Math 2150-01

11/12/25

$\boxed{\text{HW 8} \\ \text{1(d)}}$ Find y_p for

$$y'' + 3y' - 10y = 6e^{4x}$$

<u>Guess:</u> $y_p = Ae^{4x}$

$$y_p' = 4Ae^{4x}$$

$$y_p'' = 16Ae^{4x}$$

Plug it in to get:

$$(16Ae^{4x}) + 3(4Ae^{4x}) - 10(Ae^{4x}) = 6e^{4x}$$

$$18Ae^{4x} = 6e^{4x}$$

$$A = \frac{1}{3}$$

So, $\boxed{y_p = \frac{1}{3}e^{4x}}$

HW 9
1(b)

Solve $y'' + y = \sin(x)$

Step 1: Solve $y'' + y = 0$

$$r^2 + 1 = 0$$

$$r = \frac{-0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} = \frac{\pm\sqrt{-4}}{2}$$

$$= \frac{\pm\sqrt{4}\sqrt{-1}}{2} = \pm\frac{2}{2}i = \pm i$$

$$= \underbrace{0 \pm 1 \cdot i}_{\alpha \pm \beta i}$$

$\alpha = 0, \beta = 1$

$$y_h = c_1 e^{0x}\cos(1\cdot x) + c_2 e^{0x}\sin(1\cdot x)$$

$$= c_1 \boxed{\cos(x)} + c_2 \boxed{\sin(x)}$$

$\boxed{e^{0x} = e^0 = 1}$

$\boxed{y_1}$ $\boxed{y_2}$

Step 2: Find y_p for $y'' + y = \sin(x)$.

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix}$$

$$= \cos^2(x) - (-\sin^2(x))$$

$$= 1$$

$$v_1 = \int \frac{-y_2 b(x)}{W(y_1, y_2)} dx = \int \frac{-\sin(x)\sin(x)}{1} dx$$

$$= -\int \sin^2(x) dx = -\int \left[\frac{1}{2} - \frac{1}{2}\cos(2x)\right] dx$$

$$= -\left[\frac{1}{2}x - \frac{1}{2}\left(\frac{1}{2}\sin(2x)\right)\right]$$

$$= \boxed{-\frac{1}{2}x + \frac{1}{4}\sin(2x)}$$

$$V_2 = \int \frac{y_1 b(x)}{W(y_1, y_2)}\,dx = \int \frac{\cos(x)\sin(x)}{1}\,dx$$

$$= \int \underbrace{\cos(x)}\,\underbrace{\sin(x)}_{u=\sin(x)}\,\underbrace{dx}$$

$$du = \cos(x)\,dx$$

$$= \int u\,du = \frac{1}{2}u^2 = \frac{1}{2}\sin^2(x)$$

$$y_p = V_1 y_1 + V_2 y_2$$

$$= \left(-\frac{1}{2}x + \frac{1}{4}\sin(2x)\right)\cos(x)$$
$$+ \left(\frac{1}{2}\sin^2(x)\right)\sin(x)$$

$$\boxed{\begin{aligned} y &= y_h + y_p \\ &= c_1\cos(x) + c_2\sin(x) \\ &\quad + \left(-\frac{1}{2}x + \frac{1}{4}\sin(2x)\right)\cos(x) \\ &\quad + \left(\frac{1}{2}\sin^2(x)\right)\sin(x) \end{aligned}}$$

$\boxed{\text{HW 10 } 1(c)}$

Given that $y_1 = \ln(x)$ solves

on , find the general solution.

Divide by x to get

$$y'' + \underbrace{\frac{1}{x}}\, y' = 0$$

$$\boxed{a_1(x) = \frac{1}{x}}$$

$$y_2 = y_1 \cdot \int \frac{e^{-\int a_1(x)\,dx}}{y_1^2}\, dx$$

$$= \ln(x) \cdot \int \frac{e^{-\int \frac{1}{x} dx}}{(\ln(x))^2} dx$$

$$= \ln(x) \cdot \int \frac{e^{-\ln|x|}}{(\ln(x))^2} dx$$

$$= \ln(x) \cdot \int \frac{e^{\ln(x^{-1})}}{(\ln(x))^2} dx$$

$$\boxed{-\ln|x| = -\ln(x) = \ln(x^{-1})}$$

$$\boxed{\begin{array}{c} I = (0, \infty) \\ x > 0 \\ |x| = x \end{array}}$$

$$= \ln(x) \cdot \int \frac{x^{-1}}{(\ln(x))^2}\,dx$$

$$\boxed{e^{\ln|A|} = A}$$

<u>Note:</u> $\displaystyle \int \frac{x^{-1}}{(\ln(x))^2}\,dx = \int \frac{1}{u^2}\,du$

$$\boxed{\begin{aligned} u &= \ln(x) \\ du &= \tfrac{1}{x}\,dx = x^{-1}dx \end{aligned}}$$

$$= \int u^{-2}\,du = \frac{u^{-1}}{-1} = -\frac{1}{u} = \frac{-1}{\ln(x)}$$

$$= \ln(x) \cdot \left[-\frac{1}{\ln(x)}\right] = -1$$

So, $y_1 = \ln(x)$, $y_2 = -1$

$y_h = c_1 \ln(x) + c_2(-1)$ is

the general solution to

$xy'' + y' = 0$ on $I = (0, \infty)$

$\boxed{\text{HW 11}}$

1(d) Find a power series for $f(x) = \frac{1}{x}$ centered at $x_0 = 1$.

Hint: Use:

$$\ln(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots$$

which has radius of convergence $r = 1$

Differentiate the $\ln(x)$ formula to get:

$$\frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 + \cdots$$

with radius of convergence $r = 1$.

$\boxed{\text{HW 12}}$

(2) Find the first <u>four</u> non-zero terms of a power series solution to

$$y'' - (x+1)y' + x^2 y = 0$$

$$y'(0) = 1, \quad y(0) = 1$$

What is the radius of convergence?

$$y'' - \underbrace{(x+1)}\, y' + \underbrace{x^2}\, y = 0$$

| | |

Coefficients are all polynomials
they are all analytic at $x_0=0$
and have $r=\infty$

Our solution will have radius of convergence $r=\infty$.

Let's find the solution:

$$y(x) = y(0) + y'(0)(x-0) + \frac{y''(0)}{2!}(x-0)^2 + \frac{y'''(0)}{3!}(x-0)^3 + \cdots$$

We know: $\boxed{y(0)=1,\ y'(0)=1}$

$$\boxed{\begin{aligned} &y'' - (x+1)y' + x^2 y = 0 \\ &y'' = (x+1)y' - x^2 y \end{aligned}}$$

$$y''(0) = (0+1)\left[y'(0)\right] - (0)^2\left[y(0)\right]$$

$$= (1)[1] - (0)^2[1] = 1$$

$$y'' = (x+1)y' - x^2 y$$

$$y''' = \left(1 \cdot y' + (x+1)y''\right)$$

$$- \left(2xy + x^2 y'\right)$$

$$y'''(0) = \left(\overbrace{y'(0)}^{1} + \overbrace{(0+1)}^{1}\,\overbrace{y''(0)}^{1}\right)$$

$$- \left(\underbrace{2(0)}_{0}\,\underbrace{y(0)}_{1} + \underbrace{0^2}_{0}\,\underbrace{y'(0)}_{1}\right)$$

$$= 2$$

So,

$$y(x) = y(0) + y'(0)\,x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \cdots$$

$$= 1 + 1 \cdot x + \frac{1}{2!}x^2 + \frac{2}{3!}x^3 + \cdots$$

$$= 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots$$

with radius of convergence $r = \infty$