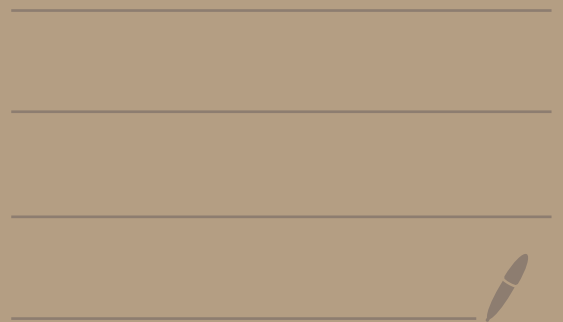


Math 2150-02

10/8/25



Hw 3 (3)

Solve the linear IVP:

$$\frac{dy}{dx} + 2xy = x$$

$$y(0) = -3$$

on $I = (-\infty, \infty)$.

$$y' + 2xy = x$$

$$A(x) = \int 2x dx = x^2$$

Multiply by $e^{A(x)} = e^{x^2}$ to get

$$\underbrace{e^{x^2} y' + e^{x^2} 2xy}_{\text{derivative of } e^{x^2} y} = \underbrace{e^{x^2} x}_{\text{right hand side}}$$

$$(e^{x^2} \cdot y)' = xe^{x^2}$$

So,

$$(e^{x^2} \cdot y)' = xe^{x^2}$$

Integrate to get

$$e^{x^2} \cdot y = \underbrace{\int xe^{x^2} dx}$$

$$\int xe^{x^2} dx = \int \frac{1}{2} e^u du$$

$$u = x^2$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$= \frac{1}{2} e^u + C$$

$$= \frac{1}{2} e^{x^2} + C$$

So,

$$e^{x^2} \cdot y = \frac{1}{2} e^{x^2} + C$$

Thus,

$$y = \frac{1}{2} + \frac{C}{e^{x^2}}$$

Let's find C using

$$\boxed{y(0) = -3.}$$

$x = 0, y = -3$

Plugging in we get

$$-3 = \frac{1}{2} + \frac{C}{e^{0^2}}$$

$$-7/2 = C$$

Answer:

$$y = \frac{1}{z} - \frac{7/2}{e^{x^2}}$$

HW 4

①(d) Solve the separable IVP:

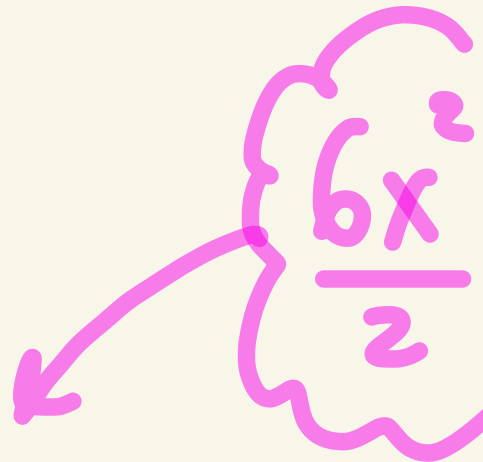
$$\frac{dy}{dx} = 6y^2x$$

$$y(0) = \frac{1}{12}$$

$$\frac{dy}{dx} = 6y^2x$$

$$\frac{dy}{y^2} = 6x dx$$

$$\int y^{-2} dy = \int 6x dx$$


$$\frac{6x^2}{2}$$

$$\frac{y^{-1}}{-1} = 3x^2 + C$$

$$-\frac{1}{y} = 3x^2 + C$$

Thus,

$$y = \frac{-1}{3x^2 + C}$$

Plug in $y(0) = \frac{1}{12}$
to get:

$$\frac{1}{12} = \frac{-1}{3(0)^2 + C}$$

$$\frac{1}{12} = \frac{-1}{C}$$

$$C = -12$$

$$\begin{aligned} x &= 0 \\ y &= 1/12 \end{aligned}$$

Answer

$$y = \frac{-1}{3x^2 - 12}$$

HW 5 ① (e)

Consider

$$\underbrace{(2y^2x - 3)}_M + \underbrace{(2yx^2 + 4)}_N y' = 0$$

Check if exact.

If so, solve it.

Check:

$$\frac{\partial M}{\partial y} = 4yx$$

$$\frac{\partial N}{\partial x} = 4yx$$

Same

It's exact.

Solve it:

$$\begin{aligned}\frac{\partial f}{\partial x} &= M \\ \frac{\partial f}{\partial y} &= N\end{aligned}$$



$$\begin{aligned}\frac{\partial f}{\partial x} &= 2y^2x - 3 & (1) \\ \frac{\partial f}{\partial y} &= 2yx^2 + 4 & (2)\end{aligned}$$

Integrate (1) with respect to x :

$$f(x, y) = 2y^2 \cdot \frac{x^2}{2} - 3x + \underbrace{C(y)}_{\text{constant with respect to } x}$$

$$f(x, y) = y^2 x^2 - 3x + C(y)$$

constant
with
respect
to x

Integrate (2) with respect to y :

$$f(x,y) = y^2 x^2 + 4y + \underbrace{D(x)}_{\text{constant with respect to } y}$$

Set the above equal:

$$\cancel{y^2 x^2} - 3x + C(y) = \cancel{y^2 x^2} + 4y + D(x)$$

$$\underbrace{-3x + C(y)}_{\text{green bracket}} = \underbrace{4y}_{\text{orange bracket}} + \underbrace{D(x)}_{\text{green bracket}}$$

Diagram showing the matching of terms: a green bracket under $-3x + C(y)$ and $D(x)$, and an orange bracket under $4y$. Arrows indicate the correspondence between $C(y)$ and $4y$, and between $-3x$ and $D(x)$.

Set $C(y) = 4y$, $D(x) = -3x$.

So,

$$\begin{aligned} f(x,y) &= y^2 x^2 + 4y + D(x) \\ &= y^2 x^2 + 4y - 3x \end{aligned}$$

Answer:

$$y^2 x^2 + 4y - 3x = c$$

where c is any constant

HW 6 $z(e,f)$ given $z(a,b,c,d)$

②(e)

Suppose you know that the general solution to

$$x^2 y'' - 5xy' + 8y = 0$$

is

$$y_h = c_1 x^2 + c_2 x^4$$

and a particular solution to

$$x^2 y'' - 5xy' + 8y = 2^4$$

is $y_p = 3$.

What is the general solution to
 $x^2 y'' - 5xy' + 8y = 24$?

Answer:

$$y = y_h + y_p = c_1 x^2 + c_2 x^4 + 3$$

Solve: (2f)

$$x^2 y'' - 5xy' + 8y = 24$$

$$y'(1) = 0, y(1) = -1$$

General sol. to $x^2 y'' - 5xy' + 8y = 24$
is $y = c_1 x^2 + c_2 x^4 + 3$.

Let's make it solve

$$y'(1) = 0, y(1) = -1.$$

We have

$$y = c_1 x^2 + c_2 x^4 + 3$$

$$y' = 2c_1 x + 4c_2 x^3$$

$$y'(1) = 0$$

$$y(1) = -1$$

→

$$2c_1(1) + 4c_2(1)^3 = 0$$

$$c_1(1)^2 + c_2(1)^4 + 3 = -1$$



$$2c_1 + 4c_2 = 0$$

$$c_1 + c_2 = -4$$

①

②

② gives $c_1 = -4 - c_2$.

Plug into ① to get

$$2(-4 - c_2) + 4c_2 = 0$$

$$-8 - 2c_2 + 4c_2 = 0$$

$$2c_2 = 8$$

$$c_2 = 4$$

$$\text{So, } c_1 = -4 - c_2 = -4 - 4 = -8$$

Thus,

$$y = c_1 x^2 + c_2 x^4 + 3$$

$$y = -8x^2 + 4x^4 + 3$$

Answer

HW 7

① (d) Solve

$$y'' - 2y' + 2y = 0$$

The characteristic equation

$$r^2 - 2r + 2 = 0$$

has roots

$$r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm \sqrt{4} \sqrt{-1}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$\sqrt{-1} = i$

$$= \frac{2}{2} \pm \frac{2}{2} i$$

$$= 1 \pm i$$

$$\left\{ \begin{array}{l} \alpha \pm \beta i \\ \alpha = 1 \\ \beta = 1 \end{array} \right\}$$

Answer:

$$\begin{aligned} y_h &= c_1 e^{\alpha x} \cos(\beta x) + c_2 e^{\alpha x} \sin(\beta x) \\ &= c_1 e^x \cos(x) + c_2 e^x \sin(x) \end{aligned}$$