## Flipped IN-CLASS Lesson Plan Template

Topic or concept: Trigonometric Integrals

Basic objectives for preparatory work:

- Review derivatives of Trigonometric Functions
- Review integrals involving \$\sin ax\$ and \$ $\$ \cos$ ax\$
- Review Pythagorean Identities and variations

Advanced objectives for classwork \& after class work:

- Integrate powers of $\$ \backslash \sin x \$$ or $\$ \backslash \cos x \$$
- Integrate products of powers of $\$ \backslash \sin x \$$ and $\$ \backslash \cos x \$$
- Integrate products of powers of $\$ \backslash \tan x \$$ and $\$ \backslash \sec x \$$

|  | Time planned | Activity and rationale | Resources needed |
| :--- | :--- | :--- | :--- |
| Beginning of <br> class period | 5 mins | A quick quiz: <br> (1) Find the derivative of $\$ \backslash \sin x \$$ <br> (2) Find the derivative of $\$ \backslash \cos x \$$ <br> (3) Find the integral of $\$ \backslash \sin x \$$ <br> (4) Find the integral of $\$ \backslash \cos x \$$ <br> (5) Write down Pythagorean identities you read <br> in the book | Paper |


|  | Time planned | Activity and rationale | Resources needed |
| :--- | :--- | :--- | :--- |
| Middle of <br> period | 20 mins | Lecture on how to integrate powers of $\$ \backslash \sin x \$$ or $\$ \backslash \cos$ <br> $x \$$ | Lecture prep/slides |
| Middle of <br> period (use if <br> needed) | 40 mins | $-\quad$ Lecture on how to integrate products of powers <br> of $\$ \backslash \sin x \$$ and $\$ \backslash \cos x \$$ <br> Lecture on how to integrate products of powers <br> of $\$ \backslash \tan x \$$ and $\$ \backslash \sec x \$$ | Lecture prep/slides |
| End of period | 10 mins | Summarize the different substitutions in different cases | handouts |

## Flipped AFTER CLASS Work Plan Template

| Advanced learning objective | Activity and rationale | Instructions to students |
| :---: | :---: | :---: |
| - Integrate powers of $\$ \backslash \sin x \$$ or $\$ \backslash \cos x \$$ <br> - Integrate products of powers of $\$ \backslash \sin x \$$ and $\$ \backslash \cos x \$$ <br> - Integrate products of powers of \$ $\backslash \tan \mathrm{x}$ \$ and \$ sec x \$ | Ask students to review the lecture notes and repeat examples in the notes. In this way, they have basic ideas of this lesson, and will help them solve homework problems. <br> Also, ask them to create and solve their own problems for different cases in this lesson, and students will more actively understand the technique learned in this lesson. | 1. Review the lecture notes, rewrite each example in the notes with solutions. <br> 2. Create your own problems (must be different than examples in class) about integration involving powers of trigonometric functions: <br> (1) $\$ \backslash \sin ^{\wedge} n x \backslash \cos ^{\wedge} m x \$$, $\$ n \$$ is odd, <br> (2) $\$ \backslash \sin ^{\wedge} n x \backslash \cos ^{\wedge} m x \$$, $\$ n \$$ is odd, <br> (3) $\$ \backslash \tan ^{\wedge} n x \backslash \sec ^{\wedge} m x \$$, $\$ n \$$ is odd, <br> (4) $\$ \backslash \tan ^{\wedge} n x \backslash \sec ^{\wedge} m x \$$, $\$ m$ is even <br> 3. Solve one of the examples you created in each case. <br> 4. Use Google, and find a way to solve the integration involving \$ $\backslash \tan ^{\wedge} n x \backslash \sec ^{\wedge} m$ $x \$$ with $\$ n \$$ even and $\$ m \$$ odd. |

## Guided Practice

Class: MATH 2120 Calculus II Sec 7.3
Date assigned: Friday, June 7, 2019
Date due: Sunday, June 9, 2019
Time estimate to complete this assignment: 30 min

## Overview/Introduction

What is this lesson about? Why do we care?
At the moment, our inventory of integrals involving trigonometric functions is rather limited. For example, we can integrate $\$ \backslash \sin$ ax\$, and \$\cos ax\$, where \$a\$ is a constant, but missing from the list are integrals of $\$ \tan \mathrm{ax}, \backslash \cot a x \$$ and $\$ \backslash \csc a x \$$. It turns out that integrals of powers of trigonometric functions, such as $\$$ int $\cos 5 x d x \$$ and $\$$ int $\cos 2 x \sin 4 x d x \$$, are also important. The goal of this section is to develop techniques for evaluating integrals involving trigonometric functions. These techniques are indispensable when we encounter trigonometric substitutions in the next section.

## Learning Objectives

Basic objectives

- Review derivatives of Trigonometric Functions
- Review integrals involving \$\sin ax\$ and \$\cos ax\$
- Review Pythagorean Identities and variations


## Advanced objectives

- Integrate powers of $\$ \backslash \sin x \$$ or $\$ \backslash \cos x \$$
- Integrate products of powers of $\$ \backslash \sin x \$$ and $\$ \backslash \cos x \$$
- Integrate products of powers of $\$ \backslash \tan x \$$ and $\$ \backslash \sec x \$$


## Preparatory Activities and Resources:

1. Resources

Reading:
(1) read Sec 3.5 on page 163 in Calculus - Early Transcendentals
(2) read Sec 5.4 on page 377 in Calculus - Early Transcendentals
(3) read the Pythagorean identities in Calculus - Early Transcendentals
2. Exercises

These exercises can be done during or after your reading. They are intended to help you review the derivatives and integrals involving basic trigonometric functions.
(6) Find the derivative of $\$ \backslash \sin x \$$
(7) Find the derivative of $\$ \backslash \cos x \$$
(8) Find the integral of $\$ \backslash \sin x \$$
(9) Find the integral of $\$ \backslash \cos x \$$
(10)Write down 3 Pythagorean identities you read in the book

Exercises: Please complete by _Sunday, June 9, 2019

- Please submit your work via Piazza.
- Reponses are due at 11:59 pm on Sunday.


## Questions?

## Advanced Practice

This is given for students to complete after the class meeting in which they work together.

## Class: MATH 2120 Calculus II Sec 7.3

Date assigned: June 10, 2019
Date due: June 12, 2019
Time estimate to complete this assignment: 75 min

## Learning Objectives

## Advanced objectives

- Integrate powers of \$ $\backslash \sin x \$$ or $\$ \backslash \cos x \$$
- Integrate products of powers of $\$ \backslash \sin x \$$ and $\$ \backslash \cos x \$$
- Integrate products of powers of $\$ \backslash \tan x \$$ and $\$ \backslash \sec x \$$


## Activities \& deliverables

5. Review the lecture notes, rewrite each example in the notes with solutions.
6. Create your own problems (must be different than examples in class) about integration involving powers of trigonometric functions:
(5) $\$ \backslash \sin ^{\wedge} n x \backslash \cos ^{\wedge} m x \$, \$ n \$$ is odd,
(6) $\$ \backslash \sin ^{\wedge} n \times \backslash \cos ^{\wedge} m \times \$, \$ n \$$ is odd,
(7) $\$ \backslash \tan ^{\wedge} n \times \backslash \sec ^{\wedge} m \times \$$, $\$ n \$$ is odd,
(8) $\$ \backslash \tan ^{\wedge} \mathrm{n} x \backslash \sec ^{\wedge} \mathrm{m} x \$$, $\$ \mathrm{~m}$ \$ is even
7. Solve one of the examples you created in each case.
8. Use Google to find a way to solve the integration involving \$\tan^n $x \backslash \sec ^{\wedge} m \times \$$ with $\$ n \$$ even and \$m\$ odd.

## Resources:

1. Lecture notes and handouts on Piazza.

## Questions?

