EE3079 Experiment: Chaos in nonlinear systems

Background:

The theory of nonlinear dynamical systems and Chaos is an intriguing area of mathematics that has received considerable attention in the recent past largely due to our ability to now analyze and describe chaotic behavior that for instance can result from the simplest of iterative maps based on basic algebraic equations. Evidently, nature’s complex patterns of behavior can sometimes be described by simple equations running in chaotic mode and possibly leading to observed Fractal patterns. The theory of Chaos has found applications in a wide range of areas from multi-level pseudo-random sequences that may be used in communications and Radar applications to reconfigurable logic gates. It turns out that simple thresholding of inputs and outputs of some basic Chaotic systems can lead to familiar logic behavior on binary inputs leading to familiar logic gates. We note that the fundamental system remains Chaotic, it is only when thresholding is applied that the logical pattern of behavior emerges.

Some of the Navy's highest priorities, such as improved communications, increased bandwidth, improved sensors, and more effective countermeasures for dealing with improvised explosive devices, are currently being addressed by nonlinear dynamics technology.

One application example in NAVY is a nonlinear sensor for magnetic detection. For this application, a variant of stochastic resonance is applied in the design of a nonlinear fluxgate magnetometer to detect the metal in objects that range in size from guns and rifles to the hull of a submarine.

Yet another application of the theory is in design of nonlinear filters to deal with interference and multipath in submarine communication systems. A submarine's ultra high frequency satellite communication (UHF SATCOM) antenna is constrained by the size of the submarine mast and must operate a few inches above the ocean surface where sea states can create dynamic multipath reflections. In addition, UHF SATCOM channels are frequently unusable due to in-band, co-site narrowband interference. For this application, a nonlinear adaptive filter is designed to remove both the interference and multipath signals, thereby increasing the number of usable UHF SATCOM channels while maximizing the data rate.
The Experiment:

The basic ideas of bifurcation and chaos can easily be demonstrated in a simple laboratory experiment with a diode providing the basic nonlinear map. A simple circuit consisting of an inductor, resistor, and diode exhibits chaotic behavior even if the input driving voltage is periodic:

![Circuit Diagram]

The circuit parameter values for this experiment are as follows:

- Resistance: 200 Ω, 5% tolerance (if not 5% it is O.K.)
- Inductance: 25 mH
- Diode: any silicon diode would do (e.g., 1N..)

The diode exhibits two capacitive effects, one due to charge in depletion layer denoted Junction capacitance $C_j$, one due to time dependence of the injected charge across the depletion layer under forward bias denoted diffusion capacitance of $C_d$. These are usually modeled as being in parallel but $C_j$ dominates under reverse bias while $C_d$ under forward bias.

The diode's capacitance in conjunction with the resistive and inductive circuit elements produce an RLC resonant circuit. When the driving potential is tuned to this resonance frequency the diode potential exhibits bifurcation as a function of the amplitude of the driving potential.

Experimental Procedure:

1. Setup the circuit shown above on a breadboard. The power supply should be set to 1 KHz sinusoidal AC, initially at 100 mV Peak to Peak (PP). All measurements in the rest of this experiment will be based on PP voltages. Make sure the AC signal has no DC level (DC offset should be zero);

2. Attach the oscilloscope channel-1 probe across the diode and increase the frequency until the voltage across the diode is maximum. That frequency is the circuit resonance frequency. Record this frequency;

3. Decrease the frequency from resonance until the output is 0.707 of maximum value at resonance, call this frequency $f_i$. Next increase the frequency above resonance till the output is again 0.707 of the maximum value at resonance, call this frequency $f_u$. The RLC
bandwidth is \((f_u - f_l)\). Finally, set the frequency to the value at resonance and for the rest of the experiment, keep the frequency at this level;

4. Attach the oscilloscope channel-2 probe across the source. You will be measuring the PP voltage at the input and output (across the diode). It is the plot of the PP voltage at output versus input that shows Bifurcation which is characteristic of Chaotic systems. A typical plot after measurements may look like this:

5. Increase the input voltage amplitude from the 100 mV PP level in increments of 200 mV PP and measure the PP output voltage across the diode. Typical pictures that identify various Bifurcation levels are shown below.

The First level Bifurcation sample picture is shown below (occurs at input of about 1.8 V PP):

To measure the bifurcation levels, use the guide below:
The 2nd level bifurcation picture and the corresponding levels is shown below (occurs at input voltage of about 4.8 V PP):

![Image of 2nd level bifurcation]

The Third level Bifurcation (occurs at input voltage level of about 5.6 V PP) may not produce a steady single trace picture. Nonetheless, it is possible to clearly identify splitting of the levels to produce 8 potential levels. The lowest level is very near zero. The other low levels show up as

![Image of Third level bifurcation]
dips whose amplitudes define the levels. The top and bottom part of the trace are shown separately for better clarity on the level splitting:

At onset of Chaos (occurs at input voltage level of about 6.2 V PP), blurring of the levels occur as shown below. The top and bottom of the trace are shown separately for clarity:
At deep Chaos, the levels can be all over the place:
Analysis:

The circuit equivalent of the diode under Forward Bias (FB) and Reverse Bias (RB) when inserted into the overall circuit diagram leads to the following configurations:

(a) Diode forward bias.  
(b) Diode reverse bias.

Under FB, the diode behaves like a constant voltage source with voltage level $V_f$, while under RB it acts like a capacitance of value $C_j$. Analysis of the circuit under FB with sinusoidal excitation leads to the following; from KVL under FB we get:

$$L \frac{dI}{dt} + RI = V_o \sin(\omega t) + V_f$$

The solution of this differential equation yields:
\[ I(t; A) = \frac{V_0}{Z_a} \cos(\omega t - \theta) + \frac{V_f}{R} + Ae^{-Rt/L} \]

Where, \( V_0 \) is the peak amplitude of the input sinusoid, \( \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \), \( Z_a \) is the forward bias impedance given by \( Z_a = \sqrt{R^2 + \omega^2 L^2} \) and \( A \) is a constant to be determined from initial conditions. The KVL under RB condition gives:

\[ L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{1}{C_j} I = V_0 \omega \sin(\omega t) \]

The solution to this equation is given by:

\[ I(t; B, \phi) = \frac{V_0}{Z_b} \cos(\omega t - \theta_B) + Be^{-Rt/2L} \cos(\omega_B t - \phi) \]

Where, \( B \) and \( \phi \) are constants to be determined from initial conditions and

\[ \theta_B = \tan^{-1}\left(\frac{L(\omega^2 - \omega_0^2)}{R \omega}\right), \quad \omega_0^2 = \frac{1}{LC_j}, \quad \omega_B^2 = \omega_0^2 - \frac{R^2}{2L^2} \]

\[ Z_b = \sqrt{R^2 + \frac{L^2}{R^2}(\omega^2 - \omega_0^2)} \]

These equations hold valid when the diode drive current is not very large. The nonlinear behavior that leads to Chaos arises due to the fact that the diode cannot switch from FB to RB and vice versa instantaneously and indeed the diode continues to conduct for a period of time \( \tau \) after the instant of switching. This recovery time is actually dependent on the magnitude of the forward current in the diode \( |I_m| \) and is given by:

\[ \tau = \tau_m (1 - e^{-|I_m|/I_C}) \]

Where, \( \tau_m \) and \( I_C \) are constants that depend on the diode in use.

The figure below illustrates the mechanism of first Bifurcation:
When the circuit is operated at the resonant frequency, some reverse current will flow through the diode in every reverse bias cycle due to the finite recovery time of the diode. If the peak current $|I_m|$ is large in the conducting cycle (interval 'a'), the diode will turn off with a certain delay (interval 'b') due to the finite recovery time and so will allow a current to flow even in the reverse-bias cycle. This reverse bias current, in turn, will prevent the diode from instantly switching on in the forward bias cycle and the diode will turn on with a delay (interval 'c'). This will keep the forward peak current smaller than in the previous forward bias cycle, hence leading to two distinct peaks of the forward current. Since it takes two cycles of the driving signal in this process to get back to the initial scenario, we identify this as a period-doubling bifurcation. As the input is further increased, another period doubling Bifurcation occurs and now four possible current levels can exist in the diode. This process continues until Chaos where a multitude of levels are possible.

**Calculations and Results:**

1. From measured resonance and upper and lower frequencies determine the RLC quality factor $Q$ given by $Q = \frac{f_o}{(f_u-f_l)}$, and compare the value to the theory given by $Q = \frac{\omega_o L}{R}$;
2. From the measured resonance frequency, determine the junction capacitance from $\omega_o = \frac{1}{\sqrt{LC}}$;
3. Plot the Bifurcation diagram (i.e., PP output versus input voltage) up to the edge of third Bifurcation as shown in the figure above;
4. From two measured recovery times after the first Bifurcation, determine the constants $r_m$ and $l_C$ for the diode you are using.