

## Math 2550 - Homework # 9

### Column space, nullspace, rank-nullity theorem

1. Determine whether  $\vec{b}$  is in the column space of  $A$ . If so, express  $\vec{b}$  as a linear combination of the columns of  $A$ .

(a)  $A = \begin{pmatrix} 1 & 3 \\ 4 & -6 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -2 \\ 10 \end{pmatrix}$

(b)  $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

(c)  $A = \begin{pmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$

2. For each matrix  $A$ , (i) find a basis for the nullspace of  $A$ , (ii) calculate the nullity of the matrix, (iii) find a basis for the column space of  $A$ , (iv) state the rank of  $A$ , (v) verify that the rank-nullity theorem is true for  $A$ .

(a)  $A = \begin{pmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{pmatrix}$

(b)  $A = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$

(c)  $A = \begin{pmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{pmatrix}$

3. Suppose that  $A$  is a  $4 \times 5$  matrix. Suppose that the nullity of  $A$  is 3. What is the rank of  $A$  ?

4. Suppose that  $A$  is a matrix where a basis for its column space is

$$\left\{ \begin{pmatrix} 2 \\ -3 \\ 1 \\ 8 \\ 7 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ 1 \\ -9 \\ 6 \end{pmatrix} \right\}$$

Also suppose that a basis for the nullspace of  $A$  contains exactly 2 vectors. How many columns does  $A$  have? Justify your answer.

5. For each set of vectors: (i) Find a subset of the given vectors that forms a basis for the space spanned by the vectors, and then (ii) express each vector that is not in the basis as a linear combination of the basis vectors.

(a)  $\vec{v}_1 = \langle 2, -1 \rangle$ ,  $\vec{v}_2 = \langle 5, -7 \rangle$ ,  $\vec{v}_3 = \langle 1, 1 \rangle$

(b)  $\vec{v}_1 = \langle 1, 0, 1 \rangle$ ,  $\vec{v}_2 = \langle 0, 1, 2 \rangle$ ,  $\vec{v}_3 = \langle 1, 1, 1 \rangle$

6. Suppose that  $A$  is a  $3 \times 3$  matrix. Suppose that the nullity of  $A$  is 0.

Show that every vector  $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$  is in the span of the columns of  $A$ .