Math 2550-01
4/18/24

Hw 7 - Part 1

(2) (b) Is $\langle 3,1,5\rangle$ in the span of $\vec{u}=\langle 0,-2,2\rangle$ and $\vec{v}=\langle 1,3,-1\rangle$? If so, write it as a lin. combo. of $\vec{u}$ and $\vec{v}$.

We want to know if we can solve

$$\langle 3,1,5\rangle = \underbrace{c_1\langle 0,-2,2\rangle + c_2\langle 1,3,-1\rangle}_{c_1\vec{u}+c_2\vec{v}}$$

we get

$$\langle 3,1,5\rangle = \langle 0,-2c_1,2c_1\rangle + \langle c_2,3c_2,-c_2\rangle$$

$$\langle 3,1,5\rangle = \langle c_2,-2c_1+3c_2,2c_1-c_2\rangle$$

This becomes

$$\boxed{\begin{aligned} c_2 &= 3 \\ -2c_1 + 3c_2 &= 1 \\ 2c_1 - c_2 &= 5 \end{aligned}}$$

$\leftarrow$ $c_2 = 3$

$\leftarrow$ $c_1 = 4,\ c_2 = 3$

$\leftarrow$ $c_1 = 4,\ c_2 = 3$

Thus,

$$\langle 3,1,5\rangle = \underbrace{4\langle 0,-2,2\rangle + 3\langle 1,3,-1\rangle}$$

$$\langle 3,1,5\rangle = 4\vec{u} + 3\vec{v}$$

So, $\langle 3,1,5\rangle$ is in the span of $\vec{u}$ and $\vec{v}$ and

$$\langle 3,1,5\rangle = 4\vec{u} + 3\vec{v}.$$

HW 7 - Part 1

(4) (c) Are the following vectors linearly independent or linearly dependent?

$\vec{v}_1 = \langle -3, 0, 4 \rangle$

$\vec{v}_2 = \langle 5, -1, 2 \rangle$

$\vec{v}_3 = \langle 1, 1, 3 \rangle$

We need to find the solutions to the equation

$$c_1 \vec{v}_1 + c_2 \vec{v}_2 + c_3 \vec{v}_3 = \vec{0}$$

If the only solution is $c_1 = 0, c_2 = 0, c_3 = 0$ then the vectors are linearly independent

If there are more solutions, then the vectors are linearly dependent.

Heres our equation:

$$c_1\langle -3,0,4\rangle + c_2\langle 5,-1,2\rangle + c_3\langle 1,1,3\rangle = \langle 0,0,0\rangle$$

we get

$$\langle -3c_1, 0, 4c_1\rangle + \langle 5c_2, -c_2, 2c_2\rangle + \langle c_3, c_3, 3c_3\rangle = \langle 0,0,0\rangle$$

which gives

$$\langle -3c_1+5c_2+c_3,\ -c_2+c_3,\ 4c_1+2c_2+3c_3\rangle = \langle 0,0,0\rangle$$

This gives

$$\boxed{\begin{aligned} -3c_1+5c_2+c_3&=0\\ -c_2+c_3&=0\\ 4c_1+2c_2+3c_3&=0 \end{aligned}}$$

$$\left(\begin{array}{ccc|c} -3 & 5 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 4 & 2 & 3 & 0 \end{array}\right) \xrightarrow{R_3 + R_1 \to R_1} \left(\begin{array}{ccc|c} 1 & 7 & 4 & 0 \\ 0 & -1 & 1 & 0 \\ 4 & 2 & 3 & 0 \end{array}\right)$$

$$\xrightarrow{-4R_1 + R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 7 & 4 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -26 & -13 & 0 \end{array}\right)$$

$$\xrightarrow{-R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & 7 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -26 & -13 & 0 \end{array}\right)$$

$$\xrightarrow{26R_2 + R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 7 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -39 & 0 \end{array}\right)$$

$$\xrightarrow{-\frac{1}{39}R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 7 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

This gives:

$$\boxed{\begin{aligned} c_1 + 7c_2 + 4c_3 &= 0 \quad (1) \\ c_2 - c_3 &= 0 \quad (2) \\ c_3 &= 0 \quad (3) \end{aligned}}$$

no free variables

Solving:

(3) $c_3 = 0$

(2) $c_2 = c_3 = 0$

(1) $c_1 = -7c_2 - 4c_3 = -7(0) - 4(0) = 0$

Thus the only sol. to

$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$$

is $c_1 = 0$, $c_2 = 0$, $c_3 = 0$.

Thus, $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent.

$\boxed{\text{Hw 6 – Part 2}}$

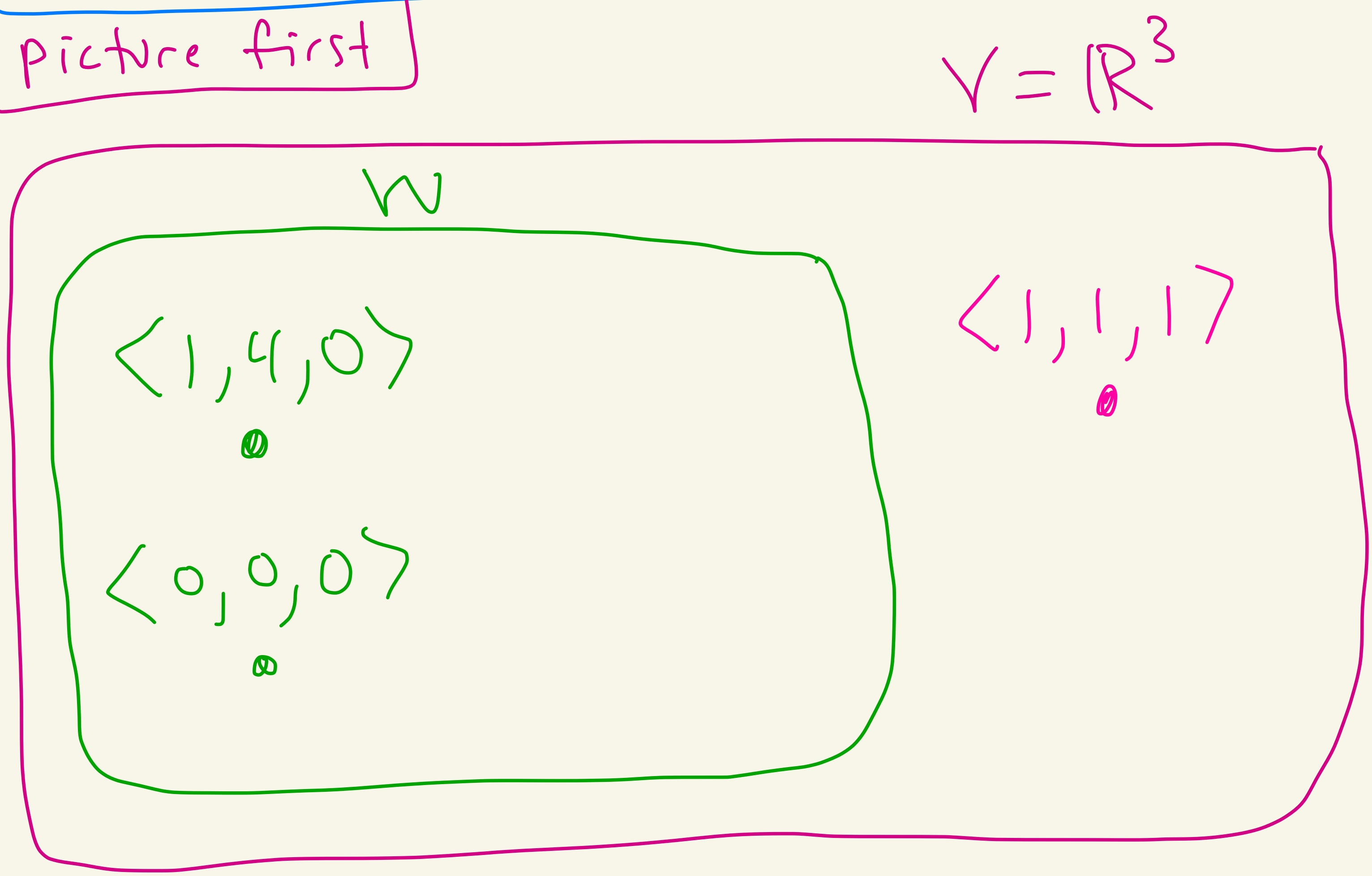
Let $V = \mathbb{R}^3$, $F = \mathbb{R}$.

Let

$$W = \{ \langle a,b,c \rangle \mid 4a - b + 2c = 0,\ a,b,c \in \mathbb{R} \}$$

Show that W is a subspace of $\mathbb{R}^3$.

$\boxed{\text{picture first}}$



$\langle 1,4,0 \rangle$ is in W since $4(1) - 4 + 2(0) = 0$

$\langle 0,0,0 \rangle$ is in W since $4(0) - 0 + 2(0) = 0$

$\langle 1,1,1\rangle$ is not in W since $4(1)-1+2(1) \neq 0$

Proof that W is a subspace:

Let's write W this way

$$W=\{\langle a,b,c\rangle \mid \boxed{b=4a+2c},\ a,b,c\in\mathbb{R}\}$$

① (zero vector)

Let $a=0, b=0, c=0$

Then, $b=4a+2c$ is true.

Thus, $\vec{0}=\langle 0,0,0\rangle$ is in W.

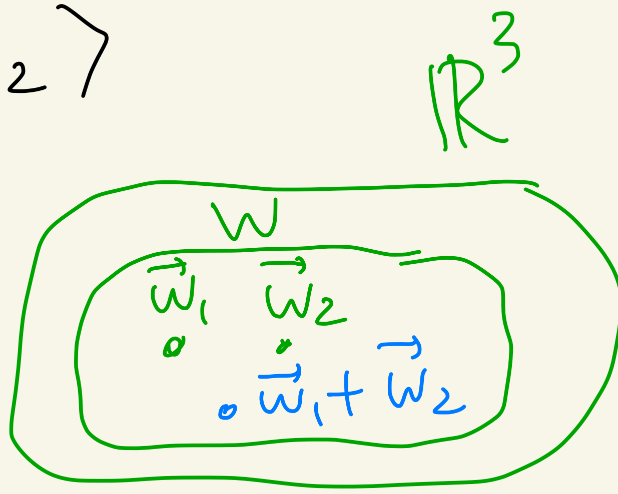
② (closed under +)

Let $\vec{w}_1=\langle a_1,b_1,c_1\rangle$

and $\vec{w}_2=\langle a_2,b_2,c_2\rangle$

both be in W.

Then, $b_1=4a_1+2c_1$



and $\boxed{b_2 = 4a_2 + 2c_2.}$

Then,

$\vec{w}_1 + \vec{w}_2 = \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$

and

$$(b_1 + b_2) = 4a_1 + 2c_1 + 4a_2 + 2c_2$$
$$= 4(a_1 + a_2) + 2(c_1 + c_2).$$

So, $\vec{w}_1 + \vec{w}_2$ is in W.

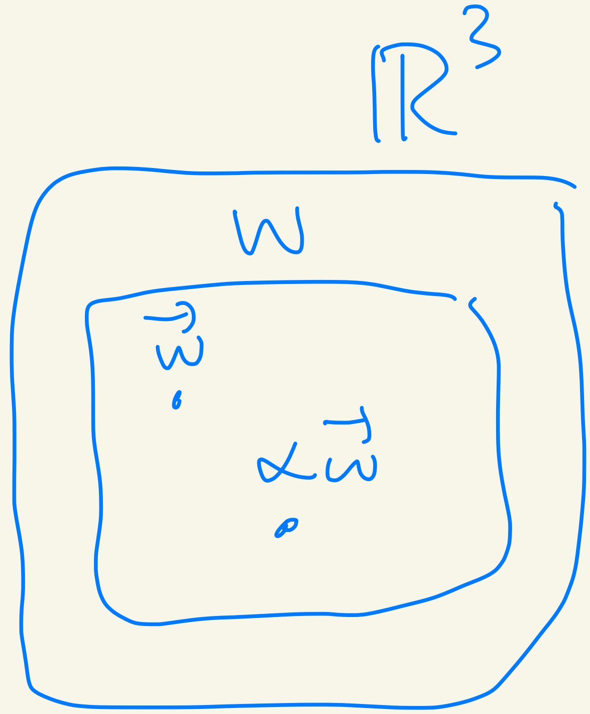
3 (closed under scaling)

Let $\vec{w} = \langle a, b, c \rangle$ be in W
and α be a scalar/number.

Then, $\boxed{b = 4a + 2c}$

We have

$$\alpha\vec{w} = \langle \alpha a, \alpha b, \alpha c \rangle$$



and

$$\alpha b = \alpha(4a+2c)$$
$$= 4(\alpha a) + 2(\alpha c).$$

So, $\alpha\vec{w}$ is in W.

By ①, ②, ③ W is a subspace.

$\blacksquare$

HW 4

(3) (c) Find the inverse of $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if it exists.

$$\left(\begin{array}{ccc|ccc} \multicolumn{3}{c|}{\overbrace{\hphantom{1 \quad 0 \quad 1}}^{A}} & \multicolumn{3}{c}{\overbrace{\hphantom{1 \quad 0 \quad 0}}^{I_3}} \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{-R_1 + R_3 \to R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{-R_2 + R_3 \to R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{-\frac{1}{2}R_3 \to R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{array}\right)$$

$$\xrightarrow[-R_3+R_2 \to R_2]{-R_3+R_1 \to R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{array}\right)$$

$$\underbrace{}_{I_3} \quad \underbrace{\phantom{A^{-1} A^{-1}}}_{A^{-1}}$$

So,

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Ex: Solve

$$\boxed{\begin{aligned} x \quad\quad\; &+ z = 1 \\ y &+ z = -2 \\ x + y \quad\; &\;\;\; = 4 \end{aligned}}$$

by inverting the coefficent matrix

The above system can be written

$$\underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$$

$$\cancel{\underbrace{\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}}_{A^{-1}} \underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_{A}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}}_{A^{-1}} \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (\frac{1}{2})(1) + (-\frac{1}{2})(-2) + (\frac{1}{2})(4) \\ (-1/2)(1) + (\frac{1}{2})(-2) + (\frac{1}{2})(4) \\ (\frac{1}{2})(1) + (\frac{1}{2})(-2) + (-\frac{1}{2})(4) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} + 1 + 2 \\ -\frac{1}{2} - 1 + 2 \\ \frac{1}{2} - 1 - 2 \end{pmatrix} = \begin{pmatrix} 3.5 \\ 0.5 \\ -2.5 \end{pmatrix}$$

So, $x = 3.5$, $y = 0.5$, $z = -2.5$

is the solution.