# Math 4680 - Homework # 6

## Contour Integrals

1. Compute the following integrals by hand by using the formula

$$\int_\gamma f(z)dz = \int_a^b f(\gamma(t))\gamma'(t)dt$$

Do not use the Fundamental Theorem of Calculus (FTOC).

   (a) Evaluate the integral $\int_\gamma (z^2 + 2)\ dz$ where $\gamma$ is the line segment joining 1 to $2 + i$.

   (b) Evaluate the integral $\int_\gamma (z^3 + z)\ dz$ where $\gamma$ is the circle of radius 3 centered at 0 traveled once counterclockwise.

   (c) Evaluate the integral $\int_\gamma (x - y)\ dz$ where $\gamma$ is the line segment joining 1 to $1 + i$.

2. Compute the following integrals in whatever way you wish.

   (a) Evaluate the integral $\int_\gamma \cos(2z)\ dz$ where $\gamma$ is the line segment joining $i + 1$ to $-i$.

   (b) Evaluate the integral $\int_\gamma ze^{z^2}\ dz$ where $\gamma$ is the unit circle traveled once counterclockwise.

   (c) Evaluate the integral $\int_\gamma \dfrac{1}{z-1}\ dz$ where $\gamma$ is the circle of radius 2 centered at 1 traveled once counterclockwise.

3. Let $\gamma$ be the circle $|z| = 2$ oriented in the counter-clockwise direction. Show that

$$\left| \int_\gamma \frac{dz}{z^2 + 1} \right| \leq \frac{4\pi}{3}$$

4. Let $\gamma$ be the bottom half of the unit circle going in a counterclockwise direction. Show that
$$\left| \int_\gamma \frac{z+1}{z^2-8} \right| \leq \frac{2\pi}{7}$$

5. (a) Let $\gamma$ be a closed piecewise smooth curve lying entirely in $A = \mathbb{C} - \{z \mid \mathrm{Re}(z) \leq 0\}$. Show that $\int_\gamma (1/z)\, dz = 0$.

   (b) Give some conditions on a closed piecewise smooth curve that guarantee that $\int_\gamma (1/z)\, dz = 0$.