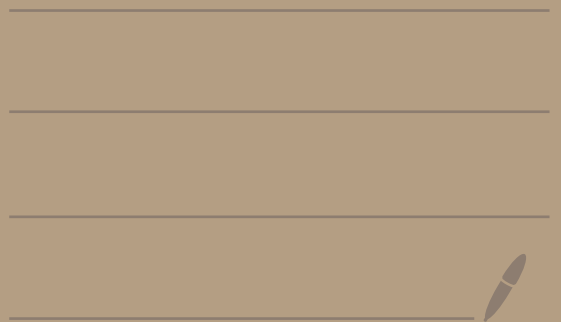


Math 2150-01

11/12/25



HW 8  
1(d)

Find  $y_p$  for

$$y'' + 3y' - 10y = 6e^{4x}$$

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Guess:  $y_p = Ae^{4x}$

$$y_p' = 4Ae^{4x}$$

$$y_p'' = 16Ae^{4x}$$

Plug it in to get:

$$(16Ae^{4x}) + 3(4Ae^{4x}) - 10(Ae^{4x}) = 6e^{4x}$$

$$18Ae^{4x} = 6e^{4x}$$

$$A = \frac{1}{3}$$

So,  $y_p = \frac{1}{3}e^{4x}$

HW 9  
1(b)

Solve  $y'' + y = \sin(x)$

Step 1: Solve  $y'' + y = 0$

$$r^2 + 1 = 0$$

$$r = \frac{-0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} = \frac{\pm \sqrt{-4}}{2}$$

$$= \frac{\pm \sqrt{4} \sqrt{-1}}{2} = \pm \frac{2}{2} \bar{i} = \pm \bar{i}$$

$$= \frac{0 \pm 1 \cdot \bar{i}}{\alpha \pm \beta \bar{i}}$$

$$\alpha \pm \beta \bar{i}$$

$$\alpha = 0, \beta = 1$$

$$y_h = c_1 e^{0x} \cos(1 \cdot x) + c_2 e^{0x} \sin(1 \cdot x)$$

$$= c_1 \boxed{\cos(x)} + c_2 \boxed{\sin(x)}$$

$$\boxed{e^{0x} = e^0 = 1}$$

$$\boxed{y_1}$$

$$\boxed{y_2}$$

Step 2: Find  $y_p$  for  $y'' + y = \sin(x)$ .

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix}$$

$$= \cos^2(x) - (-\sin^2(x))$$

$$= 1$$

---

$$V_1 = \int \frac{-y_2 b(x)}{W(y_1, y_2)} dx = \int \frac{-\sin(x)\sin(x)}{1} dx$$

$$= - \int \sin^2(x) dx = - \int \left[ \frac{1}{2} - \frac{1}{2} \cos(2x) \right] dx$$

$$= - \left[ \frac{1}{2}x - \frac{1}{2} \left( \frac{1}{2} \sin(2x) \right) \right]$$

$$= \boxed{-\frac{1}{2}x + \frac{1}{4} \sin(2x)}$$

$$V_2 = \int \frac{y_1 b(x)}{W(y_1, y_2)} dx = \int \frac{\cos(x) \sin(x)}{1} dx$$

$$= \int \underbrace{\cos(x)}_{\substack{u = \sin(x) \\ du = \cos(x) dx}} \sin(x) dx$$

$$= \int u du = \frac{1}{2} u^2 = \frac{1}{2} \sin^2(x)$$

$$y_p = V_1 y_1 + V_2 y_2$$

$$= \left(-\frac{1}{2}x + \frac{1}{4}\sin(2x)\right) \cos(x) \\ + \left(\frac{1}{2}\sin^2(x)\right) \sin(x)$$

---

$$y = y_h + y_p \\ = c_1 \cos(x) + c_2 \sin(x) \\ + \left(-\frac{1}{2}x + \frac{1}{4}\sin(2x)\right) \cos(x) \\ + \left(\frac{1}{2}\sin^2(x)\right) \sin(x)$$

Hw 10  
1(=)

Given that  $y_1 = \ln(x)$  solves

on  
general solution. find the

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Divide by  $x$  to get

$$y'' + \underbrace{\frac{1}{x}}_{a_1(x)} y' = 0$$

$$a_1(x) = \frac{1}{x}$$

$$y_2 = y_1 \cdot \int \frac{e^{-\int a_1(x) dx}}{y_1^2} dx$$

$$= \ln(x) \cdot \int \frac{e^{-\int \frac{1}{x} dx}}{(\ln(x))^2} dx$$

$$= \ln(x) \cdot \int \frac{e^{-\ln|x|}}{(\ln(x))^2} dx$$

$$= \ln(x) \cdot \int \frac{e^{\ln(x^{-1})}}{(\ln(x))^2} dx$$



$$-\ln|x| = -\ln(x) = \ln(x^{-1})$$



$$\begin{aligned} I &= (0, \infty) \\ x &> 0 \\ |x| &= x \end{aligned}$$

$$= \ln(x) \cdot \int \frac{x^{-1}}{(\ln(x))^2} dx$$



$$e^{\ln(A)} = A$$

Note:

$$\int \frac{x^{-1}}{(\ln(x))^2} dx = \int \frac{1}{u^2} du$$

$$u = \ln(x) \\ du = \frac{1}{x} dx = x^{-1} dx$$

$$= \int u^{-2} du = \frac{u^{-1}}{-1} = -\frac{1}{u} = -\frac{1}{\ln(x)}$$

$$= \ln(x) \cdot \left[ -\frac{1}{\ln(x)} \right] = -1$$

$$\text{So, } y_1 = \ln(x), y_2 = -1$$

$$y_h = c_1 \ln(x) + c_2(-1) \text{ is}$$

the general solution to

$$xy'' + y' = 0 \text{ on } I = (0, \infty)$$

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HW 11

(d) Find a power series for  
 $f(x) = \frac{1}{x}$  centered at  $x_0 = 1$ .

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Hint: Use:

$$\ln(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots$$

which has radius of convergence  $r = 1$

---

Differentiate the  $\ln(x)$  formula to get:

$$\frac{1}{x} = 1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots$$

with radius of convergence  $r=1$ .

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## HW 12

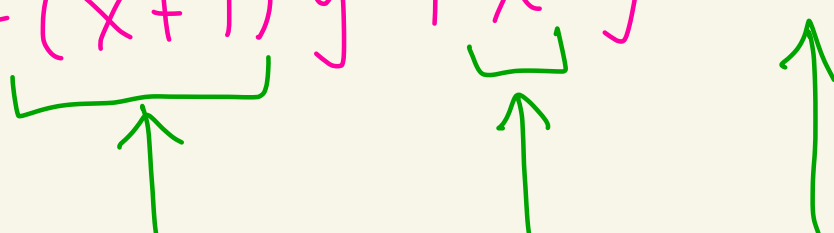
(2) Find the first four non-zero terms of a power series solution to

$$y'' - (x+1)y' + x^2y = 0$$

$$y'(0) = 1, y(0) = 1$$

What is the radius of convergence?

---

$$y'' - (x+1)y' + x^2y = 0$$


1                      1                      1  
Coefficients are all polynomials  
they are all analytic at  $x_0 = 0$   
and have  $r = \infty$

Our solution will have radius  
of convergence  $r = \infty$ .

Let's find the solution:

$$y(x) = y(0) + y'(0)(x-0) + \frac{y''(0)}{2!}(x-0)^2 + \frac{y'''(0)}{3!}(x-0)^3 + \dots$$

We know:  $y(0) = 1, y'(0) = 1$

$$y'' - (x+1)y' + x^2y = 0$$

$$y'' = (x+1)y' - x^2y$$

$$\begin{aligned}
 y''(0) &= (0+1)[y'(0)] - (0)^2[y(0)] \\
 &= (1)[1] - (0)^2[1] = 1
 \end{aligned}$$

$$y'' = (x+1)y' - x^2y$$

$$y''' = (1 \cdot y' + (x+1)y'')$$

$$- (2xy + x^2y')$$

$$y'''(0) = \left( \overbrace{y'(0)}^1 + \overbrace{(0+1)}^1 \overbrace{y''(0)}^1 \right)$$

$$- \left( \underbrace{2(0)}_0 \underbrace{y(0)}_1 + \underbrace{0^2}_0 \underbrace{y'(0)}_1 \right)$$

$$= 2$$

So,

$$y(x) = y(0) + y'(0)x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \dots$$

$$= 1 + 1 \cdot x + \frac{1}{2!}x^2 + \frac{2}{3!}x^3 + \dots$$

$$= 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \dots$$

with radius of convergence  $r = \infty$

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