Math 2150-02
8/25/25

Topic 1 - What is a differential equation?

Ex: $\boxed{y' = 5y}$

This is an example of a differential equation with unknown function y.

Let $y = e^{5x}$.

Then $y' = 5e^{5x}$

Note: $y' = 5y$

So, $y = e^{5x}$ solves $y' = 5y$

<u>Def:</u>

- An equation relating an unknown function and one or more of its derivatives is called a <u>differential equation</u>

- If a differential equation only has regular derivatives of a single function then its called an <u>ordinary differential equation</u> or ODE.

  If the differential equation contains partial derivatives then its called a <u>partial differential equation</u> or PDE.

- The <u>order</u> of a differential equation is the order of the highest derivative that occurs in the equation.

Ex: $y' = 5y$ ODE order 1

$10y''' - 2y = 7$ ODE order 3

$xy'' + y = \sin(x)$ ODE order 2

Here we are thinking of y as a function of x.
So, y is a function,
x is a number.
It means: $x \cdot y''(x) + y(x) = \sin(x)$

(Laplace's Equation)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

PDE order 2

Here $u = u(x,y)$ is a function of x and y

In this class we solve ODE's. In math 4030 you solve PDE's. This class is a prereq. for 4030

<u>Def:</u> An n^{th} order ODE is called <u>linear</u> if it is of the form:

$$a_n(x)\,y^{(n)} + a_{n-1}(x)\,y^{(n-1)} + \dots + a_1(x)\,y' + a_0(x)\,y = b(x)$$

these coefficients only have #'s and x's in them

<u>Recall:</u>

$y^{(k)}$ means the k-th derivative

Ex: $y^{(3)} = y'''$

<u>Summary:</u> Two y terms are never multiplied.

Ex:

$$\underbrace{5x}_{}y''' + \underbrace{10}_{}y'' = \underbrace{\tan(2x)}_{}$$

#'s and x's

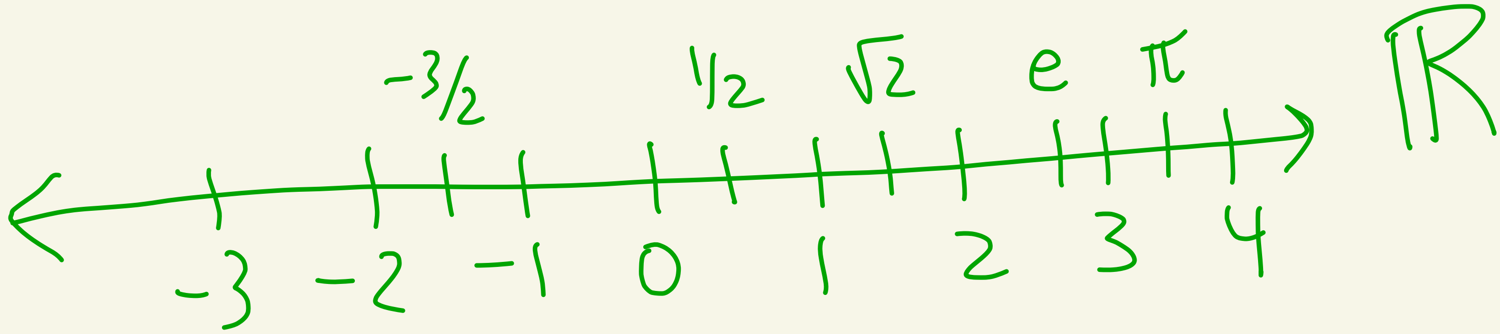
linear ODE of order 3

Ex: $10y'' + \underbrace{yy'}_{\text{two y-terms multiplied}} = 2$

two y-terms multiplied

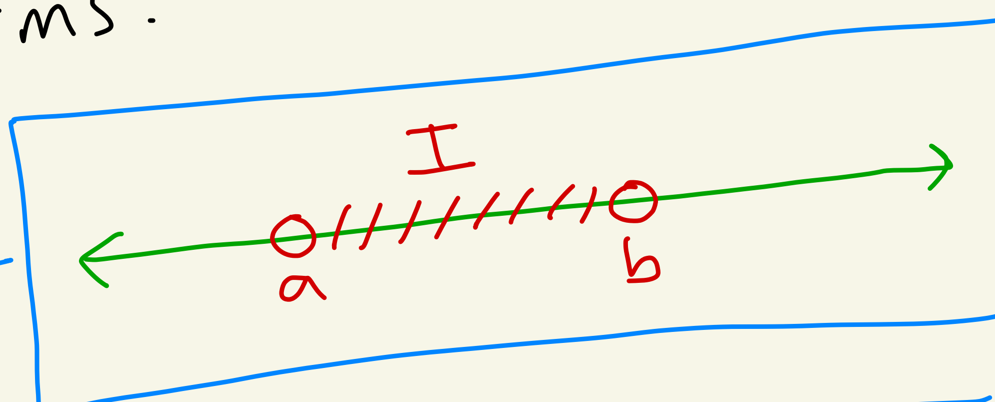
non-linear ODE of order 2

Def: The set of real numbers is denoted by $\mathbb{R}$.

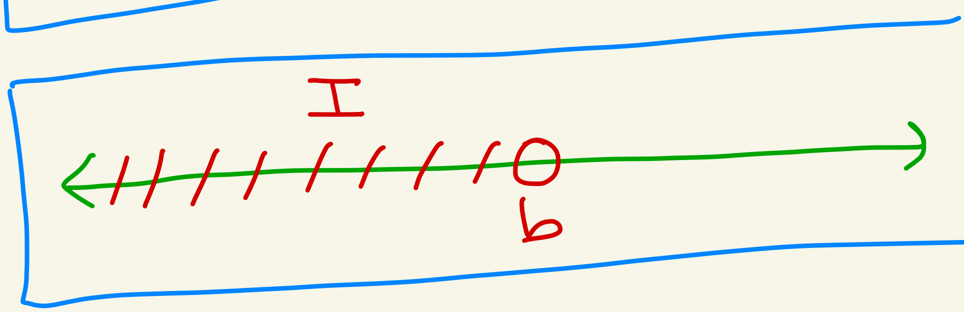


Def: An open interval I is an interval of one of the following forms:

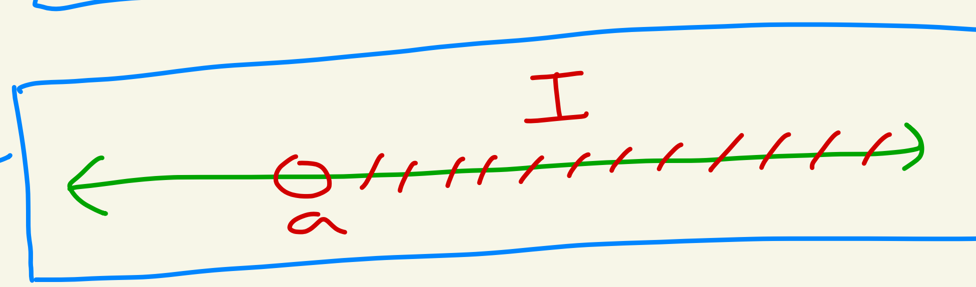
$I = (a, b)$



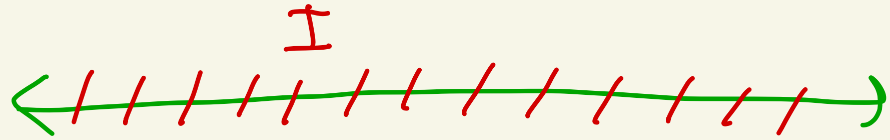
$I = (-\infty, b)$



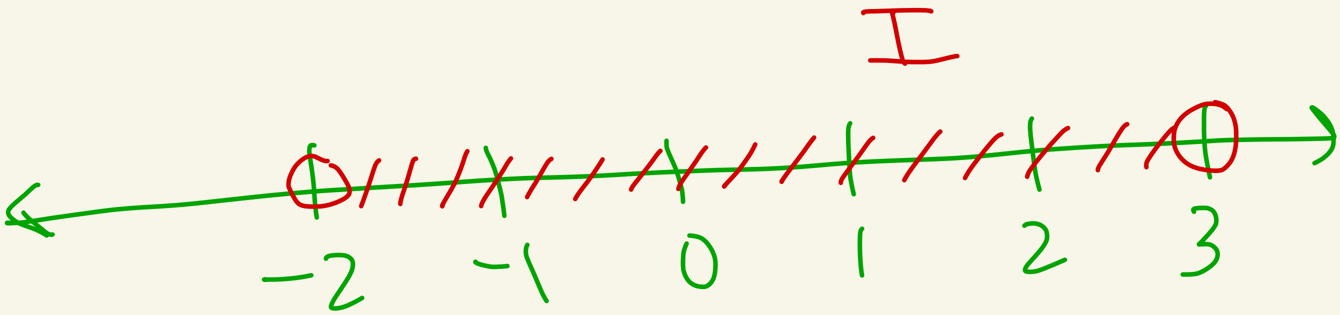
$I = (a, \infty)$



$I = (-\infty, \infty)$



Ex: $I = (-2, 3)$



Def: A function f is a **solution** to an n-th order ODE on an open interval I if:

1. $f, f', f'', \ldots, f^{(n)}$ exist on I

and

2. when you plug f and it's derivatives into the ODE, they solve the ODE for all x in I.

In addition, sometimes one is given what

$$f(x_0), f'(x_0), \ldots, f^{(n-1)}(x_0)$$

must equal for some x_0 in I. This turns the ODE into an **initial-value problem** (IVP)

<u>Ex:</u> Let's find a solution to $y'' = -y$ on $I = (-\infty, \infty)$.

Let $y = \sin(x)$.

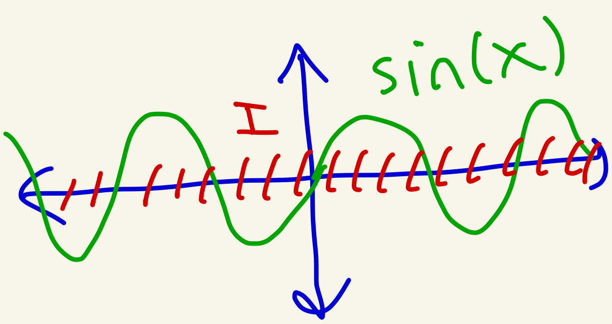
Then, $y' = \cos(x)$

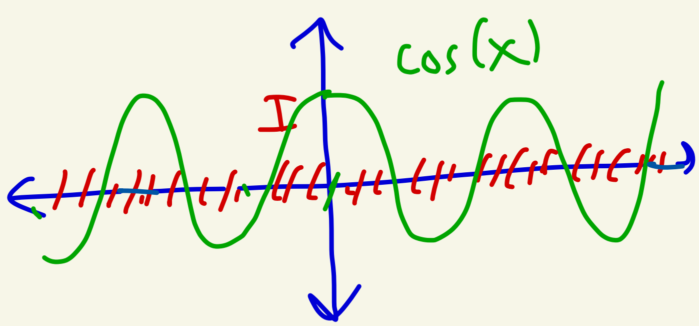
$y'' = -\sin(x)$

$y'' = -y$

So, $y'' = -y$.

Also, these functions exist on $I = (-\infty, \infty)$





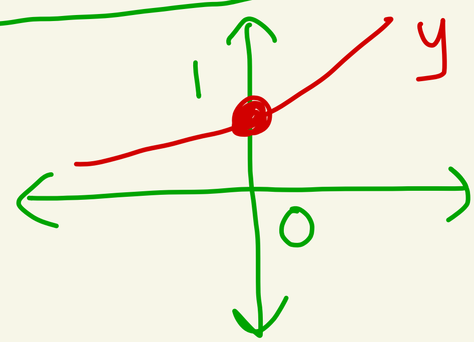
So, $y = \sin(x)$ solves $y'' = -y$ on $I = (-\infty, \infty)$

Ex: Let's find a solution to the initial-value problem

$$\boxed{\begin{aligned} y' &= y^2 \\ y(0) &= 1 \end{aligned}}$$

1st order non-linear ODE

condition on the solution



Let $y = \dfrac{1}{1-x}$

Then, $y = (1-x)^{-1}$

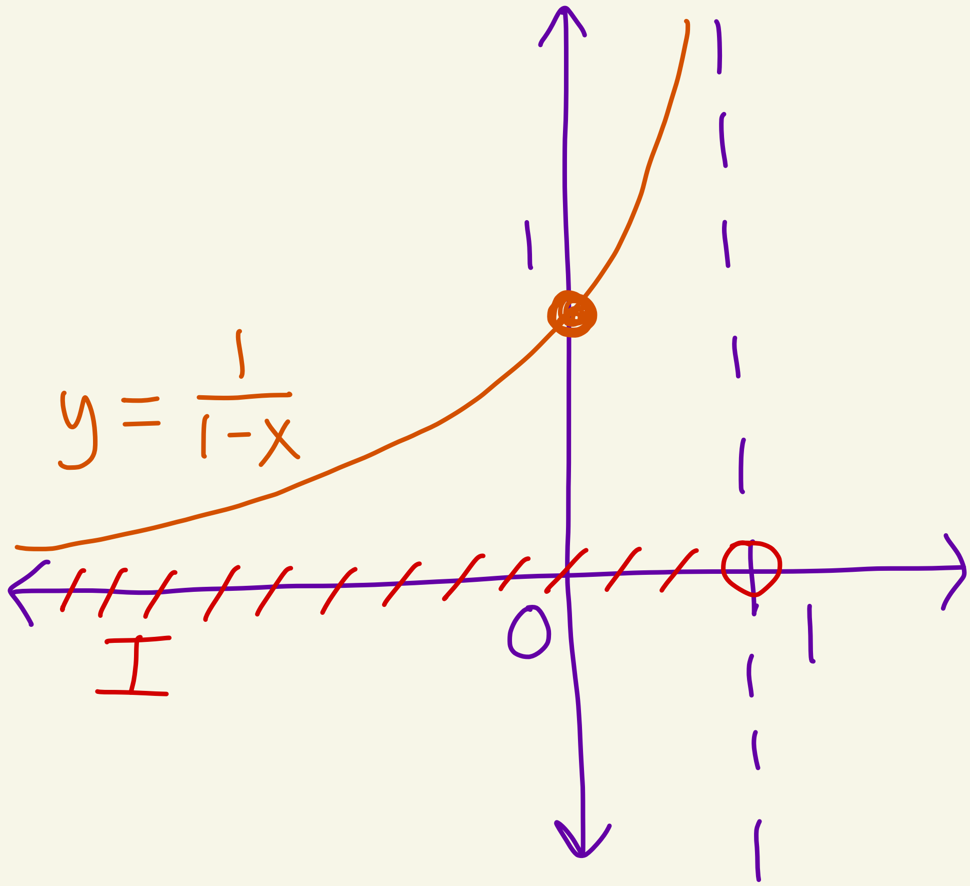
$$y' = -(1-x)^{-2} \cdot (-1)$$

$$= (1-x)^{-2} = \frac{1}{(1-x)^2}$$

$y' = y^2$

Thus, $y' = y^2$

Also, $y(0) = \frac{1}{1-0} = 1$



$y = \frac{1}{1-x}$ solves

$$\boxed{\begin{aligned} y' &= y^2 \\ y(0) &= 1 \end{aligned}}$$

on the open interval $I = (-\infty, 1)$