Math 455

Homework # 5 - Symmetric Group

1. Consider the group S_3.

   (a) Compute the group table for S_3.

   (b) Compute the orders of all the elements in S_3.

   (c) Compute the inverse of each element in S_3.

2. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$ be elements of S_5.

   (a) Draw a picture of σ.

   (b) Draw a picture of τ.

   (c) Compute $\sigma\tau$.

   (d) Compute σ^2.

   (e) Compute σ^{-1} and τ^{-1}.

   (f) Compute $\tau\sigma^2\tau^3$.

   (g) Compute the order of σ and $\langle\sigma\rangle$.

   (h) Compute the order of τ.

3. Given the following elements of S_n, decompose the permutation as the product of disjoint cycles, and then as a product of transpositions. Is the permutation even or odd?

   (a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 7 & 4 & 3 & 1 & 6 & 5 & 8 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 5 & 2 & 7 & 8 & 6 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 6 & 5 & 3 & 7 & 1 & 4 & 2 & 9 & 8 & 10 \end{pmatrix}$

4.

(a) Let G be a group and let $g \in G$. Define $\phi_g : G \to G$ by $\phi_g(x) = g^{-1}xg$. Prove that ϕ_g is an isomorphism.

(b) Let $G = \mathbb{Z}_5$ and $g = \overline{3}$. Draw a picture of ϕ_g from part 1 above.

(c) Let $G = D_{10}$ and $g = r$. Draw a picture of ϕ_g from part 1 above.

(d) Let $G = D_{10}$ and $g = s$. Draw a picture of ϕ_g from part 1 above.

5. Use the technique from Cayley's theorem to find a subgroup of $S_{\mathbb{Z}_5}$ that $\mathbb{Z}_5$ is isomorphic to.

6. Use the technique from Cayley's theorem to find a subgroup of S_{D_4} that D_4 is isomorphic to.

7. Prove that S_n is not abelian when $n \geq 3$.