Significant Figures

When writing a number, it is important to know how well the value of the number is known.

Examples
If I say I have 12 donuts, I usually mean I have exactly 12 donuts because donuts do not come in fractions of a unit.

If I say I have 12 grams of salt, does that mean exactly 12 grams or something between 11.5 and 12.5 grams. Because the mass of an object in grams may be expressed in fractions of grams, I must specify as precisely as possible what the mass is.

Significant figures indicate the precision with which a number has been determined.

Significant Figures

When expressing a value, the number of digits used indicates the number of digits that could be measured accurately.
The final digit in the value is an estimate of the least precise number.

Length = 5.73 units

Significant Figures

The manner in which a number is written indicates the number of significant figures it has.

Examples
4000 1 significant figure
the zeros are simply place holders and do not indicate precision of the measurement.
in scientific notation: $4 \times 10^3$

Significant Figures

4000. 4 significant figures
By placing a decimal point after the last zero, this indicates that the zeros were measured.
in scientific notation: $4.000 \times 10^3$

Significant Figures

4000.000 7 significant figures
zeros following the decimal point with no other digits behind them also indicate precision of measurement, so they count as significant figures.
Significant Figures

.0004   1 significant figure
for numbers less than one, zeros following
the decimal point, but before the first digit,
are simply place holders and do not
indicate precision
in scientific notation: $4 \times 10^{-4}$

.000400  3 significant figures
zeros following digits in numbers less than
1 indicate precision and are significant.
in scientific notation: $4.00 \times 10^{-4}$

Mathematical Rules for Significant Figures

Addition/Subtraction
When adding or subtracting numbers, the
number of significant figures in the result is
determined from the position relative to the
decimal point of the least significant figure
of the numbers being added or subtracted

Add 472.1, 3.192, and 5000.86

\[
\begin{array}{c}
472.1 \\
3.192 \\
+ 5000.86 \\
\hline
5476.152
\end{array}
\]

Because 472.1 has only 1
sig. fig. to the right of the
decimal point, the final
answer can have only 1
sig. fig. to the right of the
decimal point—the correct
answer is $5476.2$

Mathematical Rules for Significant Figures

Subtract 126.5419 from 8000:

\[
\begin{array}{c}
8000 \\
- 126.5419 \\
\hline
7873.4581
\end{array}
\]

Because 8000 has only 1
sig. fig. four places to the left
of the decimal point, the
least significant figure in the
final answer must also be
four places to the left of the
decimal point—correct
answer is 8000

Mathematical Rules for Significant Figures

Multiplication/Division
The number of significant figures in the
result is determined from the number of
significant figures in the least significant
value used in the calculation
Mathematical Rules for Significant Figures

Multiply 88.037 by .00721

\[
\begin{array}{c}
88.037 \\
x \quad .00721 \\
\hline
0.63474677
\end{array}
\]

88.037 has 5 sig. figs. and .00721 has 3 sig. figs. This limits the result to a total of three sig. figs. — the correct answer is 0.635.

Logarithms/Exponentiation

Remember the definitions of logarithms:

\[ x = 10^y \quad \log(x) = y \]

\[
1000 = 10^3 \quad \log(1000) = 3
\]

Logarithms have a characteristic and a mantissa:

\[ \log(87.21) = 1.9406 \]

The characteristic determines the appropriate power of ten for the result, and the mantissa relates to the proper value of the number.

Example:

\[ \log(2.18 \times 10^{-6}) = -5.662 \]

The original number has 3 sig. figs., so the resulting mantissa should also have 3 sig. figs., and the result has 4 sig. figs.

-5, the characteristic, indicates the number of 0s serving as place holders.

Example:

\[ 10^{4.5} = 3 \times 10^4 \]

the mantissa of 4.5 has only 1 sig. fig.

the characteristic = 4 and indicates the power of ten in the result.

the result will have only 1 sig. fig.
Accuracy and Precision

Accuracy and precision have very different meaning in quantitative analysis.

Definitions

Accuracy—how well a result represents the “true” value for a measurement.

Precision—how well a set of measurements can be repeated with the same result.

Ideally, we want both accuracy and precision in our lab work.

Types of Experimental Errors

Determinate (systematic) errors

Determinate errors are errors that result from a flaw in the experimental procedure:

- Miscalibrated instrument
- Operator mistakes

Determinate errors occur the same way every time an analysis is made.

They can be corrected by calibrating instruments or training operators.

Random errors

Random errors result from the physical variability of measurements.

They have an equal chance of being either positive or negative:

- Noise in instruments
- Uncertainty in making readings

Uncertainty and Errors

If we assume that determinate errors have been eliminated, the uncertainty in any quantitative result will come from random errors.

The uncertainty of a single measurement can be determined either:

- From the equipment used
  - A buret has an uncertainty of ± 0.02 mL
  - An analytical balance has an uncertainty of approximately ± 0.0003 g
Uncertainty and Errors
The uncertainty of a single measurement can be determined either:

- performing repeated measurements and calculating the standard deviation when using an atomic absorption spectrometer, measure the absorbance of a sample five times

Uncertainty in a result can be expressed as either absolute uncertainty \( (e) \) or relative uncertainty \( (\%e) \) (for now, we will use \( e \) to represent the uncertainty in a quantity).

**Absolute uncertainty**

\[
m = 3.125 \pm 0.037 \text{ g}
\]

\( e = 0.037 \text{ g} \) is uncertainty in mass

absolute uncertainty has same units as the result

**Relative uncertainty**

Relative uncertainty is calculated by dividing the absolute uncertainty by the result and multiplying by 100%.

\[
m = 3.125 \pm 0.037 \text{ g}
\]

\[
\%e = \frac{0.037}{3.125} \times 100\% = 1.18\%
\]

\[
m = 3.125 \text{ g} \pm 1.18\%
\]

Relative uncertainty is always expressed as a percentage.

Uncertainty of a calculated result is determined depending on the calculations performed:

**Addition/Subtraction**—use absolute uncertainties

\[
x_{\text{final}} = x_1 + x_2 + x_3
\]

\[
x_1 \pm e_1 \quad x_2 \pm e_2 \quad x_3 \pm e_3
\]

\[
e_{\text{final}} = e_1^2 + e_2^2 + e_3^2
\]

\[
x_{\text{final}} \pm e_{\text{final}}
\]

**Multiplication/Division**—use relative uncertainties

\[
x_{\text{final}} = (x_1)(x_2)(x_3)
\]

\[
x_1 \pm \%e_1 \quad x_2 \pm \%e_2 \quad x_3 \pm \%e_3
\]

\[
\%e_{\text{final}} = \sqrt{\%e_1^2 + \%e_2^2 + \%e_3^2}
\]

\[
x_{\text{final}} \pm \%e_{\text{final}}
\]
The uncertainty of a calculated result is determined depending on the calculations performed:

**Multiplication/Division**

\[
\text{Multiplication/Division} \\
\text{If } x_1 = 2.34 \pm 1.12 \% \quad x_2 = 8.34 \times 10^{-4} \pm 3.83 \% \quad x_3 = 1280 \pm 0.239 \%
\]

\[
\%e_{\text{final}} = \sqrt{(1.12)^2 + (3.83)^2 + (0.239)^2} = 3.99 \%
\]

\[
x_{\text{final}} = 1.52 \times 10^{-6} \pm 3.99 \%
\]

\[
x_{\text{final}} = (1.52 \pm 0.06) \times 10^{-6}
\]

---

**Exponentiation and Logarithms**

For \( y = x^a \) \( x \pm %e_x \)

\[
%e_y = a(\%e_x)
\]

\[
y = x^3 \quad x = 5.981 \pm 2.13\%
\]

\[
%e_y = 3(2.13\%) = 6.39\%
\]

\[
y = 214.0 \pm 6.39\%
\]

---

For \( y = \log(x) \) \( x \pm e_x \)

\[
e_y = \frac{1}{\ln 10} \times e_x
\]

\[
y = \log(x) \quad x = 3.17 \pm 0.28
\]

\[
e_y = (0.43429)(0.28)/(3.17) = 0.0384
\]

\[
y = 0.501 \pm 0.038
\]

---

For \( y = \ln(x) \) \( x \pm e_x \)

\[
e_y = \frac{e_x}{x}
\]

\[
y = \ln(x) \quad x = 7.432 \times 10^{-5} \pm 0.917 \times 10^{-5}
\]

\[
e_y = (0.917 \times 10^{-5})/(7.432 \times 10^{-5}) = 0.123
\]

\[
y = -9.507 \pm 0.123
\]

---

A summary of the rules for determining uncertainties and their propagation through a series of operation is given in Table 3-1