Math 2150-02
10/1/25

Topic 8 continued...

Ex: Solve
$$y'' - y' + y = 2\sin(3x)$$

Step 1: Last time we found that the general solution to
$$y'' - y' + y = 0$$
is
$$y_h = c_1 e^{x/2}\cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 e^{x/2}\sin\left(\frac{\sqrt{3}}{2}x\right)$$

Step 2: We need a particular

Solution y_p to $y''-y'+y=2\sin(3x)$

We guess:

$$y_p = A\sin(3x) + B\cos(3x)$$

Let's plug it in to find A & B.

We get:

$$y_p' = 3A\cos(3x) - 3B\sin(3x)$$

$$y_p'' = -9A\sin(3x) - 9B\cos(3x)$$

Plug into $y''-y'+y = 2\sin(3x)$ to get:

$$\left(-9A\sin(3x) - 9B\cos(3x)\right) \leftarrow y_p''$$

$$-\left(3A\cos(3x) - 3B\sin(3x)\right) \leftarrow -y_p'$$

$$+\left(A\sin(3x) + B\cos(3x)\right) \leftarrow +y_p$$

$$= 2\sin(3x)$$

We have:

$$\underbrace{(-8A+3B)}_{2}\sin(3x) + \underbrace{(-3A-8B)}_{0}\cos(3x)$$

$$= 2\sin(3x)$$

We need

$$\boxed{\begin{aligned} -8A+3B &= 2 \\ -3A-8B &= 0 \end{aligned}} \quad \begin{matrix} 1 \\ 2 \end{matrix}$$

2 gives $B = -\frac{3}{8}A$.

Plug into 1 to get:

$$-8A + 3\left(-\frac{3}{8}A\right) = 2$$

We get $\left(-8-\frac{9}{8}\right)A = 2$

So, $A = 2\left(\frac{8}{-73}\right) = \boxed{\frac{-16}{73}}$

Then, $B = -\frac{3}{8}A = \left(-\frac{3}{8}\right)\left(\frac{-16}{73}\right) = \boxed{\frac{6}{73}}$

$$\boxed{\therefore \quad y_p = \frac{-16}{73}\sin(3x) + \frac{6}{73}\cos(3x)}$$

"Therefore"

1659 book Johann Rahn

Google AI says

Step 3: Thus, the general solution to $y'' - y' + y = 2\sin(3x)$ is

$$y = y_h + y_p$$
$$= c_1 e^{x/2}\cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 e^{x/2}\sin\left(\frac{\sqrt{3}}{2}x\right)$$

$$-\frac{16}{73}\sin(3x) + \frac{6}{73}\cos(3x)$$

Ex: Solve $y'' + 3y = xe^{3x}$

Step 1: Solve $y'' + 3y = 0$

The characteristic poly is

$$r^2 + 3 = 0$$

The roots are

$$r = \frac{-0 \pm \sqrt{0^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{\pm\sqrt{-12}}{2} = \frac{\pm\sqrt{12}\sqrt{-1}}{2}$$

$i=\sqrt{-1}$

$$= \frac{\pm 2\sqrt{3}\, i}{2}$$

$$= \pm\sqrt{3}\, i$$

$$= \underbrace{0 \pm \sqrt{3}\, i}$$

$\alpha \pm \beta i$

$\alpha = 0,\ \beta = \sqrt{3}$

topic 7

$$y_h = c_1 e^{\alpha x}\cos(\beta x) + c_2 e^{\alpha x}\sin(\beta x)$$

$$= c_1 e^{0x}\cos(\sqrt{3}x) + c_2 e^{0x}\sin(\sqrt{3}x)$$

$$= c_1\cos(\sqrt{3}x) + c_2\sin(\sqrt{3}x)$$

$\boxed{e^{0x} = e^0 = 1}$

Step 2: We need a particular solution y_p to $y'' + 3y = xe^{3x}$.

We guess

$$y_p = (Ax+B)e^{3x}$$

We have

$$y_p = Axe^{3x} + Be^{3x}$$

$$y_p' = \left[Ae^{3x} + Ax\left(3e^{3x}\right)\right] + 3Be^{3x}$$

$$= Ae^{3x} + 3Axe^{3x} + 3Be^{3x}$$

$$y_p'' = 3Ae^{3x} + 3\left[Ae^{3x} + 3Axe^{3x}\right] + 9Be^{3x}$$

$$= 6Ae^{3x} + 9Axe^{3x} + 9Be^{3x}$$

Plug into $y'' + 3y = xe^{3x}$ to get:

$$\overbrace{\left(6Ae^{3x} + 9Axe^{3x} + 9Be^{3x}\right)}^{y_p''} + 3\underbrace{\left(Axe^{3x} + Be^{3x}\right)}_{y_p} = xe^{3x}$$

Combine like terms to get:

$$\underbrace{(6A + 12B)}_{0}e^{3x} + \underbrace{(12A)}_{1}xe^{3x} = xe^{3x}$$

We need

$$\boxed{\begin{aligned} 6A + 12B &= 0 \quad ① \\ 12A &= 1 \quad ② \end{aligned}}$$

② gives $\boxed{A = \frac{1}{12}}$

Plug into ① to get: $6\left(\overbrace{\frac{1}{12}}^{A}\right) + 12B = 0$

We get $\boxed{B = \left(-\frac{1}{2}\right)\left(\frac{1}{12}\right) = -\frac{1}{24}}$

Thus,

$$\boxed{y_p = (Ax + B)e^{3x} = \left(\frac{1}{12}x - \frac{1}{24}\right)e^{3x}}$$

<u>Step 3:</u> The general solution to $y'' + 3y = xe^{3x}$ is

$$y = y_h + y_p$$

$$= c_1 \cos(\sqrt{3}x) + c_2 \sin(\sqrt{3}x) + \left(\frac{1}{12}x - \frac{1}{24}\right)e^{3x}$$