Math 2150-01
11/10/25

$\boxed{\text{HW 8 - 1(c)}}$

Solve

$$y'' + 4y' + 4y = 4x^2 - 8x.$$

Step 1: Solve $y'' + 4y' + 4y = 0$.

$$r^2 + 4r + 4 = 0$$

$$(r+2)(r+2) = 0$$

$$r = -2, -2$$

$$y_h = c_1 e^{-2x} + c_2 x e^{-2x}$$

Step 2: Find y_p.

Guess: $y_p = Ax^2 + Bx + C$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

Plug into $y'' + 4y' + 4y = 4x^2 - 8x$ to get

$$\underbrace{(2A)}_{y_p''} + 4\underbrace{(2Ax+B)}_{y_p'} + 4\underbrace{(Ax^2+Bx+c)}_{y_p} = 4x^2 - 8x$$

$$2A + 8Ax + 4B + 4Ax^2 + 4Bx + 4C = 4x^2 - 8x$$

$$\underbrace{4A}\,x^2 + \underbrace{(8A+4B)}\,x + \underbrace{(2A+4B+4C)}_{0} = 4x^2 - 8x$$

Need

$$\boxed{\begin{aligned} 4A &= 4 \\ 8A + 4B &= -8 \\ 2A + 4B + 4C &= 0 \end{aligned}}$$

$$\boxed{A = 1}$$

$$\boxed{\begin{aligned} 8(1) + 4B &= -8 \\ B &= -4 \end{aligned}}$$

$$\boxed{\begin{aligned} 2(1) + 4(-4) + 4C &= 0 \\ C = \frac{14}{4} &= \frac{7}{2} \end{aligned}}$$

So, $y_p = x^2 - 4x + \frac{7}{2}$

$$\boxed{\begin{aligned} &\underline{\text{Answer:}} \\ &y = y_h + y_p = c_1 e^{-2x} + c_2 x e^{-2x} \\ &\qquad + x^2 - 4x + 7/2 \end{aligned}}$$

$\boxed{\text{HW 9 - 1(a)}}$

Find the general solution to

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

given that the general solution to

$$y'' - 4y' + 4y = 0$$

is

$$y_h = c_1 \boxed{e^{2x}} + c_2 \boxed{x e^{2x}}$$

$$\boxed{y_1 = e^{2x}} \qquad \boxed{y_2 = xe^{2x}}$$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 1 \cdot e^{2x} + x(2e^{2x}) \end{vmatrix}$$

$$= (e^{2x})(e^{2x} + 2xe^{2x})$$

$$- (2e^{2x})(xe^{2x})$$

$$= e^{4x} + 2xe^{4x} - 2xe^{4x}$$

$$= e^{4x}$$

$$v_1 = \int \frac{-y_2 b(x)}{W(y_1, y_2)} \, dx = \int \frac{-xe^{2x}(x+1)e^{2x}}{e^{4x}} \, dx$$

$$= \int \frac{-x(x+1)\cancel{e^{4x}}}{\cancel{e^{4x}}} \, dx = \int (-x^2 - x) \, dx$$

$$= \boxed{-\frac{1}{3}x^3 - \frac{1}{2}x^2}$$

$$v_2 = \int \frac{y_1 b(x)}{W(y_1, y_2)} \, dx = \int \frac{e^{2x}(x+1)e^{2x}}{e^{4x}} \, dx$$

$$= \int \frac{\cancel{e^{4x}}(x+1)}{\cancel{e^{4x}}} \, dx = \int (x+1) \, dx$$

$$= \boxed{\frac{1}{2}x^2 + x}$$

Then,

$$y_p = v_1 y_1 + v_2 y_2$$
$$= \left(-\frac{1}{3}x^3 - \frac{1}{2}x^2\right)e^{2x} + \left(\frac{1}{2}x^2 + x\right)xe^{2x}$$

$$\boxed{\begin{aligned}&\underline{\text{Answer:}}\\ &y = y_h + y_p\\ &\quad = c_1 e^{2x} + c_2 x e^{2x}\\ &\quad\quad + \left(-\frac{1}{3}x^3 - \frac{1}{2}x^2\right)e^{2x} + \left(\frac{1}{2}x^2 + x\right)xe^{2x}\end{aligned}}$$

$\boxed{\text{HW 10}}$ – #1(a)

Given that $y_1 = x^4$ is a solution to

$$x^2 y'' - 7xy' + 16y = 0$$

on $I = (0, \infty)$, find the general solution.

Divide by x^2 to get:

$$y'' \boxed{- \frac{7}{x}} y' + \frac{16}{x^2} y = 0$$

$a_1(x) = -\frac{7}{x}$

We have

$$y_2 = y_1 \cdot \int \frac{e^{-\int a_1(x)\,dx}}{y_1^2}\,dx$$

$$= x^4 \int \frac{e^{-\int -\frac{7}{x} dx}}{(x^4)^2} dx$$

$$= x^4 \int \frac{e^{7\int \frac{1}{x} dx}}{x^8} dx$$

$$= x^4 \int \frac{e^{7\ln|x|}}{x^8} dx$$

$I = (0, \infty)$

$x > 0$

$|x| = x$

$$= x^4 \int \frac{e^{7\ln(x)}}{x^8} dx$$

$A\ln(B)$
$= \ln(B^A)$

$$= x^4 \int \frac{e^{\ln(x^7)}}{x^8}\,dx$$

$$e^{\ln(A)} = A$$

$$= x^4 \int \frac{x^7}{x^8}\,dx$$

$$= x^4 \int \frac{1}{x}\,dx$$

$$= x^4 \ln|x|$$

$x > 0$

$|x| = x$

$$= x^4 \ln(x)$$

$$\boxed{\begin{array}{l}\underline{\text{Answer:}} \\ y_h = c_1 y_1 + c_2 y_2 = c_1 x^4 + c_2 x^4 \ln(x)\end{array}}$$

$$\boxed{\text{HW 11 - 1(a)}}$$

Find the power series for

$f(x) = x^3 + x$

centered at $x_0 = 1$.

$f(x) = x^3 + x \longrightarrow f(1) = 1^3 + 1 = 2$

$f'(x) = 3x^2 + 1 \longrightarrow f'(1) = 3(1)^2 + 1 = 4$

$f''(x) = 6x \longrightarrow f''(1) = 6(1) = 6$

$f'''(x) = 6 \longrightarrow f'''(1) = 6$

$f^{(k)}(x) = 0 \longrightarrow f^{(k)}(1) = 0$

$k \geq 4$ $\qquad$ $k \geq 4$

$$f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \cdots$$

$$= 2 + 4(x-1) + \frac{6}{2!}(x-1)^2$$
$$+ \frac{6}{3!}(x-1)^3 + \underbrace{0 + 0 + 0 + \cdots}_{0}$$
$$= \boxed{2 + 4(x-1) + 3(x-1)^2 + (x-1)^3}$$

Hw 12 #1

Find the first four non-zero terms for a power series solution to $\boxed{y'' - xy = 0,\ y'(0) = 1,\ y(0) = 2}$

What's the radius of convergence of the solution?

$$y'' - xy = 0$$

coefficients: $-x, 0$

Coefficients:

$\left.\begin{matrix} -x \\ 0 \end{matrix}\right\}$ both polynomials that are are analytic at $x_0 = 0$ with radius of convergence $r = \infty$

Our answer will have radius of convergence $r = \infty$.

Let's find it.

$\boxed{\underline{\text{Given:}}\ y(0) = 2,\ y'(0) = 1}$

<u>Find: $y''(0)$</u>:

$$y'' - xy = 0$$

$$y'' = xy$$

$$y''(0) = (0)[y(0)]$$
$$= 0(2)$$
$$= 0$$
$$y''(0) = 0$$

<u>Find $y'''(0)$:</u>

$$y'' = xy$$
$$y''' = 1 \cdot y + x \cdot y'$$
$$y'''(0) = y(0) + 0 \cdot [y'(0)]$$
$$= 2 + 0 \cdot [1]$$
$$= 2$$
$$y'''(0) = 2$$

<u>Find $y''''(0)$:</u>

$$y''' = y + xy'$$
$$y'''' = y' + 1 \cdot y' + x \cdot y''$$
$$y'''' = 2y' + xy''$$

$$y''''(0) = 2[y'(0)] + (0)[y''(0)]$$
$$= 2[1] + 0$$
$$= 2$$
$$y''''(0) = 2$$

Then,

$$y(x) = y(0) + y'(0)\cdot x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + \frac{y''''(0)}{4!}x^4 + \cdots$$

$$= 2 + 1\cdot x + \frac{0}{2!}x^2 + \frac{2}{3!}x^3 + \frac{2}{4!}x^4 + \cdots$$

$$= 2 + x + \frac{1}{3}x^3 + \frac{1}{12}x^4 + \cdots$$