Math 2150-02

10/8/25

$\boxed{\text{HW } 3}$ 3

Solve the linear IVP:

$$\frac{dy}{dx} + 2xy = x$$

$$y(0) = -3$$

on $I = (-\infty, \infty)$.

$$y' + 2xy = x$$

$$A(x) = \int 2x \, dx = x^2$$

multiply by $e^{A(x)} = e^{x^2}$ to get

$$\underbrace{e^{x^2} y' + e^{x^2} 2xy} = \underbrace{e^{x^2} x}$$

$$\left(e^{A(x)} \cdot y\right)' = xe^{x^2}$$

So,

$$\left(e^{x^2} \cdot y\right)' = xe^{x^2}$$

Integrate to get

$$e^{x^2} \cdot y = \int xe^{x^2}\,dx$$

$$\int xe^{x^2}\,dx = \int \frac{1}{2}e^u\,du$$

$$= \frac{1}{2}e^u + C$$

$$= \frac{1}{2}e^{x^2} + C$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

So,

$$e^{x^2} \cdot y = \frac{1}{2} e^{x^2} + C$$

Thus,

$$y = \frac{1}{2} + \frac{C}{e^{x^2}}$$

Let's find C using

$\underbrace{y(0) = -3.}_{x=0,\ y=-3}$

Plugging in we get

$$-3 = \frac{1}{2} + \frac{C}{e^{0^2}}$$

$$-7/2 = C$$

Answer:

$$y = \frac{1}{2} - \frac{7/2}{e^{x^2}}$$

HW 4

① (d) Solve the separable IVP:

$$\frac{dy}{dx} = 6y^2x$$

$$y(0) = \frac{1}{12}$$

$$\frac{dy}{dx} = 6y^2x$$

$$\frac{dy}{y^2} = 6x\,dx$$

$$\frac{6x^2}{2}$$

$$\int y^{-2}\,dy = \int 6x\,dx$$

$$\frac{y^{-1}}{-1} = 3x^2 + C$$

$$\boxed{-\frac{1}{y} = 3x^2 + C}$$

Thus,

$$y = \frac{-1}{3x^2 + C}$$

Plug in $y(0) = \frac{1}{12}$ to get:

$x = 0$
$y = 1/12$

$$\frac{1}{12} = \frac{-1}{3(0)^2 + c}$$

$$\frac{1}{12} = \frac{-1}{c}$$

$$c = -12$$

Answer

$$\boxed{y = \frac{-1}{3x^2 - 12}}$$

HW 5 | ① (e)

Consider

$$\underbrace{(2y^2x - 3)}_{M} + \underbrace{(2yx^2 + 4)}_{N}y' = 0$$

Check if exact.

If so, solve it.

Check:

$$\frac{\partial M}{\partial y} = 4yx$$

$$\frac{\partial N}{\partial x} = 4yx$$

Same

It's exact.

Solve it:

$$\boxed{\begin{aligned}\frac{\partial f}{\partial x} &= M \\ \frac{\partial f}{\partial y} &= N\end{aligned}} \rightarrow \boxed{\begin{aligned}\frac{\partial f}{\partial x} &= 2y^2x - 3 \quad (1) \\ \frac{\partial f}{\partial y} &= 2yx^2 + 4 \quad (2)\end{aligned}}$$

Integrate (1) with respect to x:

$$f(x,y) = 2y^2 \cdot \frac{x^2}{2} - 3x + \underbrace{c(y)}_{\text{constant with respect to x}}$$

$$\boxed{f(x,y) = y^2x^2 - 3x + c(y)}$$

Integrate (2) with respect to y:

$$f(x,y) = y^2x^2 + 4y + \underbrace{D(x)}_{\text{constant with respect to } y}$$

Set the above equal:

$$\cancel{y^2x^2} - 3x + C(y) = \cancel{y^2x^2} + 4y + D(x)$$

$$-3x + C(y) = 4y + D(x)$$

Set $C(y) = 4y$, $D(x) = -3x$.

So,

$$f(x,y) = y^2x^2 + 4y + D(x)$$
$$= y^2x^2 + 4y - 3x$$

Answer:

$$y^2x^2 + 4y - 3x = c$$

where c is any constant

HW 6 2(e,f) given 2(a,b,c,d)

2 (e) Suppose you know that the general solution to

$$x^2y'' - 5xy' + 8y = 0$$

is

$$y_h = c_1 x^2 + c_2 x^4$$

and a particular solution to

$$x^2y'' - 5xy' + 8y = 24$$

is $y_p = 3$.

What is the general solution to

$$x^2y'' - 5xy' + 8y = 24 \text{ ?}$$

Answer:

$$y = y_h + y_p = c_1 x^2 + c_2 x^4 + 3$$

Solve: 2f

$$x^2y'' - 5xy' + 8y = 24$$

$$y'(1) = 0, \quad y(1) = -1$$

General sol. to $x^2y'' - 5xy' + 8y = 24$ is $y = c_1 x^2 + c_2 x^4 + 3$.

Let's make it solve

$y'(1) = 0$, $y(1) = -1$.

We have

$$y = c_1 x^2 + c_2 x^4 + 3$$

$$y' = 2c_1 x + 4c_2 x^3$$

$$\boxed{\begin{array}{l} y'(1) = 0 \\ y(1) = -1 \end{array}} \rightarrow \boxed{\begin{array}{l} 2c_1(1) + 4c_2(1)^3 = 0 \\ c_1(1)^2 + c_2(1)^4 + 3 = -1 \end{array}}$$

$$\boxed{\begin{array}{l} 2c_1 + 4c_2 = 0 \\ c_1 + c_2 = -4 \end{array}} \begin{array}{l} (1) \\ (2) \end{array}$$

(2) gives $c_1 = -4 - c_2$.

plug into (1) to get

$$2(-4 - c_2) + 4c_2 = 0$$

$$-8 - 2c_2 + 4c_2 = 0$$

$$2c_2 = 8$$

$$\boxed{c_2 = 4}$$

So, $\boxed{c_1 = -4 - c_2 = -4 - 4 = -8}$

Thus,

$$y = c_1 x^2 + c_2 x^4 + 3$$

$$\boxed{y = -8x^2 + 4x^4 + 3}$$ Answer

HW 7

1 (d) Solve

$$y'' - 2y' + 2y = 0$$

The characteristic equation

$$r^2 - 2r + 2 = 0$$

has roots

$$r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm \sqrt{4}\sqrt{-1}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$\sqrt{-1} = i$

$$= \frac{2}{2} \pm \frac{2}{2} i$$

$$= \underbrace{1 \pm i}_{\alpha \pm \beta i}$$

$\alpha \pm \beta i$

$\alpha = 1$

$\beta = 1$

Answer:

$$y_h = c_1 e^{\alpha x}\cos(\beta x) + c_2 e^{\alpha x}\sin(\beta x)$$

$$= c_1 e^{x}\cos(x) + c_2 e^{x}\sin(x)$$