Math 2150-01
12/3/25

$$\boxed{\text{Test 1 \#5}}$$

$$\underbrace{(2xy+3)}_{M}+\underbrace{(x^2+4y)}_{N}y'=0$$

(a)

$$\frac{\partial M}{\partial y}=2x=\frac{\partial N}{\partial x} \leftarrow \boxed{\text{exact}}$$

(b)

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x}&=2xy+3 \quad (1)\\ \frac{\partial f}{\partial y}&=x^2+4y \quad (2)\end{aligned}}$$

(1) $f(x,y)=x^2y+3x+C(y)$

(2) $f(x,y)=x^2y+2y^2+D(x)$

$$\cancel{x^2y} + 3x + C(y) = \cancel{x^2y} + 2y^2 + D(x)$$

$$3x + C(y) = 2y^2 + D(x)$$

$$C(y) = 2y^2, \quad D(x) = 3x$$

① $f(x,y) = x^2y + 3x + 2y^2$

Answer:

$$\boxed{x^2y + 3x + 2y^2 = c}$$

where c is any constant

Test 1 #7

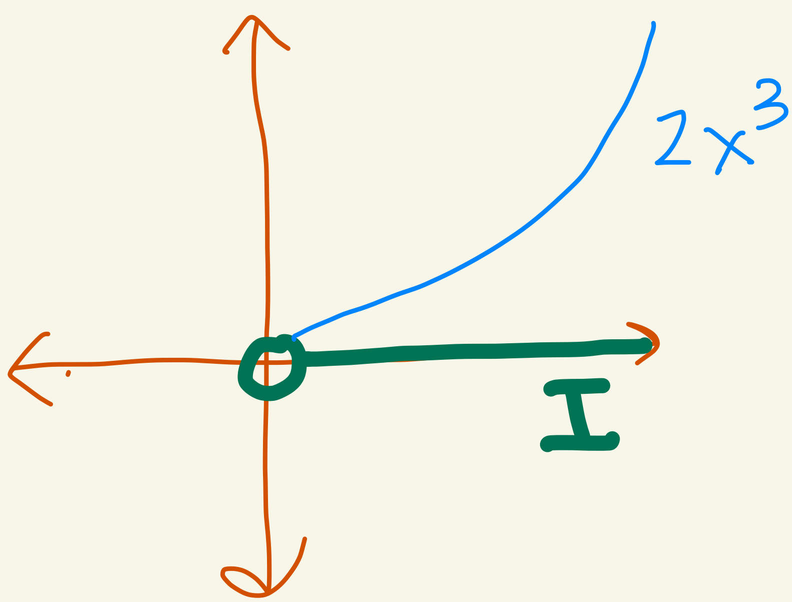
$f_1(x) = x$, $f_2(x) = x^3$, $I = (0, \infty)$.

(a) Show f_1, f_2 lin. ind. on I

$$W(f_1, f_2) = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix} = \begin{vmatrix} x & x^3 \\ 1 & 3x^2 \end{vmatrix}$$

$$= (x)(3x^2) - (1)(x^3)$$

$$= 2x^3$$



the wronskian is not the zero function on $I = (0, \infty)$. So, f_1, f_2 are lin. ind on I.

(b) Show f_1, f_2 solve

$$\boxed{x^2 y'' - 3xy' + 3y = 0}$$

We have:

$f_1 = x$, $f_1' = 1$, $f_1'' = 0$

plugging in we get:

$$x^2(0) - 3x(1) + 3(x)$$

$$= 0 - 3x + 3x = 0$$

And $f_2 = x^3$, $f_2' = 3x^2$, $f_2'' = 6x$

plugging in we get:

$$x^2(6x) - 3x(3x^2) + 3(x^3)$$

$$= 6x^3 - 9x^3 + 3x^3 = 0$$

(c) general sol. to $x^2y'' - 3xy' + 3y = 0$

is $y_h = c_1 x + c_2 x^3$

HW 8 #1(c)

Solve

$y'' + 4y' + 4y = 4x^2 - 8x$

Step 1:

Solve $y'' + 4y' + 4y = 0$

$r^2 + 4r + 4 = 0$

$(r+2)(r+2) = 0$

$r = -2, -2$

$y_h = c_1 e^{-2x} + c_2 x e^{-2x}$

Step 2: Guess y_p for

$y'' + 4y' + 4y = 4x^2 - 8x$

$$y_p = Ax^2 + Bx + C$$
$$y_p' = 2Ax + B$$
$$y_p'' = 2A$$

Plug in:

$$(2A) + 4(2Ax+B) + 4(Ax^2+Bx+C) = 4x^2 - 8x$$
$$2A + 8Ax + 4B + 4Ax^2 + 4Bx + 4C = 4x^2 - 8x$$
$$\boxed{4A}x^2 + \boxed{(8A+4B)}x + \boxed{(2A+4B+4C)} = \boxed{4}x^2 \boxed{-8}x$$

0

$$4A = 4$$
$$8A + 4B = -8$$
$$2A + 4B + 4C = 0$$

① ② ③

① $A = 1$

② $8(1) + 4B = -8$

$B = -4$

③ $2(1) + 4(-4) + 4C = 0$

$C = 14/4 = 7/2$

So,

$$y_p = Ax^2 + Bx + C$$
$$= x^2 - 4x + 7/2$$

(c) What's the general solution to $y'' - 4y' + 4y = 4x^2 - 8x$?

$$y = y_h + y_p$$
$$= c_1 e^{-2x} + c_2 x e^{-2x} + x^2 - 4x + \frac{7}{2}$$

Test 2-#5

1st 4 non-zero terms of power series solution to

$$\boxed{\begin{gathered} y'' - 2xy' - y = 0 \\ y'(0) = 1, \quad y(0) = 1 \end{gathered}}$$

center $x_0 = 0$

Since the coefficients of $y'' - 2xy' - y = 0$ are polynomials our power series solution will have radius of convergence $r = \infty$.

power series:

$$y(x) = y(0) + y'(0)(x-0) + \frac{y''(0)}{2!}(x-0)^2 + \frac{y'''(0)}{3!}(x-0)^3 + \cdots$$

$$= y(0) + y'(0)x + \frac{y''(0)}{2}x^2 + \frac{y'''(0)}{6}x^3 + \frac{y''''(0)}{24}x^4 + \cdots$$

We have $\boxed{y(0) = 1}$, $\boxed{y'(0) = 1}$.

Know:

$$\boxed{\begin{aligned} &y'' - 2xy' - y = 0 \\ &y'' = 2xy' + y \\ &y''(0) = 2(0)[y'(0)] + [y(0)] \\ &\qquad = 2(0)[1] + [1] \\ &\qquad = 0 + 1 = 1 \end{aligned}}$$

Let's find $y'''(0)$.

$$\boxed{\begin{aligned} &y'' = 2xy' + y \\ &y''' = 2y' + 2xy'' + y' \end{aligned}}$$

$$\boxed{\begin{aligned} y''' &= 3y' + 2xy'' \\ y'''(0) &= 3\left[y'(0)\right] + 2(0)\left[y''(0)\right] \\ &= 3\left[1\right] + 2(0)\left[1\right] \\ &= 3 + 0 = 3 \end{aligned}}$$

So,

$$y(x) = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \cdots$$

$$= 1 + 1\cdot x + \frac{1}{2}x^2 + \frac{3}{3!}x^3 + \cdots$$

$$= 1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \cdots$$

Test 2 - #3

$y_1 = x^2$ solves

$$x^2 y'' + 2xy' - 6y = 0$$

on $I = (0, \infty)$

Find another lin. ind. sol. y_2 and state the general sol.

Divide by x^2:

$$y'' + \underbrace{\frac{2}{x}}_{} y' - \frac{6}{x^2} y = 0$$

$$a_1 = \frac{2}{x}$$

$$y_2 = y_1 \cdot \int \frac{e^{-\int a_1(x)\,dx}}{y_1^2}\,dx$$

$$= x^2 \int \frac{e^{-\int \frac{2}{x} dx}}{(x^2)^2} dx$$

$$= x^2 \int \frac{e^{-2 \int \frac{1}{x} dx}}{x^4} dx$$

$$= x^2 \int \frac{e^{-2 \ln(x)}}{x^4} dx$$

$$= x^2 \int \frac{\cancel{e^{\ln}}(x^{-2})}{x^4} dx$$

$$= x^2 \int \frac{x^{-2}}{x^4} dx$$

$$= x^2 \int x^{-6}\,dx$$

$$= x^2 \left(\frac{x^{-6+1}}{-6+1} \right)$$

$$= \boxed{-\frac{1}{5}x^{-3}}$$

Answer:

$$y_h = c_1 y_1 + c_2 y_2 = c_1 x^2 + c_2\left(-\frac{1}{5}x^{-3}\right)$$