Math 2150-02
11/12/25

$\boxed{\text{HW 8 1(d)}}$

Find $y_p$ for

$$y'' + 3y' - 10y = 6e^{4x}$$

<u>Guess:</u> $y_p = Ae^{4x}$

$$y_p' = 4Ae^{4x}$$

$$y_p'' = 16Ae^{4x}$$

Plug it in to get:

$$(16Ae^{4x}) + 3(4Ae^{4x}) - 10(Ae^{4x}) = 6e^{4x}$$

$$16Ae^{4x} + 12Ae^{4x} - 10Ae^{4x} = 6e^{4x}$$

$$18Ae^{4x} = 6e^{4x}$$

$$A = \frac{1}{3}$$

So, $y_p = \frac{1}{3} e^{4x}$

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$\boxed{\text{Hw 9 } 1(b)}$

$\boxed{\text{Solve } y'' + y = \sin(x)}$

Step 1: Solve $y'' + y = 0$

$$r^2 + 1 = 0$$

$$r = \frac{-0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{\pm\sqrt{-4}}{2} = \frac{\pm\sqrt{4}\sqrt{-1}}{2} = \pm i$$

$$= \underbrace{0 \pm 1 \cdot i}_{\alpha \pm \beta i}$$

$$\alpha = 0, \ \beta = 1$$

$$y_h = c_1 e^{0x} \cos(1 \cdot x) + c_2 e^{0x} \sin(1 \cdot x)$$

$$\boxed{y_h = c_1 \cos(x) + c_2 \sin(x)}$$

$$\boxed{y_1 = \cos(x)} \qquad \boxed{y_2 = \sin(x)}$$

Step 2: Find $\boxed{y_p = V_1 y_1 + V_2 y_2}$

Need

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix}$$

$$= \cos^2(x) - \left(-\sin^2(x)\right)$$

$$= \cos^2(x) + \sin^2(x)$$

$$= 1$$

Then,

$$v_1 = \int \frac{-y_2 b(x)}{W(y_1, y_2)} \, dx$$

$$= \int \frac{-\sin(x)\sin(x)}{1} \, dx$$

$$= -\int \sin^2(x)\,dx$$

$$= -\int \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right) dx$$

$$= -\left[\frac{1}{2}x - \frac{1}{2}\left(\frac{1}{2}\sin(2x)\right)\right]$$

$$= \boxed{-\frac{1}{2}x + \frac{1}{4}\sin(2x)}$$

And,

$$v_2 = \int \frac{y_1 \cdot b(x)}{W(y_1, y_2)}\,dx = \int \frac{\cos(x)\sin(x)}{1}\,dx$$

$$= \int \cos(x)\sin(x)\,dx$$

$$= \int u \, du = \frac{1}{2}u^2 = \boxed{\frac{1}{2}\sin^2(x)}$$

$u = \sin(x)$
$du = \cos(x)\,dx$

So, the answer is:

$$\boxed{\begin{aligned} y &= y_h + y_p \\ &= c_1\cos(x) + c_2\sin(x) \quad \leftarrow \boxed{y_h} \\ &\quad + \left(-\frac{1}{2}x + \frac{1}{4}\sin(2x)\right)\cos(x) \\ &\quad + \left(\frac{1}{2}\sin^2(x)\right)\sin(x) \end{aligned}}$$

$y_p = v_1 y_1 + v_2 y_2$

$\boxed{\text{HW 10-1(c)}}$

Given that $y_1 = \ln(x)$ is a solution to

$$xy'' + y' = 0$$

on $I = (0, \infty)$, find the general solution.

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Divide by $x$ to get

$$y'' + \frac{1}{x} y' = 0$$

$\boxed{a_1(x) = \frac{1}{x}}$

We have

$$y_2 = y_1 \cdot \int \frac{e^{-\int a_1(x)\,dx}}{y_1^2}\,dx$$

$$= \ln(x) \int \frac{e^{-\int \frac{1}{x}dx}}{(\ln(x))^2}\,dx$$

$$= \ln(x) \int \frac{e^{-\ln|x|}}{(\ln(x))^2}\,dx$$

$$I = (0,\infty)$$
$$x > 0$$
$$|x| = x$$

$$= \ln(x) \int \frac{e^{-\ln(x)}}{(\ln(x))^2}\,dx$$

$$= \ln(x) \int \frac{e^{\ln(x^{-1})}}{(\ln(x))^2}\,dx$$

$$A\ln|x| = \ln(x^A)$$

$$= \ln(x) \int \frac{x^{-1}}{(\ln(x))^2}\,dx$$

$$e^{\ln(A)} = A$$

$$\int \frac{x^{-1}}{(\ln(x))^2}\,dx = \int \frac{1}{u^2}\,du = \int u^{-2}\,du$$

$$\boxed{\begin{aligned} u &= \ln(x) \\ du &= x^{-1}\,dx \end{aligned}}$$

$$= \frac{u^{-1}}{-1}$$

$$= -\frac{1}{u} = -\frac{1}{\ln(x)}$$

$$= \ln(x)\left[-\frac{1}{\ln(x)}\right]$$

$$= -1$$

Thus, the solution to
$xy'' + y' = 0$ on $I = (0, \infty)$ is

$$y_h = c_1 y_1 + c_2 y_2$$
$$= c_1 \ln(x) + c_2(-1)$$

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$\boxed{\text{HW 11 } 1(d)}$

Find a power series for
$f(x) = \frac{1}{x}$ centered at $x_0 = 1$

<u>Hint:</u> Use

$$\ln(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots$$

with radius of convergence $r=1$

Differentiate the $\ln(x)$ power series to get

$$\frac{1}{x} = 1 - \frac{1}{2}\left[2(x-1)^1\right] + \frac{1}{3}\left[3(x-1)^2\right] - \frac{1}{4}\left[4(x-1)^3\right] + \dots$$

So,

$$\frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots$$

with radius of convergence $r=1$

$\boxed{\text{HW 12}}$

② Find the first <u>four</u> nonzero terms for a power series solution to:

$$y'' - (x+1)y' + x^2 y = 0$$

$$y'(0) = 1, \quad y(0) = 1$$

What is the radius of convergence?

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The center of our power series will be $x_0 = 0$.

Since the coefficients are $-(x-1)$ and $x^2$ and $0$ are all polynomials, our power series solution will

have radius of convergence
$r = \infty$.

Let's find $y$.

<u>Given</u>: $y(0) = 1, \; y'(0) = 1$

Find $y''(0)$ next.

$$y'' - (x+1)y' + x^2 y = 0$$

$$y'' = (x+1)y' - x^2 y$$

$$y''(0) = (0+1)\left[\underbrace{y'(0)}_{1}\right] - (0)^2\left[\underbrace{y(0)}_{1}\right]$$

$$= (1)[1] - (0)[1] = 1$$

Find $y'''(0)$ next.

$$y'' = (x+1)y' - x^2 y$$

$$y''' = (1)y' + (x+1)y'' - \left[2xy + x^2 y'\right]$$

$$y'''(0) = \overbrace{y'(0)}^{1} + (0+1)\overbrace{\left[y''(0)\right]}^{1} - \left[2(0)\underbrace{\left[y(0)\right]}_{1} + (0)^2\underbrace{\left[y'(0)\right]}_{1}\right]$$

$$y'''(0) = 2$$

$$y(x) = y(0) + y'(0)(x-0)$$

$$+ \frac{y''(0)}{2!}(x-0)^2$$

$$+ \frac{y'''(0)}{3!}(x-0)^3 + \cdots$$

$$= 1 + 1 \cdot x + \frac{1}{2!}x^2 + \frac{2}{3!}x^3 + \cdots$$

$$= \boxed{1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots}$$

$$\boxed{\text{with } r = \infty}$$