

Math 2550-01

216124

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Ex:

$$\begin{pmatrix} 1 & -1 \\ 5 & 3 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & -5 \end{pmatrix} = \begin{pmatrix} 1+0 & -1+2 \\ 5+1 & 3-5 \end{pmatrix} \\ = \begin{pmatrix} 1 & 1 \\ 6 & -2 \end{pmatrix}$$

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$$\begin{pmatrix} 5 & 0 \\ 3 & 1 \\ 1 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ -5 & -2 \end{pmatrix} = \begin{pmatrix} 5-1 & 0-2 \\ 3-3 & 1-4 \\ 1-(-5) & -1-(-2) \end{pmatrix} \\ = \begin{pmatrix} 4 & -2 \\ 0 & -3 \\ 6 & 1 \end{pmatrix}$$

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$$(-2) \begin{pmatrix} 3 & 2 \\ 0 & -1 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} (-2)(3) & (-2)(2) \\ (-2)(0) & (-2)(-1) \\ (-2)(5) & (-2)(-3) \end{pmatrix} = \begin{pmatrix} -6 & -4 \\ 0 & 2 \\ -10 & 6 \end{pmatrix}$$

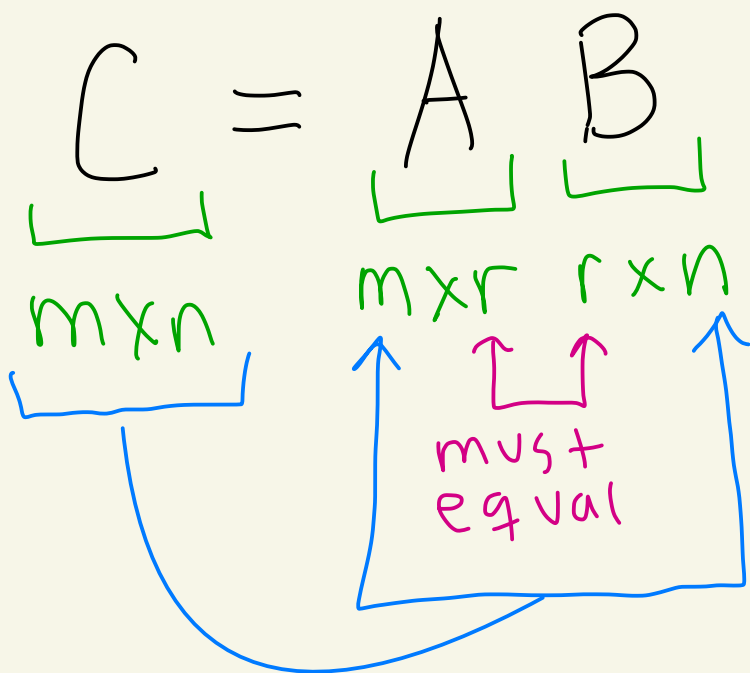
$$\begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 5 & 3 \end{pmatrix}$$

$2 \times 2$                        $3 \times 2$

is undefined  
since the  
matrices  
don't have  
the same  
size

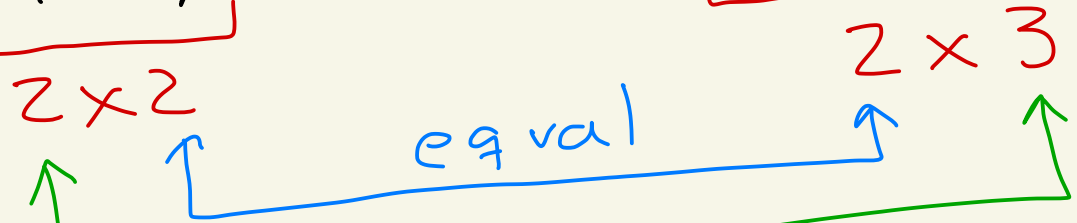
To be able to add or subtract two matrices, both matrices have to be exactly the same size. Otherwise, the operation is undefined.

Def: Let  $A$  be an  $m \times r$  matrix and let  $B$  be an  $r \times n$  matrix. We define the product of  $A$  and  $B$ , denoted by  $AB$ , as the  $m \times n$  matrix  $C$  whose entry at row  $\bar{i}$  and column  $\bar{j}$  is defined to be the dot product of row  $\bar{i}$  of  $A$  with column  $\bar{j}$  of  $B$ .



Ex: Calculate  $AB$ , if possible, where

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$



$AB$  will be  $2 \times 3$

(row 1 A) ·  
(col 1 B)

$$(1 \ 2) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(row 1 A) ·  
(col 2 B)

$$(1 \ 2) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

(row 1 A) ·  
(col 3 B)

$$(1 \ 2) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$AB =$

$$\begin{pmatrix} (1 \ 2) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ (-1 \ 0) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} (1 \ 2) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ (-1 \ 0) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} (1 \ 2) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \\ (-1 \ 0) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{pmatrix}$$

(row 2 A) ·  
(col 1 B)

(row 2 A) ·  
(col 2 B)

(row 2 A) ·  
(col 3 B)

$$= \begin{pmatrix} (1)(1) + (2)(0) & (1)(2) + (2)(1) & (1)(-1) + (2)(0) \\ (-1)(1) + (0)(0) & (-1)(2) + (0)(1) & (-1)(-1) + (0)(0) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & -1 \\ -1 & -2 & 1 \end{pmatrix}$$

Ex: Using the same matrices

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

can we calculate  $BA$  ?

$$\begin{array}{cc} \underbrace{B} & \underbrace{A} \\ 2 \times 3 & 2 \times 2 \\ \uparrow & \uparrow \\ & 3 \neq 2 \end{array}$$

since  $3 \neq 2$

$BA$  is undefined

You can also see this if you try to calculate it...

$$BA = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$$

(row 1 B) ·  
(col 1 A)

$$= \begin{pmatrix} (1 \ 2 \ -1) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} & \text{~~~~~} \\ \text{~~~~~} & \text{~~~~~} \end{pmatrix}$$

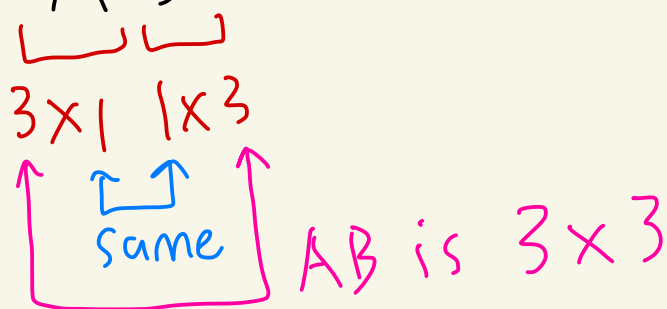
this dot product can't be calculated since the sizes don't match

Ex: Let

$$A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$\text{and } B = \begin{pmatrix} 0 & 1 & -3 \end{pmatrix}$$

Calculate  $AB$  if possible



$$AB = \begin{pmatrix} \underbrace{(\text{row 1 } A) \cdot (\text{col 1 } B)} & \underbrace{(\text{row 1 } A) \cdot (\text{col 2 } B)} & \underbrace{(\text{row 1 } A) \cdot (\text{col 3 } B)} \\ \underbrace{(\text{row 2 } A) \cdot (\text{col 1 } B)} & \underbrace{(\text{row 2 } A) \cdot (\text{col 2 } B)} & \underbrace{(\text{row 2 } A) \cdot (\text{col 3 } B)} \\ \underbrace{(\text{row 3 } A) \cdot (\text{col 1 } B)} & \underbrace{(\text{row 3 } A) \cdot (\text{col 2 } B)} & \underbrace{(\text{row 3 } A) \cdot (\text{col 3 } B)} \end{pmatrix}$$

$(1) \cdot (0)$        $(1) \cdot (1)$        $(1) \cdot (-3)$   
 $(2) \cdot (0)$        $(2) \cdot (1)$        $(2) \cdot (-3)$   
 $(-1) \cdot (0)$        $(-1) \cdot (1)$        $(-1) \cdot (-3)$



$$= \begin{pmatrix} 0 & 1 & -3 \\ 0 & 2 & -6 \\ 0 & -1 & 3 \end{pmatrix}$$

Ex: Let  $A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$  and  $B = (0 \ 1 \ -3)$

as above. Can we calculate  $BA$ ?

$$BA = (0 \ 1 \ -3) \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$\underbrace{\quad}$   $\underbrace{\quad}$   
 $1 \times 3$   $3 \times 1$   
 $\uparrow \uparrow$   
 same  
 $\uparrow \uparrow \uparrow$   
 $BA$  is  $1 \times 1$

$$= \left( (0 \ 1 \ -3) \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \right)$$

$(\text{row } 1 \text{ of } B) \cdot$   
 $(\text{col } 1 \text{ of } A)$

$$= \left( (0)(1) + (1)(2) + (-3)(-1) \right)$$

$$= (5)$$

BA is a  
1x1  
matrix

Note: We saw above that  
when  $A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$  and  $B = (0 \ 1 \ -3)$   
that  $AB \neq BA$ .

In general,  $AB = BA$  is not  
always true for matrices

Def: Let  $A$  be an  $m \times n$  matrix. The transpose of  $A$ , denoted by  $A^T$ , is defined to be the  $n \times m$  matrix that results from interchanging the rows and columns of  $A$ . That is, the  $i$ -th column of  $A^T$  is the  $i$ -th row of  $A$ . Or you could say that the  $j$ -th row of  $A^T$  is the  $j$ -th column of  $A$ .

Some people write  $A^t$  instead of  $A^T$

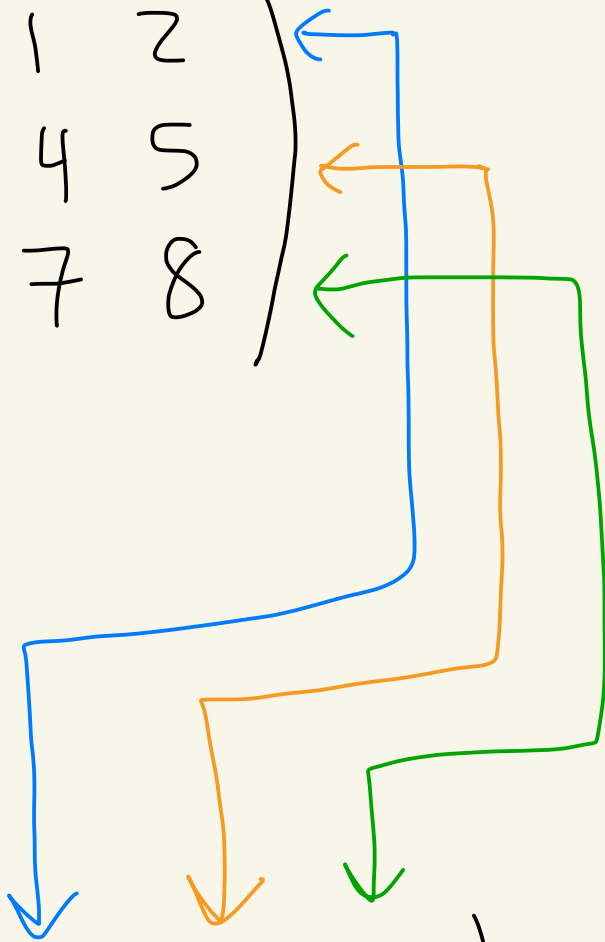
Ex:

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{pmatrix}$$

A is  
3x2

$$A^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{pmatrix}$$

A<sup>T</sup> is  
2x3



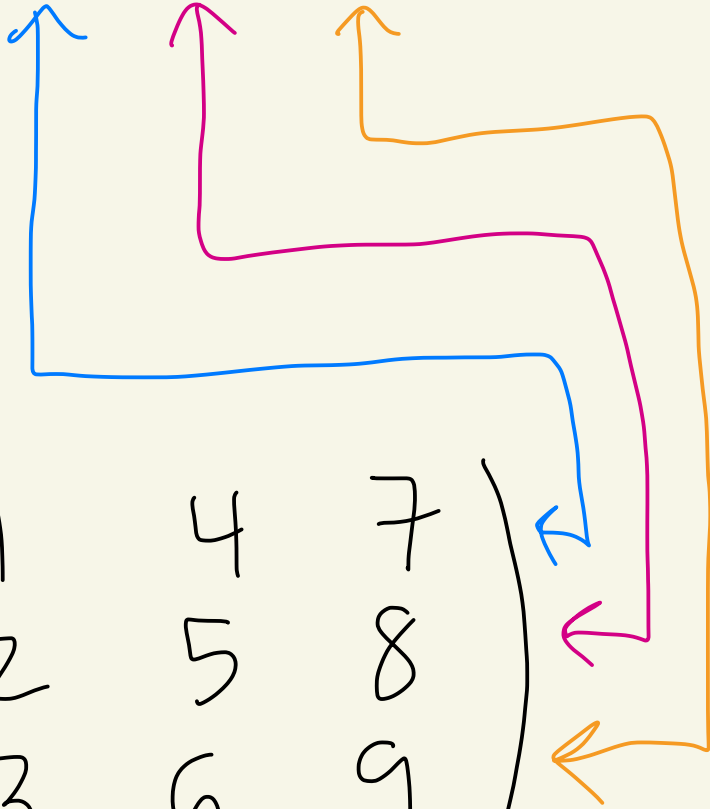
Ex:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

A is  
3x3

$$A^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

$A^T$  is  
3x3



Def: The  $m \times n$  zero matrix

is the  $m \times n$  matrix where every entry is zero.

We denote this matrix by

$O_{m \times n}$  or just by  $O$  if

we don't want to mention the size.

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Ex:  $O_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$O_{3 \times 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$O_{1 \times 6} = (0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

Ex:  $A = \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix}$

$$A + O_{2 \times 2} = \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix} = A$$

$$O_{2 \times 2} + A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 5 \\ -1 & 3 \end{pmatrix} = A$$

So,  $A + O_{2 \times 2} = A$

$O_{2 \times 2} + A = A$

So,  $O_{2 \times 2}$   
acts like  
the number  
0 does