1. Let $d$ be a metric on $\mathbb{R}^n$ such that $d$ is equivalent to the Euclidean metric. Let $U = \{ (x_1, x_2, x_3, \ldots, x_n) : \sum_{j=1}^{n} x_j^2 < 1 \}$ be the usual open ball of radius 1 centered at the origin. Show that $U$ is bounded with respect to $d$. [Hint: Consider the closure of $U$.]

2. (a) Let $\tau_1, \tau_2$ be two topologies on some set $X$. Show that $\tau_1 \cap \tau_2$ is a topology on $X$.

(b) Give an example of two topologies $\tau_1, \tau_2$ on $\{ a, b, c, d, e \}$ such that $\tau_1 \cup \tau_2$ is not a topology.

3. Let $X$ be a topological space, and let $Y$ be a Hausdorff space. Let $f, g$ be two continuous functions from $X$ to $Y$. Let $A = \{ x \in X \mid f(x) = g(x) \}$. Show that $A$ is closed in $X$.

4. Let $n$ be a positive integer. Let $S^n = \{ x \in \mathbb{R}^n : \|x\| = 1 \}$, where $\| \cdot \|$ denotes the usual Euclidean norm on $\mathbb{R}^n$. Show that $S^n$ is path connected.

5. In a topological space $X$, let $\overline{A}$ denote the closure of a subset $A$ and let $A'$ denote its set of limit (a.k.a. cluster or accumulation) points. Prove

(a) Prove $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

(b) Show that if $X$ is Hausdorff, then $A'$ is closed.

6. (a) Let $X$ be a Hausdorff space and let $A$ be a compact subset. Prove $A$ is closed.

(b) Let $X$ compact topological space and let $C$ be a closed subset. Prove $C$ is compact.

7. (a) What does it mean for a topological space $X$ to be locally connected?

(b) Give an example of a connected topological space $X$ that is not locally connected.

Clearly explain why $X$ is connected, citing appropriate theorems; similarly, explain why the definition you provide in (a) fails to be satisfied by $X$. 