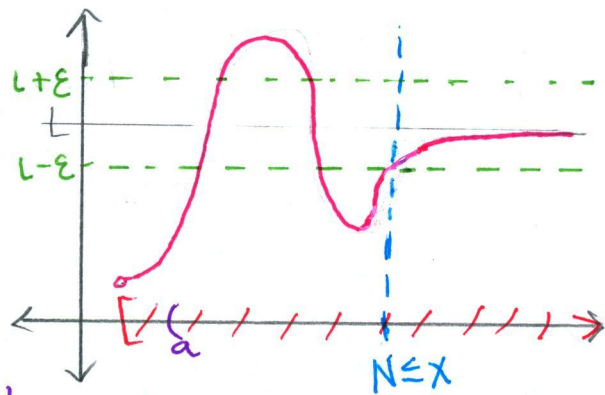


HW 3 stuff

Def: Let $f: D \rightarrow \mathbb{R}$ where $D \subseteq \mathbb{R}$ and D contains some interval (a, ∞) for some $a \in \mathbb{R}$

Let $L \in \mathbb{R}$. We say that the limit of f as x tends to infinity is L

and write $\lim_{x \rightarrow \infty} f(x) = L$ if $\forall \epsilon > 0 \exists N > 0$ where if $x \in D$ and $x \geq N$ then $|f(x) - L| < \epsilon$.



Example: $f(x) = \frac{2x-3}{5x+10}$. Let's show $\lim_{x \rightarrow \infty} \frac{2x-3}{5x+10} = \frac{2}{5}$

Proof:

Let $\epsilon > 0$, Note that

$$\left| \frac{2x-3}{5x+10} - \frac{2}{5} \right| = \left| \frac{10x-15-10x-20}{5(5x+10)} \right| = \frac{7}{5x+10}$$

$\xrightarrow{x \rightarrow \infty}$
assume $x > 0$

Note that $\frac{7}{5x+10} < \epsilon$ iff $\frac{7}{\epsilon} < 5x+10$ iff $\left(\frac{7}{\epsilon} - 10\right) < x$

Pick $N > \left(\frac{7}{\epsilon} - 10\right)$

If $x \geq N > \left(\frac{7}{\epsilon} - 10\right)$, then $\left| \frac{2x-3}{5x+10} - \frac{2}{5} \right| < \epsilon$

Example: Suppose $f: D \rightarrow \mathbb{R}$ and $g: D \rightarrow \mathbb{R}$ for some $D \subseteq \mathbb{R}$ that contains an interval of the form (a, ∞)

Assume $\lim_{x \rightarrow \infty} f(x) = F$ and $\lim_{x \rightarrow \infty} g(x) = G$

If $\alpha, \beta \in \mathbb{R}$ then,

$$\lim_{x \rightarrow \infty} [\alpha f(x) + \beta g(x)] = \alpha F + \beta G.$$

proof: Let $\epsilon > 0$

It is true that,

$$|(\alpha f(x) + \beta g(x)) - (\alpha F + \beta G)| = |\alpha(f(x) - F) + \beta(g(x) - G)|$$

$$\leq |\alpha(f(x) - F)| + |\beta(g(x) - G)| = |\alpha||f(x) - F| + |\beta||g(x) - G|$$

$$\leq (|\alpha|+1)|f(x) - F| + (|\beta|+1)|g(x) - G|$$

in case $|\alpha|=0$ in case $|\beta|=0$

Since $\lim_{x \rightarrow \infty} f(x) = F \exists N_1 > 0$ where if $x \geq N_1$, then $|f(x) - F| < \frac{\epsilon}{2(|\alpha|+1)}$

since $\lim_{x \rightarrow \infty} g(x) = G \exists N_2 > 0$ where if $x \geq N_2$ then $|g(x) - G| < \frac{\epsilon}{2(|\beta|+1)}$

Let $N = \max\{N_1, N_2\}$. If $x \geq N$, then

$$\begin{aligned} |(\alpha f(x) + \beta g(x)) - (\alpha F + \beta G)| &< (|\alpha|+1)|f(x) - F| + (|\beta|+1)|g(x) - G| \\ &< (|\alpha|+1) \frac{\epsilon}{2(|\alpha|+1)} + (|\beta|+1) \frac{\epsilon}{2(|\beta|+1)} \\ &= \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon \quad \square \end{aligned}$$

Homework 2 #3(h)

Show that $\lim_{n \rightarrow \infty} \frac{n^2}{n+1}$ does not exist.

proof: we can show that $\frac{n^2}{n+1}$ is unbounded and then the limit won't exist.

Let $M > 0$, we will show that

$$\exists n_0 > 0 \text{ where } \frac{n_0^2}{n_0+1} > M$$

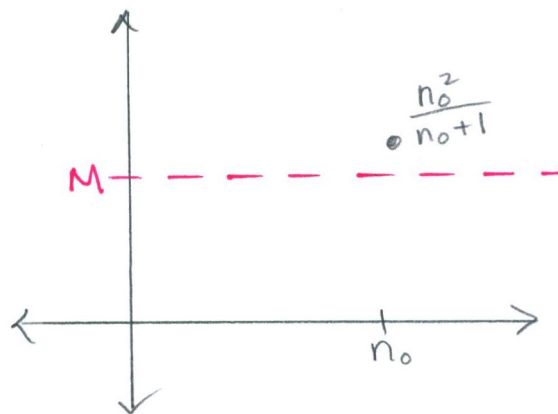
Note that if $n \geq 1$ then

$$\frac{n^2}{n+1} \geq \frac{n^2}{n+n} = \frac{n^2}{2n} = \frac{n}{2}$$

$$\text{and } \frac{n}{2} > M \text{ iff } n > 2M$$

If $n_0 > 2M$ then,

$$\frac{n_0^2}{n_0+1} \geq \frac{n_0}{2} > M \quad \square$$



Homework 1 #1(d)

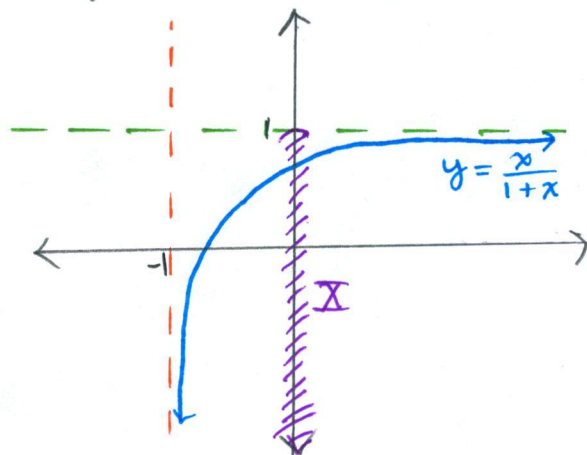
Find $\inf(\mathbb{X})$ and $\sup(\mathbb{X})$, if they exist

$$\mathbb{X} = \left\{ \frac{x}{1+x} \mid x > -1 \right\}$$

$$\mathbb{X} = (-\infty, 1)$$

$\inf(\mathbb{X})$ does not exist

$$\sup(\mathbb{X}) = 1$$



Homework 1 #1(e)

Find $\inf(\mathbb{X})$ and $\sup(\mathbb{X})$, if they exist

$$\mathbb{X} = \{x \mid x^2 + x < 3\}$$

$$y = x^2 + x - 3$$

$$x^2 + x - 3 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-3)}}{2(1)} = \frac{-1 \pm \sqrt{13}}{2} = -\frac{1}{2} \pm \frac{\sqrt{13}}{2}$$

$$\text{smallest } y' = (x^2 + x - 3)' = 0$$

$$2x + 1 = 0$$

$$x = -\frac{1}{2} \leftarrow \text{min}$$

$$\inf(\mathbb{X}) = -\frac{1}{2} - \frac{\sqrt{13}}{2}$$

$$\sup(\mathbb{X}) = -\frac{1}{2} + \frac{\sqrt{13}}{2}$$

