10/1
Tuesday

$\boxed{\text{Last time}}$ Find max/min of

$$f(x,y) = 2x^2 + y^2 + 2$$

subject to

$$g(x,y) = x^2 + 4y^2 - 4 = 0$$

$$\boxed{\begin{array}{c} \nabla f = \lambda \nabla g \\ g = k \end{array}} \to \boxed{\begin{array}{ll} ① & 2x(2-\lambda) = 0 \\ ② & 2y(1-4\lambda) = 0 \\ ③ & x^2 + 4y^2 - 4 = 0 \end{array}}$$

Sol

x=

x=

x=

x=

in of

0

Solutions:

$x=0, y=1, \lambda=\frac{1}{4}$

$x=0, y=-1, \lambda=\frac{1}{4}$

$x=2, y=0, \lambda=2$

$x=-2, y=0, \lambda=2$

Now input solutions into f:

$f(0,1) = 2(0)^2 + 1^2 + 2 = 3$ min

$f(0,-1) = 2(0)^2 + (-1)^2 + 2 = 3$

$f(2,0) = 2(2)^2 + (0)^2 + 2 = 10$ max

$f(-2,0) = 2(-2)^2 + 0^2 + 2 = 10$

f has a min of 3 at (0,1) and (0,-1)
and f has a max of 10 at (2,0) and (-2,0)
above $g(x,y) = x^2 + 4y^2 - 4 = 0$

SOLUTION Figure 12.101a shows the elliptic paraboloid $z = f(x, y)$ above the ellipse C in the xy-plane. As the ellipse is traversed, the corresponding function values on the surface vary. The goal is to find the minimum and maximum of these function values. An alternative view is given in Figure 12.101b, where we see the level curves of f and the constraint curve C. As the ellipse is traversed, the values of f vary, reaching maximum and minimum values along the way.

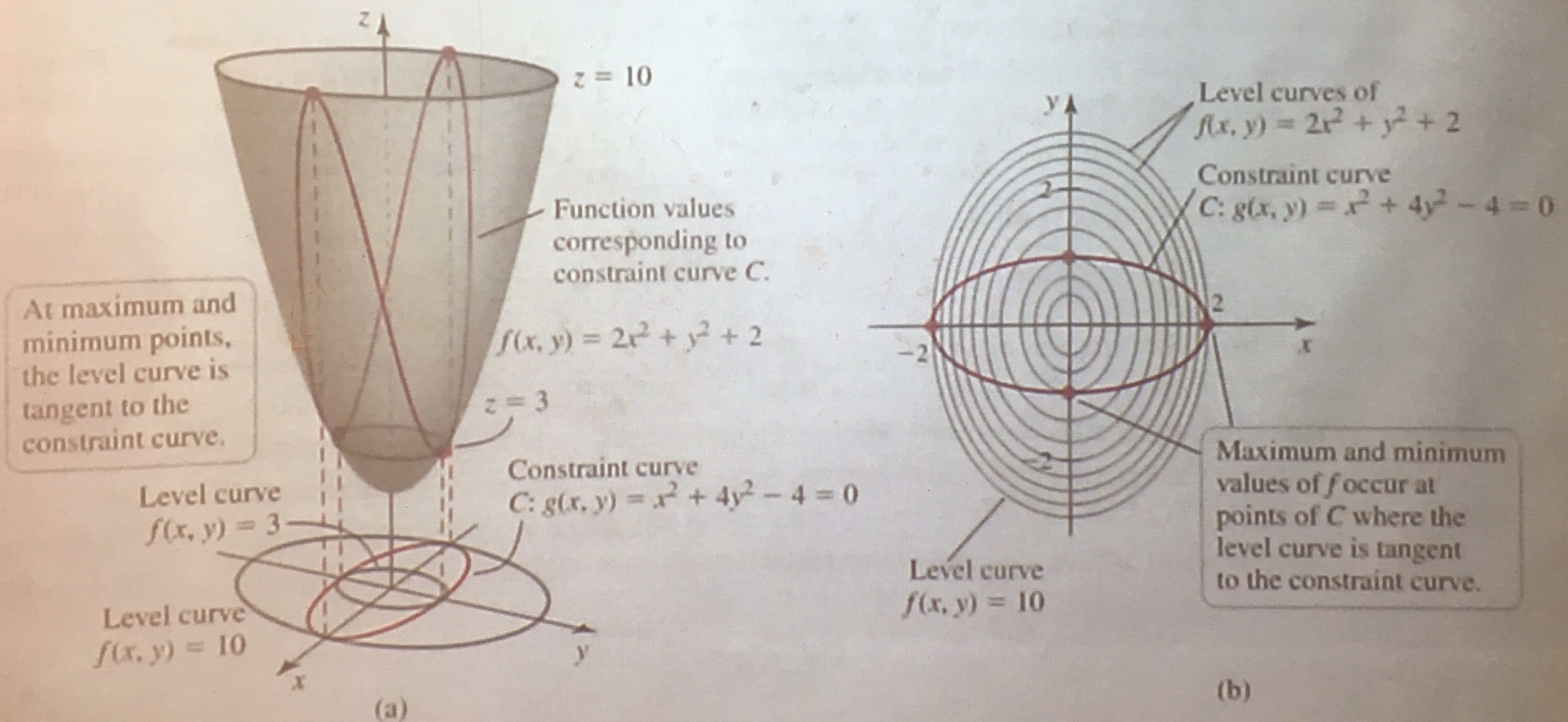


Figure 12.101

$$x^2+4y^2=1$$

$$x^2+\frac{y^2}{\left(\frac{1}{4}\right)}=1$$

$$\boxed{\frac{x^2}{1^2}+\frac{y^2}{\left(\frac{1}{2}\right)^2}=1}$$

12.9

HW #46

Find the absolute max and min of

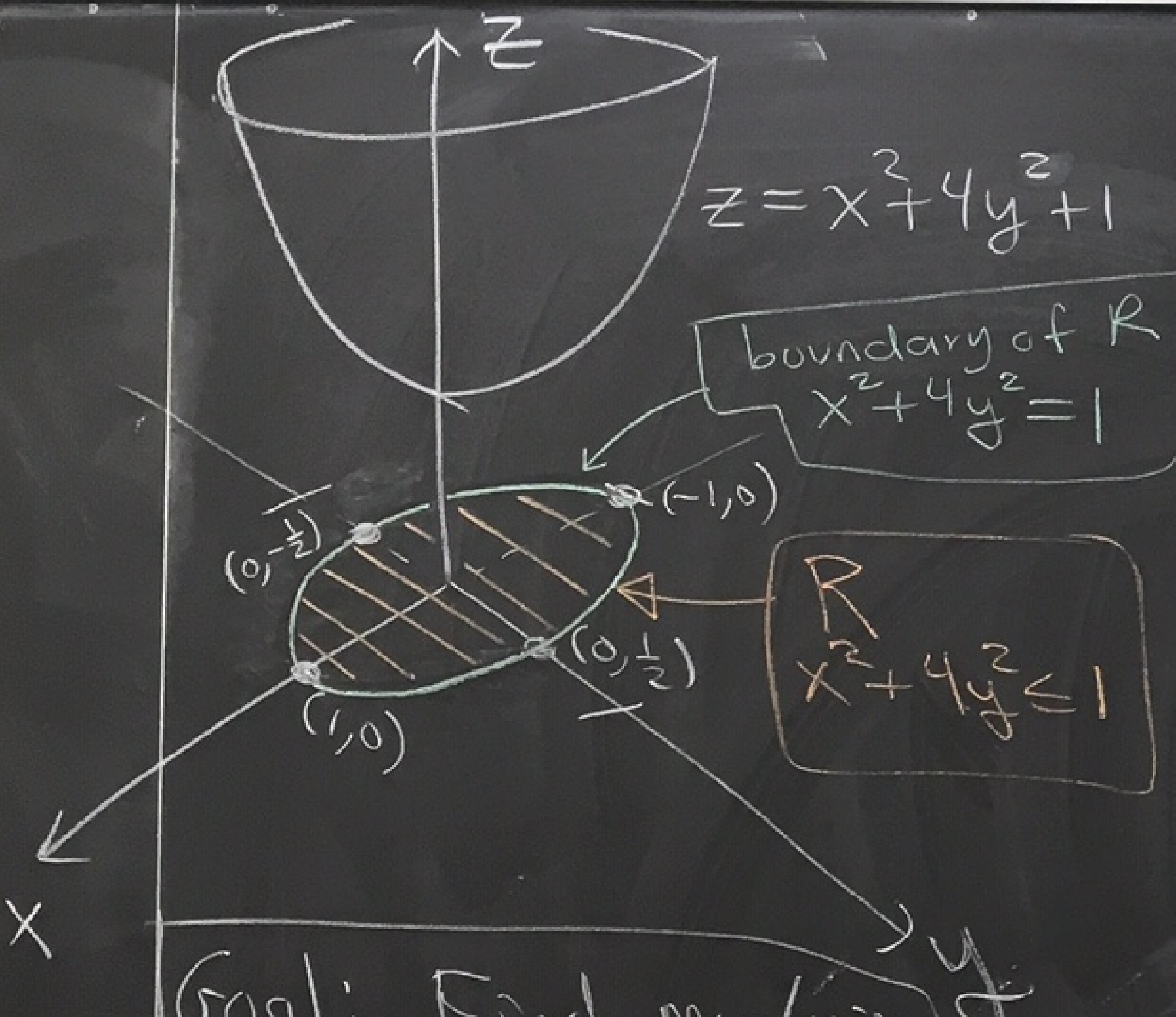
$$f(x,y)=x^2+4y^2+1$$

over

$$R=\{(x,y)\mid x^2+4y^2\le 1\}$$

$\boxed{R \text{ consists of all } (x,y) \text{ where } x^2+4y^2\le 1}$

x



$\boxed{\text{Goal: Find max/min of } f \text{ over } R}$

① Find critical pts of f inside of R

② Find max/min of f on the boundary of R

③ The max of steps 1 and 2 are the max of f over R. The min of steps 1 and 2 are the min of f over R.

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①

$$\left.\begin{aligned} f_x &= 2x = 0 \\ f_y &= 8y = 0 \end{aligned}\right\} \quad \left.\begin{aligned} x &= 0 \\ y &= 0 \end{aligned}\right\}$$

critical points in R:

$$\boxed{(x,y) = (0,0)}$$

Plug critical pts into f

$$\boxed{f(0,0) = 0^2 + 4 \cdot 0^2 + 1 = 1}$$

(2) Find max/min of $f(x,y) = x^2 + 4y^2 + 1$
over the boundary of R which is given by $x^2 + 4y^2 = 1$.
Use Lagrange!

$$\boxed{\begin{matrix} \nabla f = \lambda \nabla g \\ g = k \end{matrix}} \Longrightarrow \boxed{\begin{matrix} \nabla f = \langle 2x, 8y \rangle \\ \lambda \nabla g = \lambda \langle 2x, 8y \rangle \\ = \langle 2\lambda x, 8\lambda y \rangle \\ x^2 + 4y^2 = 1 \leftarrow \boxed{g = k} \end{matrix}} \Longrightarrow \boxed{\begin{matrix} 2x = 2\lambda x \\ 8y = 8\lambda y \\ x^2 + 4y^2 = 1 \end{matrix}}$$

$f = x^2 + 4y^2 + 1$

$g = k$ is $\underbrace{x^2 + 4y^2}_{g(x,y) = x^2 + 4y^2} = 1$

① $2x(1-\lambda)=0$

② $8y(1-\lambda)=0$

③ $x^2+4y^2=1$

Start with eqn ①

Solutions to ① are $x=0$ or $\lambda=1$.

case 1: $X=0$

Plug $x=0$ into ③ to get $y=\pm\frac{1}{2}$.

① and ③ are now solved.

To solve ② plug $y=\pm\frac{1}{2}$ into ②.

$y=\frac{1}{2}$: $8\left(\frac{1}{2}\right)(1-\lambda)=0$ ← eqn ②

$\boxed{\lambda=1}$

$y=-\frac{1}{2}$: $8\left(-\frac{1}{2}\right)(1-\lambda)=0$ ← eqn ②

$\boxed{\lambda=1}$

case 1 gives these solutions:

$x=0, y=\frac{1}{2}, \lambda=1$

$x=0, y=-\frac{1}{2}, \lambda=1$

Case 2: $\lambda = 1$

Oops!

1 & 2 are solved for all x, y with $\lambda = 1$.

Step 2

Start over

max/min

$f(x,y) = x^2 + 4y^2 + 1$

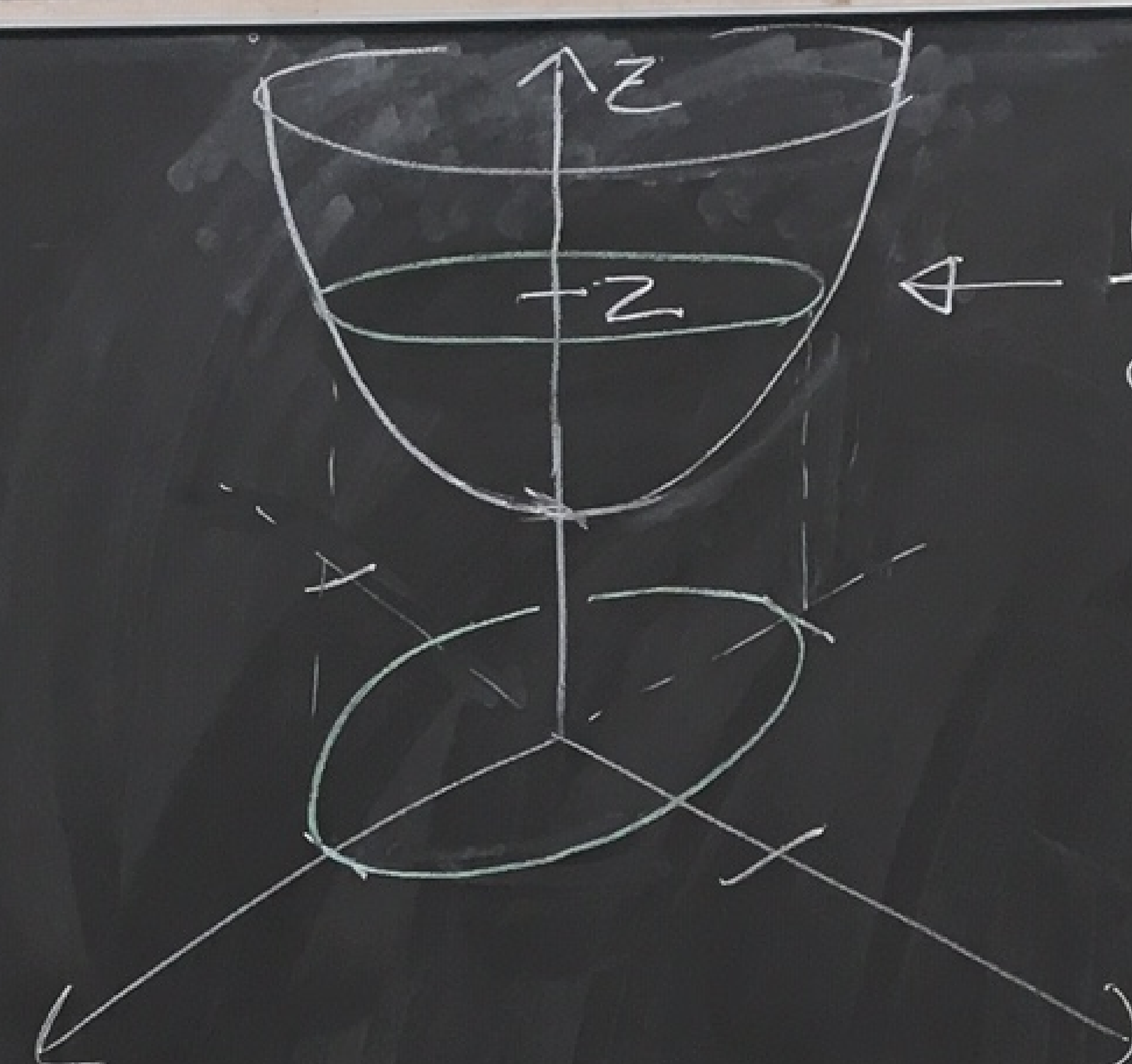
subject to

$x^2 + 4y^2 = 1$

Step ②

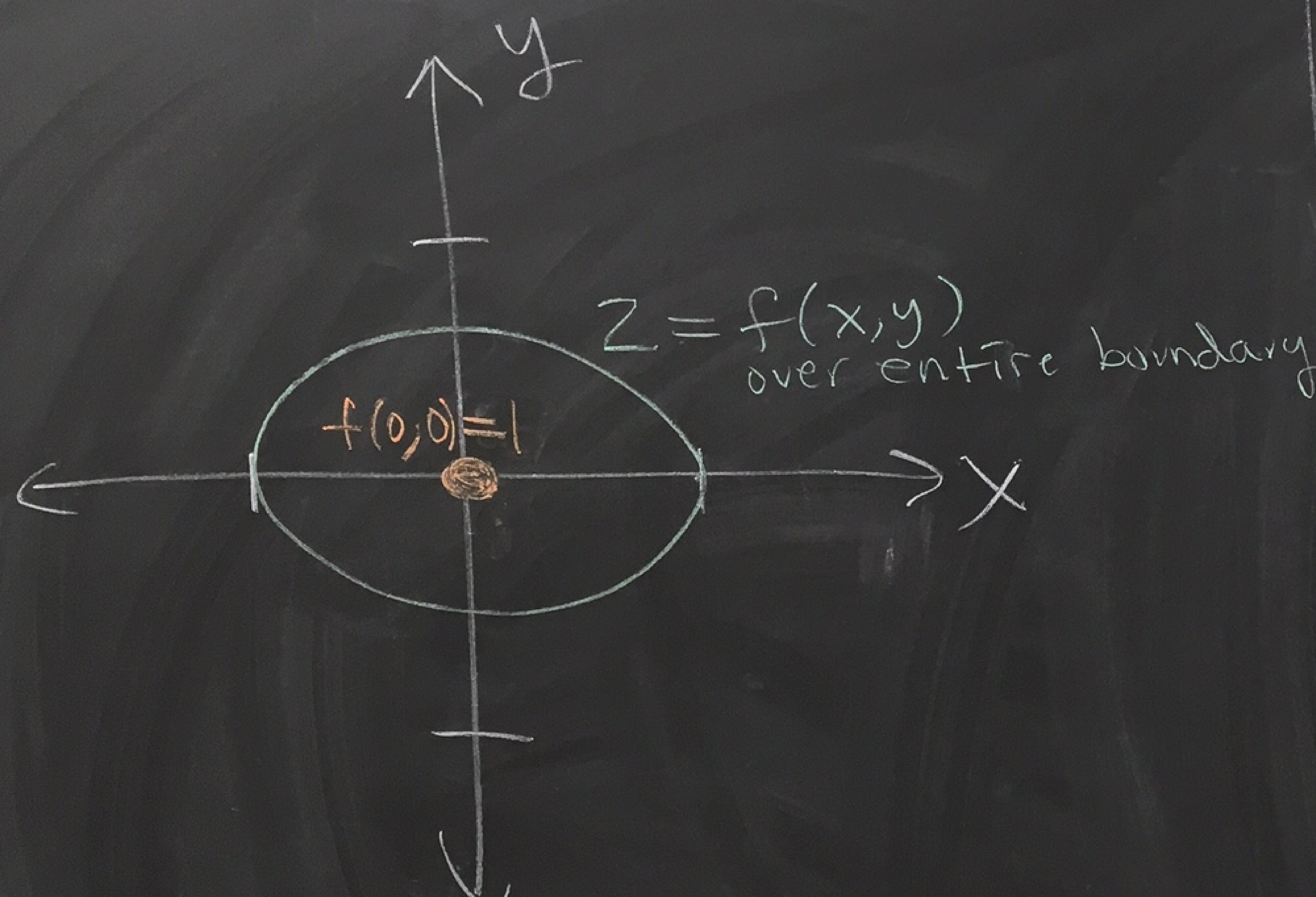
Plug $x^2+4y^2=1$ into f. Over $x^2+4y^2=1$ we get

$$f(x,y) = \underbrace{x^2+4y^2}_{1}+1 = 2$$



← $f(x,y) = 2$ everywhere above
$x^2+4y^2=1$ ← (boundary of R)

Step 3



Answer

- min of f over R is 1 at (0,0).
- max of f over R is 2 at every (x,y) on the boundary of R [ie at all (x,y) on $x^2+4y^2=1$].