuesday Review day is Thursday 4/18.

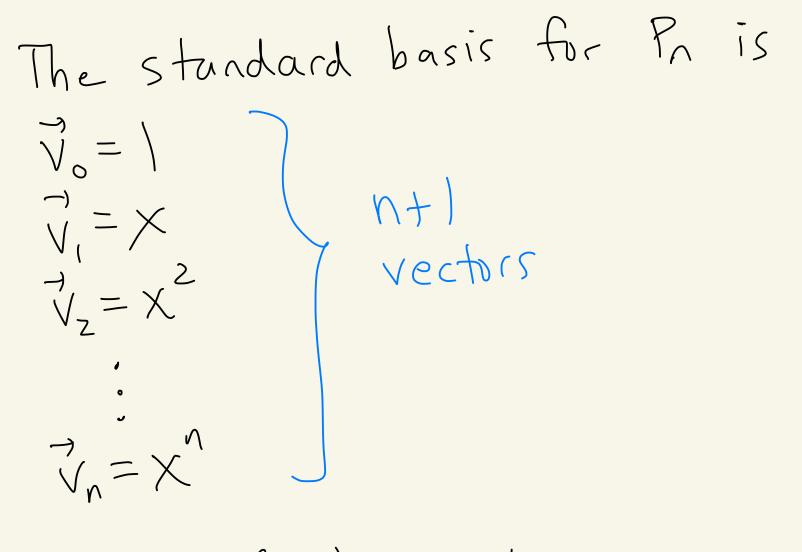
Special example:
The "trivial" vector space is

$$V = \{ \vec{O} \}$$
 the vector space
with just one
vector
It turns out that V
has no basis.
We just define its
dimension to be O.

Ex: Let V = TR'' and F = IR, The standard basis for IRⁿ is V, V2, Where Vi has a 1 in spot i and O's everywhere else. This gives $\dim(\mathbb{R}^n) = n$. dim (IRn) standard basis for IRn $Z = \langle 1, 0 \rangle, \quad \forall_2 = \langle 0, 1 \rangle$ 2 3 $\vec{v}_1 = \langle 1, 0, 0 \rangle, \quad \vec{v}_2 = \langle 0, 1, 0 \rangle$ 3 $\vec{v}_{\lambda} = \langle 0, 0, 1 \rangle$ $\begin{array}{c|c} Y_{1} = \langle 1, 0, 0, 0 \rangle \\ Y_{1} = \langle 0, 0, 0 \rangle \\ Y_{2} = \langle 0, 1, 0, 0 \rangle \end{array}$ 4

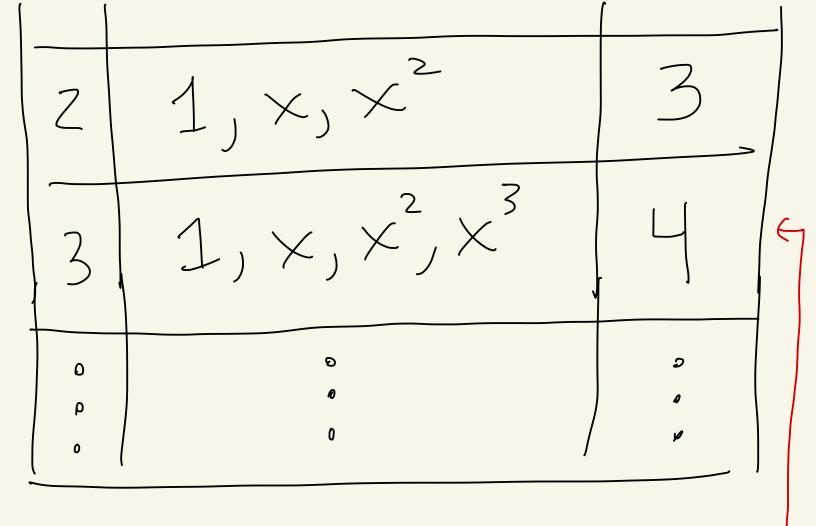
$$E_{X}: Let n be an integer
with $n \ge 0$. Then

$$P_n = \sum_{a_0} a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n | a_{o_1 \dots, a_n}^{a_0, \dots, a_n}$$$$



So, $dim(P_n) = n+1$

n	standard basis for Pr	$dim(P_n)$
0	1	
	1, ×	2



Ex: $x - x^{3} = 5 \cdot 1 + 1 \cdot x + 0 \cdot x^{2} - 1 \cdot x^{3}$

Ex: Let
$$V = M_{2,2}$$

F = R .
Note:
 $\begin{pmatrix} 1 & 2 \\ -1 & 5 \end{pmatrix} = l \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + 2 \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + (-1) \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + 5 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
One can show [Hw 7-Part 1]
that
 $T_{1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad V_{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$
 $V_{3} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad V_{4} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
is a basis for $M_{2,2}$.
Thus, dim $(M_{2,2}) = U$

Theorem: Let V be a finitedimensional vector space over a field F with dim (V)=n. So, V has a basis with J Let WIJWZJUJ Win be vectors from V. OIF M<N, then Wi, Wz j w do not Spun V. 2 If M>N, then WijWzjing Win are linearly dependent.

Ex: Let
$$V = \mathbb{R}^3$$
, $F = \mathbb{R}$.
Then, dim $(\mathbb{R}^3) = 3 + n=3$

Let

$$\vec{w}_1 = \langle 1, 5, 0 \rangle$$

 $\vec{w}_2 = \langle 2, -1, 0 \rangle$
 $m = 2$
 $m = 2$

Ex: Let
$$V = P_2$$
, $F = R$.
We know that $\dim(P_2) = 3$
basis: $1, x, x^2$
Let
 $\overrightarrow{W}_1 = 1 + x$
 $\overrightarrow{W}_2 = 1 - x^2$
 $\overrightarrow{W}_3 = x$
 $\overrightarrow{W}_4 = x^2$
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(2) from above says since we have 4 vectors in a 3-dimensional space [4>3] then winwerwing must be lincarly dependent.

Here we have $\left| \cdot \vec{w}_1 - \left| \cdot \vec{w}_2 - \vec{w}_3 - \vec{w}_4 \right| = 0 \right|$ $|+\chi - |+\chi^2 - \chi - \chi^2 = 0$ $\overline{W}_{1} = \overline{W}_{2} + \overline{W}_{3} + \overline{W}_{4}$

Ex:
$$V = P_2$$
, $F = IR$
We know (since we showed that
 $1, x, x^2$ is a basis for P_2)
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Freviously, we showed these vectors are linearly independent. Since we have 3 lin. ind. vectors in a 3-dimensional space, they form a basis for P2.