Math 2150-01

10/8/25

HW 3

(3) Solve

$$\frac{dy}{dx} + 2xy = x$$

$$y(0) = -3$$

on $I = (-\infty, \infty)$

$$y' + 2xy = x$$

$$A(x) = \int 2x\,dx = x^2$$

multiply by $e^{A(x)} = e^{x^2}$ to get

$$\underbrace{e^{x^2}y' + e^{x^2}2xy}_{(e^{A(x)}\cdot y)'} = \underbrace{e^{x^2}x}_{xe^{x^2}}$$

$$(e^{x^2}\cdot y)' = xe^{x^2}$$

Integrate to get

$$e^{x^2}\cdot y = \underbrace{\int xe^{x^2}\,dx}$$

$$\int xe^{x^2}\,dx = \int \frac{1}{2}e^u\,du = \frac{1}{2}e^u + C$$

$$= \frac{1}{2}e^{x^2} + C$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

So,

$$e^{x^2} \cdot y = \frac{1}{2} e^{x^2} + C$$

$$y = \frac{1}{2} + \frac{C}{e^{x^2}}$$

Now find C using $\underbrace{y(0) = -3}_{\substack{x=0 \\ y=-3}}$.

We get:

$$-3 = \frac{1}{2} + \frac{C}{e^{0^2}}$$

$$-3 = \frac{1}{2} + C$$

$$-7/2 = C$$

Thus,

$$\boxed{y = \frac{1}{2} - \frac{7/2}{e^{x^2}}}$$

HW 4

① (d) Solve the separable IVP:

$$\frac{dy}{dx} = 6y^2x, \quad y(0) = \frac{1}{12}$$

$$\frac{dy}{dx} = 6y^2x$$

$$\frac{dy}{y^2} = 6x\,dx$$

$$\int y^{-2}\,dy = \int 6x\,dx$$

$$\frac{y^{-1}}{-1} = 3x^2 + C$$

$$-\frac{1}{y} = 3x^2 + C$$

$$-\frac{1}{3x^2+C} = y$$

Use $\underbrace{y(0) = \frac{1}{12}}_{x=0,\ y=\frac{1}{12}}$ to find C.

We get: $-\dfrac{1}{3(0)^2+C} = \dfrac{1}{12}$

$$-\frac{1}{c} = \frac{1}{12}$$

$$c = -12$$

$$\boxed{\text{Thus, } \quad y = \frac{-1}{3x^2-12}}$$

$\boxed{\text{HW 5}}$

1 (e)

$$\underbrace{(2y^2x-3)}_{M} + \underbrace{(2yx^2+4)}_{N}y' = 0$$

Check if exact.
If so, solve.

Check $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

$$\frac{\partial M}{\partial y} = 4yx$$

equal

$$\frac{\partial N}{\partial x} = 4yx$$

It's exact.

Need to find f where

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x} &= 2y^2x - 3 \quad (1) \\ \frac{\partial f}{\partial y} &= 2yx^2 + 4 \quad (2) \end{aligned}}$$

$$\boxed{\begin{aligned} \frac{\partial f}{\partial x} &= M \\ \frac{\partial f}{\partial y} &= N \end{aligned}}$$

Integrate (1) with respect to x:

$$f(x,y) = y^2x^2 - 3x + \underbrace{C(y)}_{\text{constant with respect to } x}$$

Integrate ② with respect to y:

$$f(x,y) = y^2x^2 + 4y + \underbrace{D(x)}_{\text{constant with respect to } y}$$

Set equal:

$$\cancel{y^2x^2} - 3x + C(y) = \cancel{y^2x^2} + 4y + D(x)$$

$$-3x + C(y) = 4y + D(x)$$

Set $C(y) = 4y$, $D(x) = -3x$

So,

$$f(x,y) = y^2x^2 - 3x + C(y)$$
$$= y^2x^2 - 3x + 4y$$

$$\boxed{\begin{array}{l}\underline{\text{Answer:}} \\ \qquad y^2x^2 - 3x + 4y = c \\ \text{where } c \text{ is any constant}\end{array}}$$

HW 6 | 2(e,f) given 2(a,b,c,d)

Suppose you know the general solution to

$$x^2y'' - 5xy' + 8y = 0$$

on $I = (-\infty, \infty)$ is

$$y_h = c_1 x^2 + c_2 x^4$$

and you know that a particular solution to

$$x^2 y'' - 5xy' + 8y = 24$$

is $y_p = 3$.

(e) What is the general solution to

$$x^2 y'' - 5xy' + 8y = 24 \, ?$$

Answer:

$$\boxed{y = y_h + y_p = c_1 x^2 + c_2 x^4 + 3}$$

(f) What is a solution to

$$x^2y'' - 5xy' + 8y = 24$$
$$y(1) = -1, \quad y'(1) = 0$$

We know the general solution is

$$y = c_1 x^2 + c_2 x^4 + 3.$$

Let's make it solve

$$y(1) = -1, \quad y'(1) = 0.$$

Have

$$y = c_1 x^2 + c_2 x^4 + 3$$
$$y' = 2c_1 x + 4c_2 x^3$$

$$\boxed{\begin{array}{l} y(1) = -1 \\ y'(1) = 0 \end{array}} \longrightarrow \boxed{\begin{array}{l} -1 = c_1(1)^2 + c_2(1)^4 + 3 \\ 0 = 2c_1(1) + 4c_2(1)^3 \end{array}}$$

$$\boxed{\begin{aligned} c_1 + c_2 &= -4 \quad (1) \\ 2c_1 + 4c_2 &= 0 \quad (2) \end{aligned}}$$

(1) gives $c_1 = -4 - c_2$.

Plug into (2) to get

$$2(-4 - c_2) + 4c_2 = 0.$$

So, $2c_2 = 8$.

Thus, $\boxed{c_2 = 4.}$

So, $\boxed{c_1 = -4 - c_2 = -4 - 4 = -8.}$

$\boxed{\text{\underline{Answer:}} \quad y = -8x^2 + 4x^4 + 3}$

$\boxed{\text{HW 7}}$

$\boxed{\text{1 (d)}}$

Solve $y'' - 2y' + 2y = 0$

The characteristic poly

$$r^2 - 2r + 2 = 0$$

has roots

$$r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm \sqrt{4}\sqrt{-1}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$i = \sqrt{-1}$

$$= \frac{2}{2} \pm \frac{2i}{2}$$

$$= 1 \pm i$$

$$= \underbrace{1 \pm 1 \cdot i}_{\alpha \pm \beta i}$$

$\alpha = 1$

$\beta = 1$

So,

$$y_h = c_1 e^{\alpha x} \cos(\beta x) + c_2 e^{\alpha x} \sin(\beta x)$$

$$= c_1 e^{x} \cos(x) + c_2 e^{x} \sin(x)$$