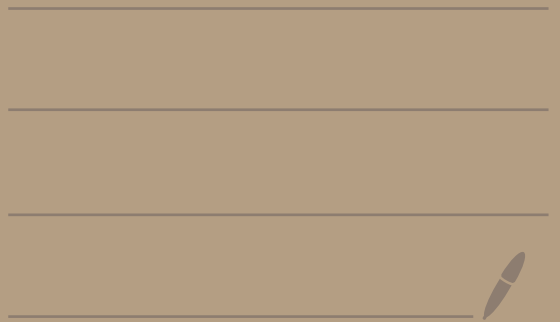


Math 2550-01

3/7/24

~



• Test 1 next Tuesday

• Today is review

HW 1 - Part 1

⑤ Compute $\frac{1}{2}\vec{u} - \vec{v}$ where

$$\vec{u} = \langle 2, 0, 8, -4, 10 \rangle$$

$$\vec{v} = \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$$

$$\begin{aligned}\frac{1}{2}\vec{u} - \vec{v} &= \frac{1}{2}\langle 2, 0, 8, -4, 10 \rangle \\ &\quad - \langle 0, \frac{1}{2}, 3, 10, -1 \rangle \\ &= \langle 1, 0, 4, -2, 5 \rangle \\ &\quad - \langle 0, \frac{1}{2}, 3, 10, -1 \rangle \\ &= \langle 1, -\frac{1}{2}, 1, -12, 6 \rangle\end{aligned}$$

$$\begin{aligned}\vec{u} \cdot \vec{v} &= \langle 2, 0, 8, -4, 10 \rangle \cdot \langle 0, \frac{1}{2}, 3, 10, -1 \rangle \\ &= (2)(0) + (0)(\frac{1}{2}) + (8)(3) \\ &\quad + (-4)(10) + (10)(-1)\end{aligned}$$

$$\begin{aligned} &= 0 + 0 + 24 - 40 - 10 \\ &= -26 \end{aligned}$$

⑧ List 2 elements from

$$S = \{ t \langle 3, 17 \rangle + s \langle -1, 5 \rangle \mid s, t \in \mathbb{R} \}$$

$s=10, t=3:$

$$\begin{aligned} 3 \langle 3, 17 \rangle + 10 \langle -1, 5 \rangle &= \langle 9, 3 \rangle + \langle -10, 50 \rangle \\ &= \langle -1, 53 \rangle \end{aligned}$$

So, $\langle -1, 53 \rangle$ is in S .

$s=0, t=0:$

$$\begin{aligned} 0 \langle 3, 17 \rangle + 0 \langle -1, 5 \rangle &= \langle 0, 0 \rangle + \langle 0, 0 \rangle \\ &= \langle 0, 0 \rangle \end{aligned}$$

So, $\langle 0, 0 \rangle$ is in S .

④ (a) $\vec{v} = \langle 1, 5, -1, 0, 3 \rangle$

Find norm / length.

$$\begin{aligned}\|\vec{v}\| &= \sqrt{(1)^2 + (5)^2 + (-1)^2 + (0)^2 + (3)^2} \\ &= \sqrt{1 + 25 + 1 + 9} \\ &= \sqrt{36} = 6\end{aligned}$$

HW 1 - Part 2

① (d) (modified to \mathbb{R}^3)

Let $\vec{u}, \vec{v} \in \mathbb{R}^3$ and $\alpha \in \mathbb{R}$.

Prove: $\alpha(\vec{u} \cdot \vec{v}) = (\alpha\vec{u}) \cdot \vec{v}$.

Proof:

Let $\vec{u}, \vec{v} \in \mathbb{R}^3$ and $\alpha \in \mathbb{R}$.

Then,

$$\vec{u} = \langle a, b, c \rangle$$

$$\text{and } \vec{v} = \langle d, e, f \rangle$$

where $a, b, c, d, e, f \in \mathbb{R}$.

Then,

$$\alpha(\vec{u} \cdot \vec{v}) = \alpha(\langle a, b, c \rangle \cdot \langle d, e, f \rangle)$$

$$= \alpha(ad + be + cf)$$

$$= \alpha ad + \alpha be + \alpha cf$$

Also,

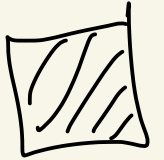
$$(\alpha \vec{u}) \cdot \vec{v} = (\alpha \langle a, b, c \rangle) \cdot \langle d, e, f \rangle$$

S
A
B
A
F

$$= \langle \alpha a, \alpha b, \alpha c \rangle \cdot \langle d, e, f \rangle$$

$$= \alpha a d + \alpha b e + \alpha c f$$

We see that $\alpha(\vec{u} \cdot \vec{v}) = (\alpha\vec{u}) \cdot \vec{v}$.



HW 2 - Part 1

Compute $2A + BC^T$

where $A = \begin{pmatrix} 1 & 3 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

$$C = \begin{pmatrix} 1 & 2 & 1 & 1 \\ -1 & 0 & -1 & 1 \end{pmatrix}$$

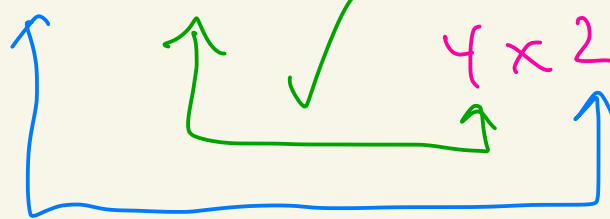
Answer: $2A + BC^T =$

$$2 \begin{pmatrix} 1 & 3 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 & 1 \\ -1 & 0 & -1 & 1 \end{pmatrix}^T$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 0 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$$

2×4

4×2



answer is 2×2

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \left(\begin{array}{cc} (1 \ 0 \ 1 \ -1) \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} & (1 \ 0 \ 1 \ -1) \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \\ (0 \ 0 \ 1 \ 1) \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} & (0 \ 0 \ 1 \ 1) \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \end{array} \right)$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1+0+1-1 & -1+0-1-1 \\ 0+0+1+1 & 0+0-1+1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & -3 \\ 2 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix}}$$

HW 2 - Part 2

① (a)

Let A, B, C be 2×2 matrices.

Prove $(B+C)A = BA + CA$

proof: Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}, C = \begin{pmatrix} i & j \\ k & l \end{pmatrix}.$$

where a, b, c, \dots, l are real #s.

Then,

$$\begin{aligned} (B+C)A &= \left[\begin{pmatrix} e & f \\ g & h \end{pmatrix} + \begin{pmatrix} i & j \\ k & l \end{pmatrix} \right] \begin{pmatrix} a & b \\ c & d \end{pmatrix} \\ &= \begin{pmatrix} e+i & f+j \\ g+k & h+l \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{aligned}$$

$$\begin{aligned}
&= \begin{pmatrix} (e+i \quad f+\bar{j}) \begin{pmatrix} a \\ c \end{pmatrix} & (e+i \quad f+\bar{j}) \begin{pmatrix} b \\ d \end{pmatrix} \\ (g+k \quad h+l) \begin{pmatrix} a \\ c \end{pmatrix} & (g+k \quad h+l) \begin{pmatrix} b \\ d \end{pmatrix} \end{pmatrix} \\
&= \begin{pmatrix} ea+\bar{i}a+fc+\bar{j}c & eb+\bar{i}b+fd+\bar{j}d \\ ga+ka+hc+lc & gb+kb+hd+ld \end{pmatrix}
\end{aligned}$$

Also,

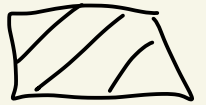
$$BA+CA = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} \bar{i} & \bar{j} \\ k & l \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} ea+fc & eb+fd \\ ga+hc & gb+hd \end{pmatrix}$$

$$+ \begin{pmatrix} \bar{i}a+\bar{j}c & \bar{i}b+\bar{j}d \\ ka+lc & kb+ld \end{pmatrix}$$

$$= \begin{pmatrix} ea+fc+ia+jc & eb+fd+ib+jd \\ ga+hc+ka+lc & gb+hd+kb+ld \end{pmatrix}$$

Comparing the two calculations above we see that $(B+C)A = BA+CA$



HW 3

① (c) Solve

$$x - y + 2z - w = -1$$

$$2x + y - 2z - 2w = -2$$

$$-x + 2y - 4z + w = 1$$

$$3x \qquad \qquad \qquad -3w = -3$$

already 1 ✓

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array} \right)$$

make 0's

make 1

$$\begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ \xrightarrow{\hspace{2cm}} \\ R_1 + R_3 \rightarrow R_3 \\ -3R_1 + R_4 \rightarrow R_4 \end{array} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array} \right)$$

$$R_2 \leftrightarrow R_3 \rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array} \right)$$

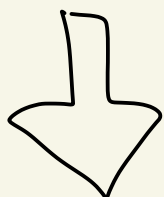
← make 0

$$\begin{array}{l} -3R_2 + R_3 \rightarrow R_3 \\ -3R_2 + R_4 \rightarrow R_4 \end{array} \rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{array}{rcl} x - y + 2z - w & = & -1 \\ y - 2z & = & 0 \\ 0 & = & 0 \\ 0 & = & 0 \end{array}$$

leading
x, y

free
z, w



$$x = -1 + y - 2z + w$$

①

$$y = 2z$$

②

$$z = s$$

③

$$w = t$$

④

④ $w = t$

③ $z = s$

② $y = 2z = 2s$

① $x = -1 + y - 2z + w = -1 + 2s - 2s + t = -1 + t$

Answer:

$$x = -1 + t$$

$$y = 2s$$

$$z = s$$

$$w = t$$

where s, t
can be any
real #'s