Math 2550-01

3/7/24

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- Test 1 next Tuesday
- Today is review

$\boxed{\text{HW 1 - Part 1}}$

5) Compute $\frac{1}{2}\vec{u} - \vec{v}$ where

$\vec{u} = \langle 2, 0, 8, -4, 10 \rangle$

$\vec{v} = \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$

$$\frac{1}{2}\vec{u} - \vec{v} = \frac{1}{2}\langle 2, 0, 8, -4, 10 \rangle - \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$$

$$= \langle 1, 0, 4, -2, 5 \rangle - \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$$

$$= \langle 1, -\frac{1}{2}, 1, -12, 6 \rangle$$

$$\vec{u} \cdot \vec{v} = \langle 2, 0, 8, -4, 10 \rangle \cdot \langle 0, \frac{1}{2}, 3, 10, -1 \rangle$$

$$= (2)(0) + (0)(\tfrac{1}{2}) + (8)(3) + (-4)(10) + (10)(-1)$$

$= 0 + 0 + 24 - 40 - 10$

$= -26$

⑧ List 2 elements from

$S = \{ t\langle 3,1\rangle + s\langle -1,5\rangle \mid s,t \in \mathbb{R} \}$

$s=10,\ t=3$:

$3\langle 3,1\rangle + 10\langle -1,5\rangle = \langle 9,3\rangle + \langle -10,50\rangle$

$= \langle -1,53\rangle$

So, $\boxed{\langle -1,53\rangle}$ is in S.

$s=0,\ t=0$:

$0\langle 3,1\rangle + 0\langle -1,5\rangle = \langle 0,0\rangle + \langle 0,0\rangle$

$= \langle 0,0\rangle$

So, $\boxed{\langle 0, 0 \rangle}$ is in S.

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④ (a) $\vec{v} = \langle 1, 5, -1, 0, 3 \rangle$

Find norm/length.

$$\|\vec{v}\| = \sqrt{(1)^2 + (5)^2 + (-1)^2 + (0)^2 + (3)^2}$$

$$= \sqrt{1 + 25 + 1 + 9}$$

$$= \sqrt{36} = 6$$

HW 1 - Part 2

① (d) (modified to $\mathbb{R}^3$)

Let $\vec{u}, \vec{v} \in \mathbb{R}^3$ and $\alpha \in \mathbb{R}$.

Prove: $\alpha(\vec{u} \cdot \vec{v}) = (\alpha \vec{u}) \cdot \vec{v}$.

Proof:

Let $\vec{u}, \vec{v} \in \mathbb{R}^3$ and $\alpha \in \mathbb{R}$.

Then,

$$\vec{u} = \langle a, b, c \rangle$$

and $$\vec{v} = \langle d, e, f \rangle$$

where $a, b, c, d, e, f \in \mathbb{R}$.

Then,

$$\alpha(\vec{u} \cdot \vec{v}) = \alpha(\langle a, b, c \rangle \cdot \langle d, e, f \rangle)$$

$$= \alpha(ad + be + cf)$$

$$= \alpha ad + \alpha be + \alpha cf$$

Also,

$$(\alpha \vec{u}) \cdot \vec{v} = (\alpha \langle a, b, c \rangle) \cdot \langle d, e, f \rangle$$

SAME

$$= \langle \alpha a, \alpha b, \alpha c \rangle \cdot \langle d, e, f \rangle$$

$$= \alpha a d + \alpha b e + \alpha c f$$

We see that $\alpha(\vec{u} \cdot \vec{v}) = (\alpha \vec{u}) \cdot \vec{v}$.

$\blacksquare$

HW 2 ~ Part 1

Compute $2A + BC^T$

where $A = \begin{pmatrix} 1 & 3 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

$$C = \begin{pmatrix} 1 & 2 & 1 & 1 \\ -1 & 0 & -1 & 1 \end{pmatrix}$$

<u>Answer:</u> $2A + BC^T =$

$$2\begin{pmatrix} 1 & 3 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 1 & 1 \\ -1 & 0 & -1 & 1 \end{pmatrix}^T$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \underbrace{\begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}}_{2 \times 4}\underbrace{\begin{pmatrix} 1 & -1 \\ 2 & 0 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}}_{4 \times 2}$$

✓

answer is 2×2

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} \begin{pmatrix} 1 & 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1+0+1-1 & -1+0-1-1 \\ 0+0+1+1 & 0+0-1+1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & -3 \\ 2 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix}}$$

HW 2 - Part 2

1 (a)

Let A, B, C be 2×2 matrices.
Prove $(B+C)A = BA + CA$

proof: Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}, C = \begin{pmatrix} i & j \\ k & l \end{pmatrix}.$$

where $a, b, c, \ldots, l$ are real #s.

Then,

$$(B+C)A = \left[\begin{pmatrix} e & f \\ g & h \end{pmatrix} + \begin{pmatrix} i & j \\ k & l \end{pmatrix}\right]\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} e+i & f+j \\ g+k & h+l \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} (e+i \quad f+j)\begin{pmatrix} a \\ c \end{pmatrix} & (e+i \quad f+j)\begin{pmatrix} b \\ d \end{pmatrix} \\ (g+k \quad h+l)\begin{pmatrix} a \\ c \end{pmatrix} & (g+k \quad h+l)\begin{pmatrix} b \\ d \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} ea+ia+fc+jc & eb+ib+fd+jd \\ ga+ka+hc+lc & gb+kb+hd+ld \end{pmatrix}$$

Also,

$$BA+CA = \begin{pmatrix} e & f \\ g & h \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} i & j \\ k & l \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} ea+fc & eb+fd \\ ga+hc & gb+hd \end{pmatrix}$$

$$+ \begin{pmatrix} ia+jc & ib+jd \\ ka+lc & kb+ld \end{pmatrix}$$

$$= \begin{pmatrix} ea+fc+ia+jc & eb+fd+ib+jd \\ ga+hc+ka+lc & gb+hd+kb+ld \end{pmatrix}$$

Comparing the two calculations above we see that $(B+C)A = BA+CA$

$\blacksquare$

HW 3

① (c) Solve

$$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\qquad - 3w &= -3 \end{aligned}$$

already 1 ✓

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right)$$

make 0's

make 1

$$\xrightarrow[\substack{R_1 + R_3 \to R_3 \\ -3R_1 + R_4 \to R_4}]{-2R_1 + R_2 \to R_2} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right)$$

make 0

$$\xrightarrow[-3R_2+R_4 \to R_4]{-3R_2+R_3 \to R_3} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\boxed{\begin{aligned} x - y + 2z - w &= -1 \\ y - 2z &= 0 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}}$$

leading
x, y

free
z, w

$\Downarrow$

$$x = -1 + y - 2z + w \quad (1)$$
$$y = 2z \quad (2)$$
$$z = s \quad (3)$$
$$w = t \quad (4)$$

(4) $w = t$

(3) $z = s$

(2) $y = 2z = 2s$

(1) $x = -1 + y - 2z + w = -1 + 2s - 2s + t$

$= -1 + t$

Answer:

$x = -1 + t$

$y = 2s$

$z = s$

$w = t$

where s, t can be any real #'s