Lemma: Suppose H is a subgroup of a group G. Then,

① the identity element of H is the same as the identity element of G.

② If $a \in H$, then the inverse of a in H is the same as the inverse of a in G.

proof: ① Suppose e_H is the identity of H and e_G is the identity of G. We know that

$$e_H * e_G = e_H = e_H * e_H \, . \quad (\star)$$

Suppose e_H^{-1} is the inverse of e_H in G. Then applying e_H^{-1} to $(\star)$ gives

$$\underbrace{e_H^{-1} * e_H * e_G}_{e_G} = \underbrace{e_H^{-1} * e_H * e_H}_{e_H} \, .$$

So, $e_G = e_H$.

② Let $a \in H$ and e be the identity of G (and H). Suppose a_H^{-1} is the inverse of a in H and a_G^{-1} is the inverse of a in G. Then,

$$a * a_H^{-1} = e = a * a_G^{-1} \, .$$

Thus, $a_H^{-1} = \underbrace{a_H^{-1} * a}_{e} * a_H^{-1} = \underbrace{a_H^{-1} * a}_{e} * a_G^{-1} = a_G^{-1}$. $\blacksquare$