## Possible topics:

Here are some ideas for topics for your final paper. Or pick your own. Pick something that you are interested in so you will be excited and motivated to learn about the topic.

<u>Rules</u>: The only rule is that you can't pick a topic that was part of a class that you already took, but it can be an extension of a class that you took; that is, a further topic that you didn't study in that class. If in doubt, please feel free to ask me.

Ideas:

- Number Theory:
  - Arithmetical functions: divisor function, Euler totient function, Mobius function, tau function.
  - The Mobius inversion formula
  - Fibonacci sequence and Lucas sequence
  - Fibonacci sequence and golden ratio
  - Magic squares, magic cubes
  - Continued fractions
  - Pythagorean triples
  - Quadratic reciprocity and it's various proofs
  - Primitive roots modulo n
  - Theory of prime numbers (basic estimates on primes, properties of primes, etc.)
  - p-adic numbers
  - Quadratic number fields
- Combinatorics:
  - Catalan numbers
  - Binomial coefficients and their properties (includes Pascal's triangle, etc)
  - Generating functions
  - Recurrence relations
- Abstract Algebra:
  - Quaternions
  - Rings and ideals of rings
  - Fields and their properties
  - Sylow's Theorems and their uses (this is group theory)
  - The class equation and its corollaries (this is group theory)
- Linear Algebra:
  - Google's PageRank algorithm
  - Theory of vector spaces
- Analysis:
  - Metric spaces
  - The theory of infinite sums
  - The theory of Reimann integration
  - Basics of measure theory

- Construction of the real number system
- Topology
  - Topological spaces and basic topology (open sets, bases, closed sets, product topology, etc.)
  - The fundamental group of a topological space
- Set Theory:
  - Topics in Cardinality of sets (for example, Cantor-Schroder-Bernstein theorem, axiom of choice, continuum hypothesis, etc.)
  - Zorn's lemma, axiom of choice, well-ordering theorem
  - ZFC (Zermelo-Fraenkel) axioms for set theory
- Graph theory:
  - Graph coloring
  - Hamiltonian cycles / Eulerian circuits
  - Planar graphs
  - Bipartite graphs and matching problems
  - Cayley graphs (these graphs are built using groups)
  - Adjacency matrices associated with graphs and their eigenvalues (adjacency matrix, incidence matrix)
  - line graphs
  - independent sets and cliques in graphs
  - homomorphisms and automorphisms of graphs and their properties
  - graph algorithms like Dijkstra and Prim
  - Ramsey Theory
- Differential Equations
  - Theorems on the existence and uniqueness of solutions to different types of equations
  - Solutions to systems of linear equations
- Cryptography
  - RSA public-key cryptography
  - Other cryptographic systems