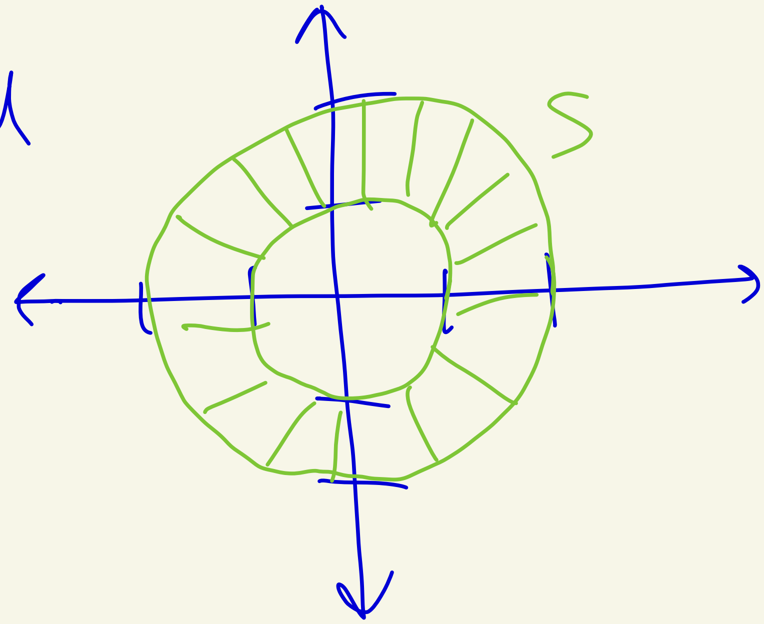
4680 - HW 7

Solutions

1 (a) $S = \{ z \mid 1 \leq |z| \leq 2 \}$

S is path connected
S is not open
So not a region



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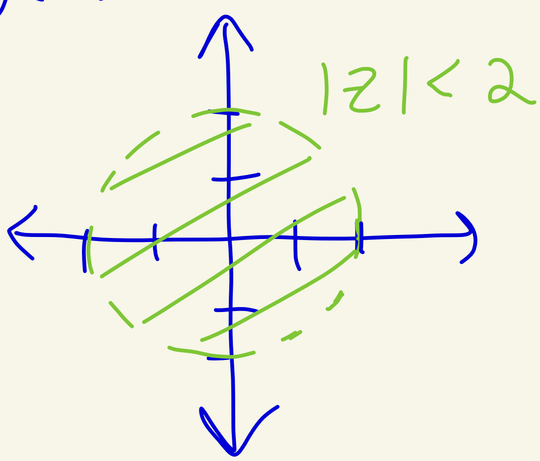
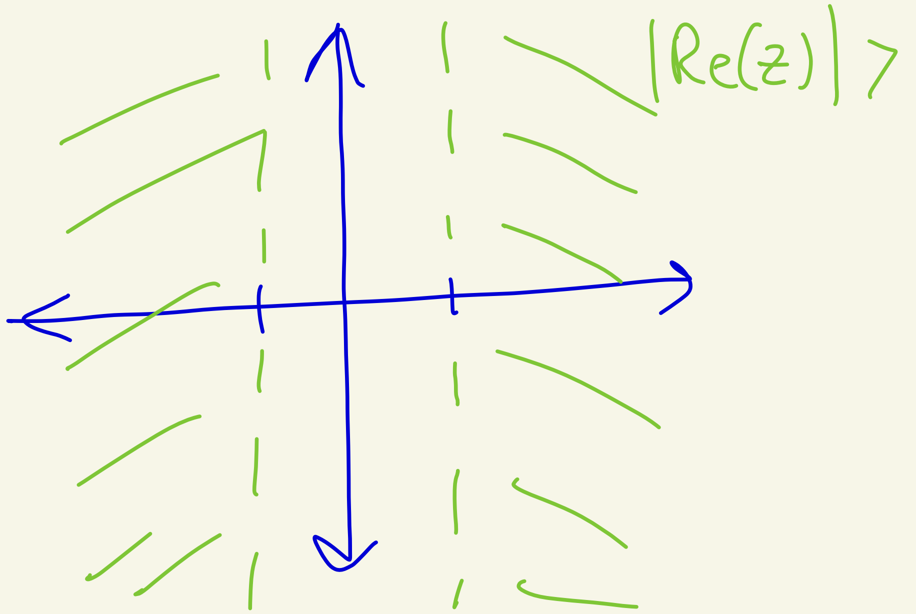
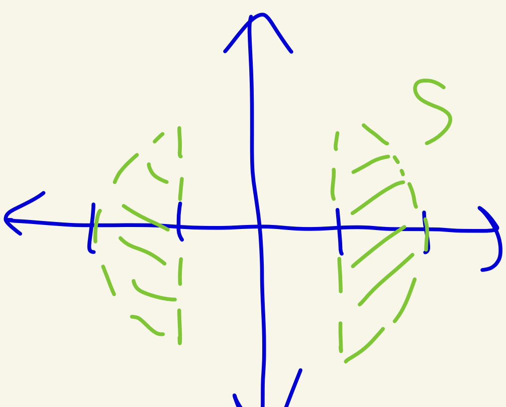
1 (b)







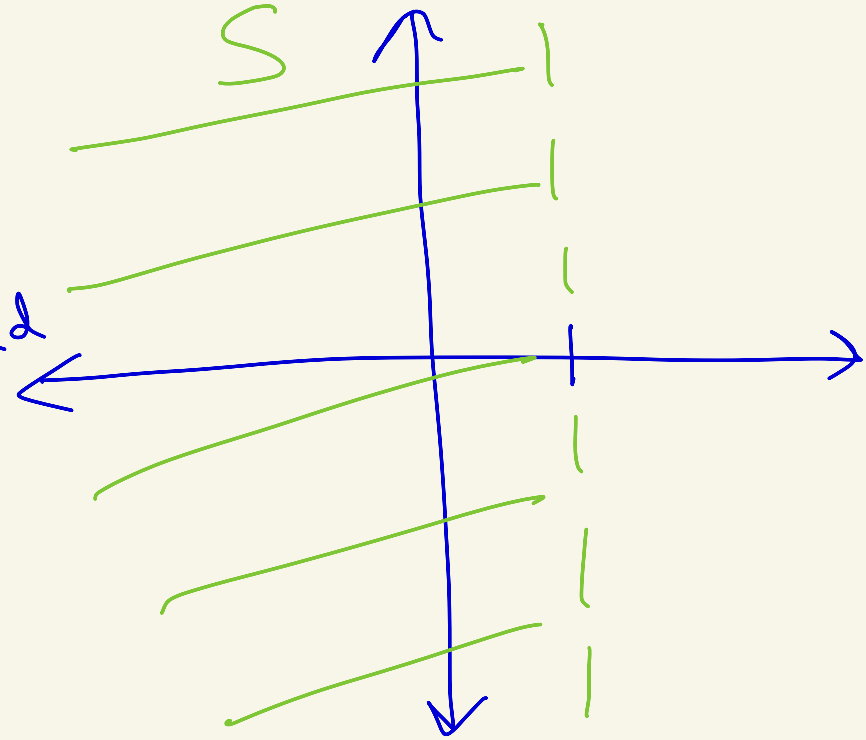
S is not path-connected
S is open
S is not a region

1(c)

S is open

S is path connected

S is a region

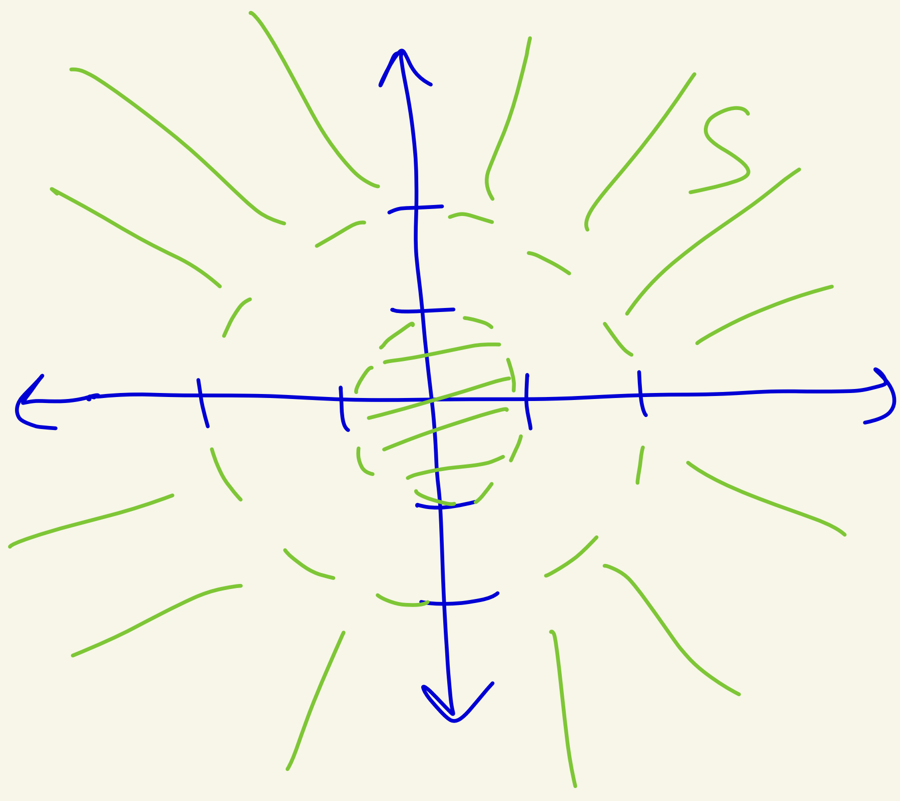


1(d)

S is not path-connected

S is open

S is not a region



(2) Let $f : A \to \mathbb{C}$ be analytic on A where A is a region (open and path-connected)

We are assuming that $f(z)$ is real for all $z \in A$,

Thus, $f(x+iy) = u(x,y) + i0$ on A for some real valued function $u(x,y)$.

By Cauchy-Riemann (with $v=0$) we have that $\frac{\partial u}{\partial x}(x,y) = \frac{\partial v}{\partial y}(x,y) = 0$ for all $x+iy \in A$. And since $v(x,y) = 0$ on A we get $\frac{\partial v}{\partial x}(x,y) = 0$ for all $x+iy \in A$.

Thus, $f'(x+iy) = \frac{\partial u}{\partial x}(x,y) + i\frac{\partial v}{\partial x}(x,y)$

$= 0 + i0 = 0$

for all $x+iy \in A$. $\blacksquare$

So, $f'(z) = 0$, $\forall z \in A$

where A is a region.

By a class theorem,

this implies that f

is constant on A.