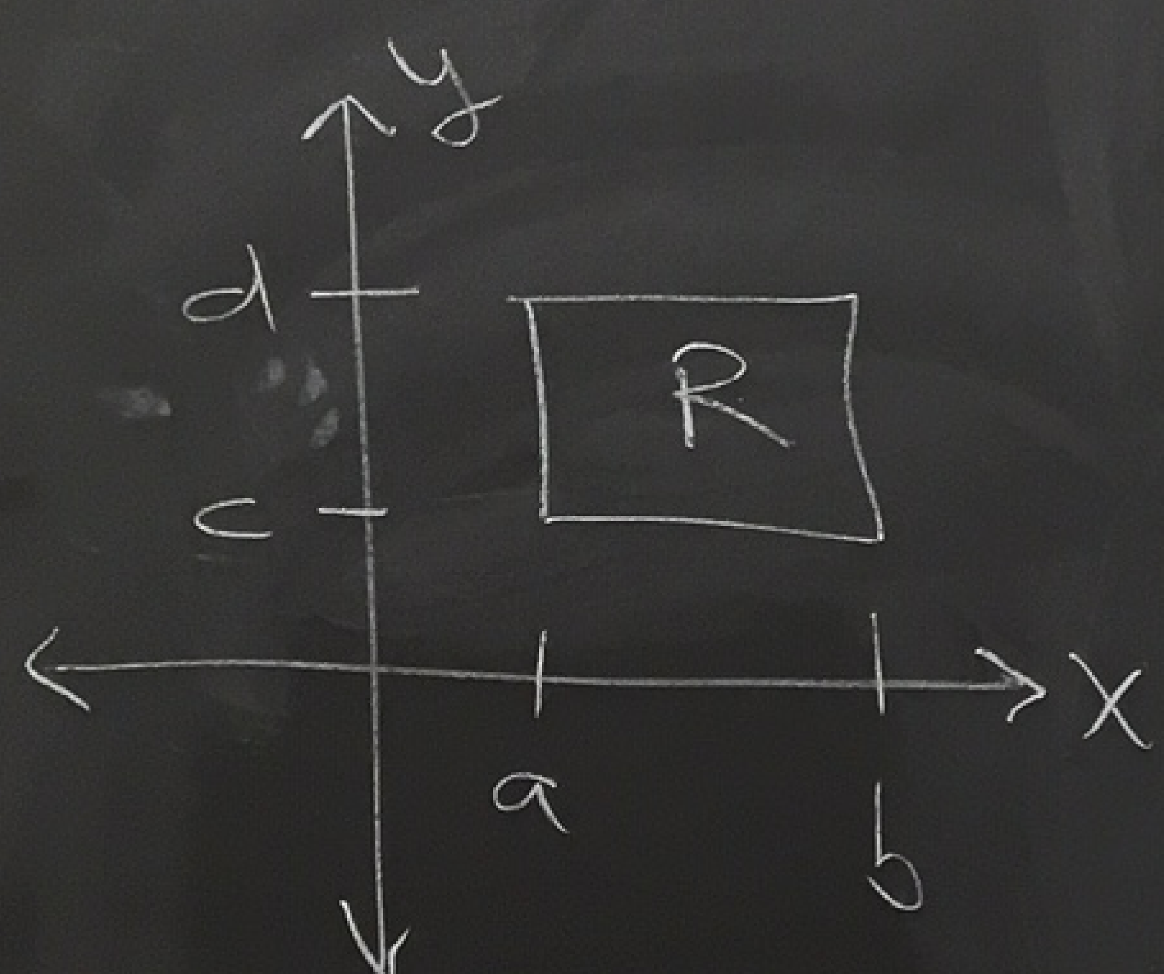
Thurs
10/10

Recall from last time

Fubini's Theorem



If $f(x,y)$ is continuous on

$$R = \left\{ (x,y) \;\middle|\; \begin{matrix} a \le x \le b \\ c \le y \le d \end{matrix} \right\}$$

then

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

Ex: Evaluate $\iint_R y\sin(xy)\,dA$

where $R = \left\{(x,y) \,\middle|\, \begin{array}{l} 1 \le x \le 2 \\ 0 \le y \le \pi \end{array}\right\}$

$$\iint_R y\sin(xy)\,dA = \int_1^2 \left[\int_0^\pi y\sin(xy)\,dy\right] dx =$$

$u \longrightarrow$

LIATE

L = log, I = inverse trig, A = algebraic, T = trig, E = exp.

$$\int y\sin(xy)\,dy = y\left(-\frac{1}{x}\cos(xy)\right) - \int -\frac{1}{x}\cos(xy)\,dy$$

$u = y \quad dv = \sin(xy)\,dy$

$du = dy \quad v = -\frac{1}{x}\cos(xy)$

$$= -\frac{y}{x}\cos(xy) + \frac{1}{x}\int \cos(xy)\,dy$$

$$= -\frac{y}{x}\cos(xy) + \frac{1}{x}\cdot\frac{1}{x}\sin(xy)$$

$$\int u\,dv = uv - \int v\,du$$

$$\int f(ct)\,dt = \frac{1}{c}F(ct)$$

$$\boxed{F' = f}$$

$$= \int_1^2 \left[-\frac{y}{x}\cos(xy) + \frac{1}{x^2}\sin(xy)\right]_{y=0}^{\pi} dx$$

$$= \int_1^2 \left[\left(-\frac{\pi}{x}\cos(\pi x) + \frac{1}{x^2}\sin(\pi x)\right) - \left(0 + \frac{1}{x^2}\sin(0)\right)\right] dx$$

$$= \int_1^2 \left(\underbrace{-\frac{\pi}{x}\cos(\pi x)}_{1} + \underbrace{\frac{1}{x^2}\sin(\pi x)}_{2}\right) dx = \underbrace{-\pi\left[\frac{1}{\pi x}\sin(\pi x)\Big|_{x=1}^{2} + \frac{1}{\pi}\int_1^2 \frac{1}{x^2}\sin(\pi x)\,dx\right]}_{1} + \underbrace{\int_1^2 \frac{1}{x^2}\sin(\pi x)\,dx}_{2}$$

$$\int \frac{1}{x}\cos(\pi x)\,dx = \frac{1}{\pi x}\sin(\pi x) - \frac{1}{\pi}\int\left(-\frac{1}{x^2}\right)\sin(\pi x)\,dx$$

$$u = x^{-1} \quad dv = \cos(\pi x)\,dx$$

$$du = -x^{-2}\,dx \quad v = \frac{1}{\pi}\sin(\pi x)$$

$$-\pi\left[\frac{1}{2\pi}\underbrace{\sin(2\pi)}_{0}-\frac{1}{\pi}\underbrace{\sin(\pi)}_{0}\right]\underbrace{-\int_1^2\frac{1}{x^2}\sin(\pi x)\,dx+\int_1^2\frac{1}{x^2}\sin(\pi x)\,dx}_{0}$$

Yes

$$=0$$

$$x\,)dx\Big]+\underbrace{\int_1^2\frac{1}{x^2}\sin(\pi x)dx}_{2}$$

Or we could change the ordering

$$\iint_R y\sin(xy)\,dA=\int_0^\pi\left[\int_1^2 y\sin(xy)\,dx\right]dy=\int_0^\pi\left[y\left(\frac{-1}{y}\cos(xy)\right)\Bigg|_{x=1}^{2}\right]dy$$

$$=\int_0^\pi\left(-\cos(2y)-(-\cos(y))\right)dy=\left[-\frac{1}{2}\sin(2y)+\sin(y)\right]\Bigg|_{y=0}^{\pi}=\left[\left(-\frac{1}{2}\sin(2\pi)+\frac{1}{2}\sin(\pi)\right)-\left(-\frac{1}{2}\sin(0)+\sin(0)\right)\right]=0$$

13.1

(20) $$\iint_R \frac{y}{\sqrt{1-x^2}}\, dA = \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \left[\int_1^2 \frac{1}{\sqrt{1-x^2}} \cdot y\, dy \right] dx$$

$$R = \left\{ (x,y) \;\middle|\; \begin{array}{l} \frac{1}{2} \le x \le \frac{\sqrt{3}}{2} \\ 1 \le y \le 2 \end{array} \right\}$$

$$= \int_{1/2}^{\frac{\sqrt{3}}{2}} \left[\frac{1}{\sqrt{1-x^2}} \cdot \frac{y^2}{2} \Bigg|_{y=1}^{y=2} \right] dx$$

$$\boxed{\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C}$$

$$= \int_{1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}} \left(\frac{2^2}{2} - \frac{1^2}{2} \right) dx =$$

$= \frac{3}{2}$

$= \frac{3}{2}$

$= \frac{3}{2}$

$= \frac{\sqrt{3}}{2}$

dx

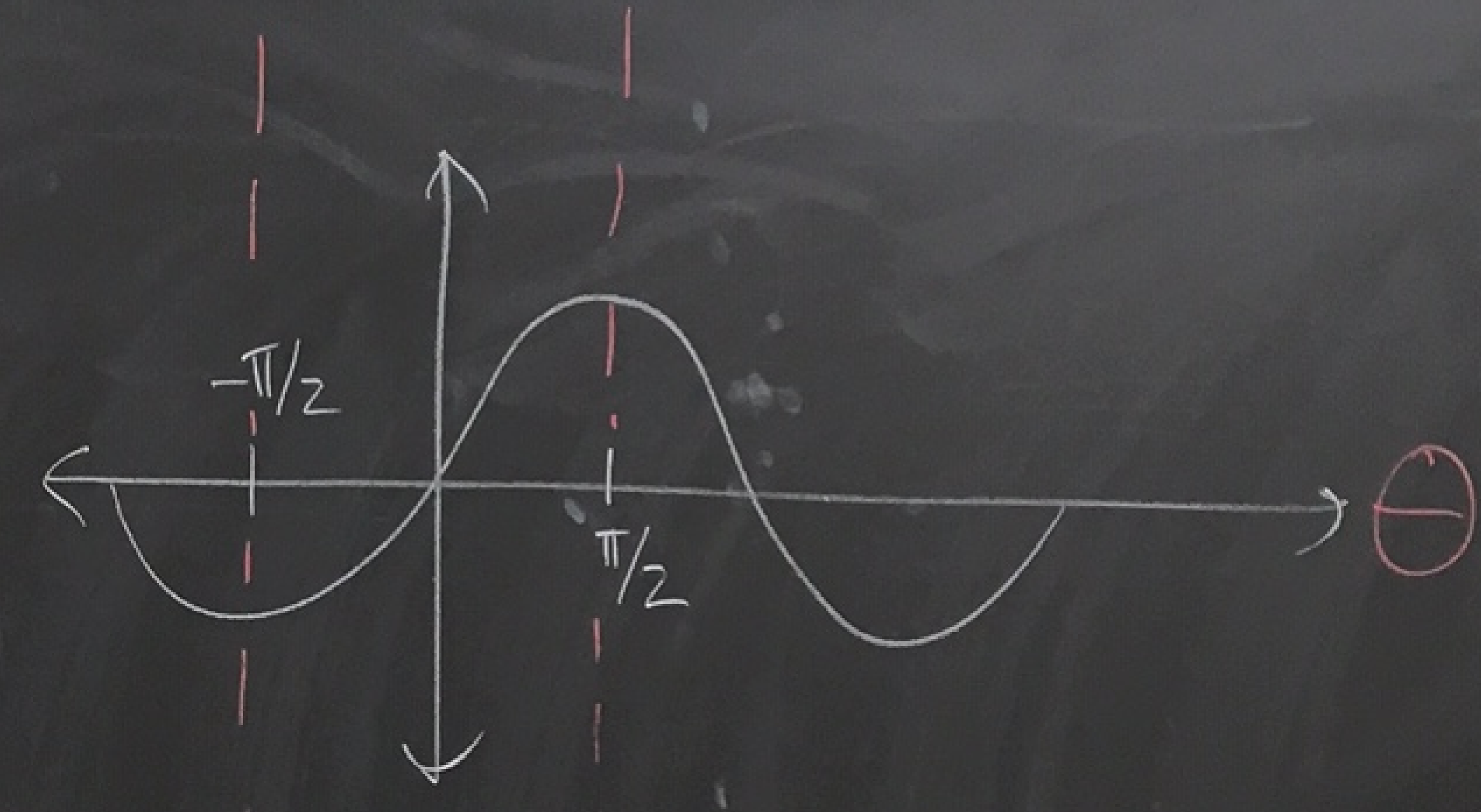
x

$$= \frac{3}{2}\int_{1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{3}{2}\left[\sin^{-1}(x)\Big|_{x=1/2}^{\sqrt{3}/2}\right]$$

$$= \frac{3}{2}\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)\right)$$

$$= \boxed{\frac{3}{2}\left[\frac{\pi}{3}-\frac{\pi}{6}\right]}$$



$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

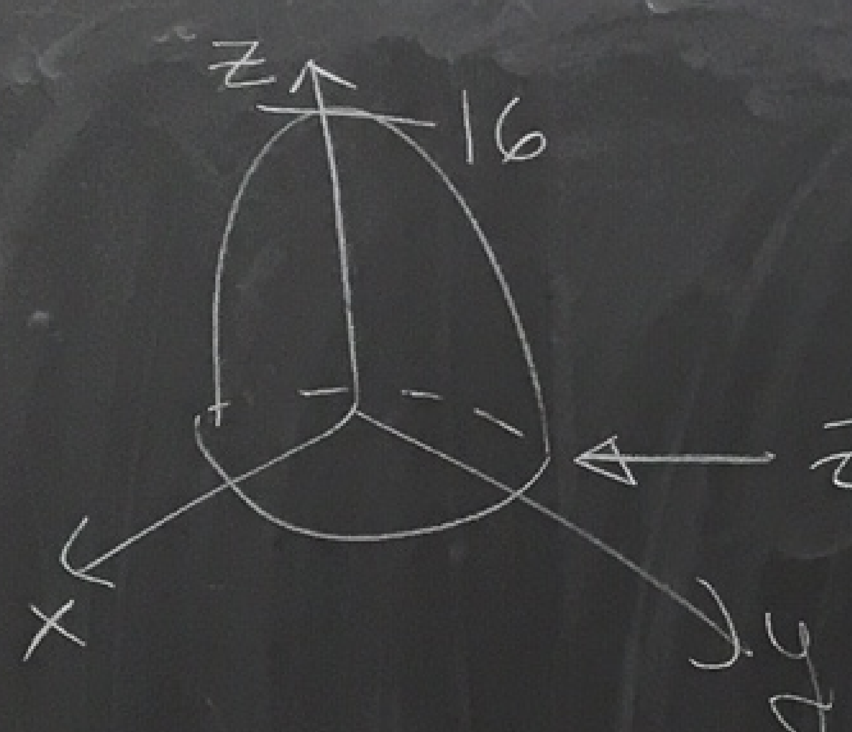
Ex: Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2+2y^2+z=16$, the planes $x=2$ and $y=2$, and the three coordinate planes (xy, yz, xz-planes)

$z=0$ $x=0$ $y=0$

$z=$

elliptic paraboloid

$$z = 16 - x^2 - 2y^2$$



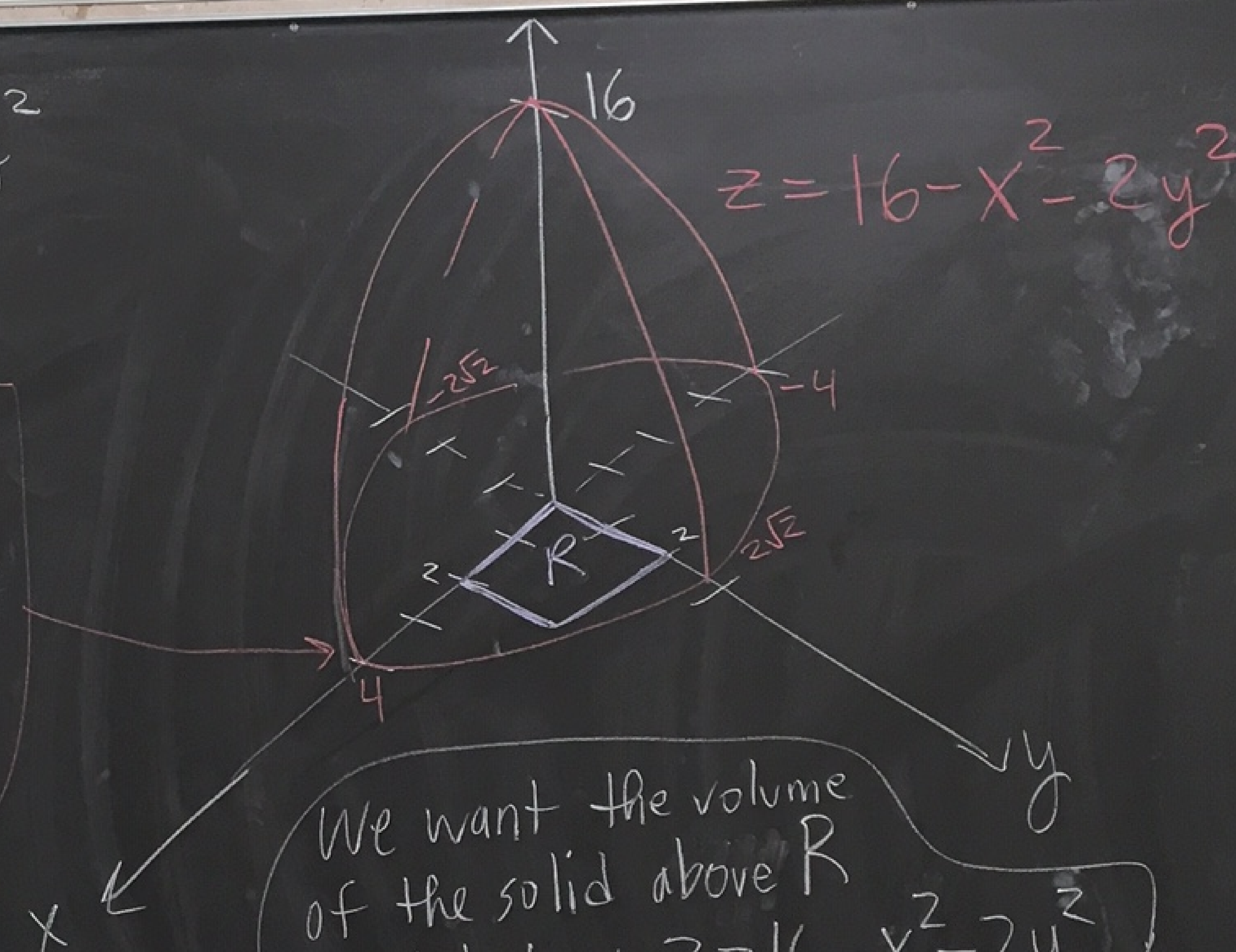
$\longleftarrow z = 0$

$$0 = 16 - x^2 - 2y^2$$

$$\frac{x^2}{16} + \frac{y^2}{8} = 1$$

$$\frac{x^2}{4^2} + \frac{y^2}{(2\sqrt{2})^2} = 1$$

$$2\sqrt{2} \approx 2.828$$



We want the volume of the solid above R and below $z = 16 - x^2 - 2y^2$

$$\iint_R (16-x^2-2y^2)\,dA$$

$$= \int_0^2 \int_0^2 (16-x^2-2y^2)\,dx\,dy$$

$$= \int_0^2 \left[16x - \frac{x^3}{3} - 2y^2 x\right]_{x=0}^{2} dy$$

$$\begin{array}{r} 32 \\ \times\ 3 \\ \hline 96 \\ -\ 8 \\ \hline 88 \end{array}$$

$$= \int_0^2 \left[\left(16(2) - \frac{2^3}{3} - 2y^2(2) \right) - \left(0 \right) \right] dy$$

$$\uparrow$$

$$x = 0$$

$$= \int_0^2 \left(32 - \frac{8}{3} - 4y^2 \right) dy$$

$$= \int_0^2 \left(\frac{88}{3} - 4y^2 \right) dy = \left(\frac{88}{3} y - 4 \frac{y^3}{3} \right) \Bigg|_0^2 = \left(\frac{88}{3}(2) - 4 \cdot \frac{2^3}{3} \right) - \left(0 \right) = \frac{1}{3}\left(176 - 32 \right)$$

$$= \frac{144}{3} = 48$$