Senior Assessment Exam

As part of MATH 490, you will take the Senior Assessment Exam (during the final exam period), which will count for at least 25% of your course grade.

The exam will consist of 25 questions (22 multiple choice or fill in the gaps/short answer) and 2-3 “free response” questions (for example to draw the graph of a function with given properties). The questions will cover core ideas from the following courses:

- MATH 102/103 (3-4) Precalculus
- MATH 206-209 (8) Calculus
- MATH 248 (2) Discrete Mathematics
- MATH 255 (2) Linear Algebra
- MATH 325 (2) Proof and Notation
- MATH 455 (2-3) Abstract Algebra
- MATH 465 (2-3) Advanced Calculus

The number in parenthesis indicates the number of multiple choice/short answer questions from each of the courses or course sequences. The three “free response” questions may come from any of the courses.

This handout contains a description of the topics that you should be familiar with and provides sample problems for each area.

Note that NO CALCULATORS or NOTES will be allowed in the exam.
Assessment Exam Syllabus for College Algebra

Operations with and simplification of polynomial, rational, and radical functions and solving simple equations involving them.

Properties of exponential and logarithmic expressions

Graphs of simple polynomial, rational, radical, exponential, and logarithmic functions.

Composition of functions, inverse of functions.

Applications of the above.
Assessment Exam Items for College Algebra

Note: Calculators are not allowed.

1. Let \( f(x) = e^x \) and \( g(x) = \ln x \). Then, for \( x > 0 \), the composition of \( f(x) \) and \( g(x) \) is

   a) \( f \circ g = 0 \)  b) \( f \circ g = 1 \)  c) \( f \circ g = e^x \)  d) \( f \circ g = \ln x \)  e) None of these

2. The inverse of the function \( y = f(x) = 3x - 6 \) is:

   a. \( y = g(x) = 3x + 6 \)  b. \( y = g(x) = (1/3)x + 6 \)  c. \( y = g(x) = (1/3)x + 2 \)

   d. \( y = g(x) = 3x + 2 \)  e. None of these

3. Suppose this sketch is the graph of the function \( y = f(x) \). Which of the following must be true about the graph of \( y = af(bx) + c \) if \( a > 1 \), \( b > 1 \), and \( c < 0 \)?

   a. The apparent maximum at \( x_1 \) will be even higher.
   b. The apparent maximum at \( x_1 \) will be moved to the right.
   c. The apparent minimum at 0 may no longer be an apparent minimum.
   d. The apparent minimum at 0 will be even lower.
   e. All of the graph will be moved down.

4. The sum of all of the real numbers solutions of \( \sqrt{x} + 2x = 1 \) is:

   a. \( 1/4 \)  b. \( 5/4 \)  c. 5  d. 1  e. None of these
Assessment Exam Syllabus for Trigonometry

Basic definitions of the six trigonometric functions and their inverses, both in terms of right triangles and as circular functions, both in terms of radian measure and degree measure. Their values for familiar angles and the relationship with isosceles right and equilateral triangles; i.e., 45°, 30°, 60°, as well as 0°, 90°, 270°.

Graphs of the basic trigonometric functions and variations on them, period and amplitude, phase shift.

Right triangle trigonometric identities, sum and difference formulas and the law of sines and the law of cosines.

Solve triangles and calculate missing but determined segments and angles of other figures.

Polar form of complex numbers, multiplication and division, DeMoivre’s Theorem, roots of a complex number.
Assessment Exam Items for Trigonometry

Note: No calculator allowed.

1. \( \sin(x + \frac{\pi}{4}) \) is equal to
   a) \( \cos(x - \frac{\pi}{4}) \)  \\ b) \( \cos(\frac{\pi}{4} + x) \)  \\ c) \( \cos(x + \frac{\pi}{2}) \)  \\ d) \( \sin(x - \frac{\pi}{4}) \)  \\ e) None of these

2. The amplitude, period, and phase shift respectively of \( \sin(2\pi x - 1) \) are:
   a) \( 1, 2\pi, -1 \)  \\ b) \( 1, 1, 1 \)  \\ c) \( 1, 1, \frac{1}{2\pi} \)  \\ d) \( 1, 2, -\frac{\pi}{2} \)  \\ e) None of these

3. \( \sin(\cos^{-1}(-\frac{4}{5})) \) is equal to
   a) \( \frac{3}{5} \)  \\ b) \( -\frac{3}{5} \)  \\ c) \( \frac{3}{4} \)  \\ d) \( -\frac{3}{4} \)  \\ e) None of these

4. In the triangle on the right, side \( x \) has length
   a) \( 5 \)  \\ b) \( 8 \)  \\ c) \( \sqrt{13} \)  \\ d) \( \sqrt{37} \)  \\ e) None of these

5. A point in the plane has polar coordinates \( (r, \theta) = (2, \frac{2\pi}{3}) \). Then its rectangular coordinates \( (x, y) \) are
   a) \( (1, \sqrt{3}) \)  \\ b) \( (\sqrt{3}, 1) \)  \\ c) \( \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) \)  \\ d) \( \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) \)  \\ e) \( (1, -\sqrt{3}) \)
Assessment Exam Syllabus for Calculus

Functions: Definition of a function, domain and range. Computation of limits, to determine points of continuity and discontinuity.

Derivative of a function: Definition of derivative; derivative as the slope of the tangent line and as the rate of change; computation of derivatives (using e.g., chain rule, product rule and quotient rule); implicit differentiation.

Applications of derivative of a function: Max/min problems and curve sketching; Mean-value theorem, Intermediate value theorem.

Integration: Evaluation of definite and indefinite integrals using substitution and integration by parts; Integrals of basic functions such as \( x^n, \ e^x, \ \sin x, \ \cos x, \ \ln x \).

Application of integrals: Area of a plane region bounded by two curves; volumes of solids of revolution.

Indeterminate forms: L’Hospital’s rule; improper integrals.

Vectors in 3-D: Dot product, cross product and their geometrical meanings; applications to planes, lines.

Sequences and series: Arithmetic and geometric sequences and series; Tests for convergence of a series (e.g., ratio test, integral test); Taylor series.

Assessment Exam Problems for Calculus

1. The domain of the real-valued function \( f(x) = \frac{\sqrt{x}}{x-2}, \ x \in \mathbb{R} \) is
   a. All real numbers.
   b. All real numbers except 2.
   c. All non-negative real numbers except 2.
   d. All real numbers greater than 2.
   e. None of the above.

2. If \( f(x) = \frac{x}{x^2 + 1} \), then \( f'(x) = \frac{(1-x)(1+x)}{(x^2 + 1)^2} \).
   Which of the following statements is true?
   a. There is a local maximum at \( x = 0 \).
   b. There is a local minimum at \( x = 0 \).
   c. There is a local maximum at \( x = 1 \)
   d. There is a local minimum at \( x = 1 \).
   e. None of the above is true.
3. Let \( f(x) \) be a continuous function for which \( \int_0^4 f(x) \, dx = 5 \). Then, the total area enclosed by the curve \( y = f(x) \), the x-axis, and the lines \( x = 0 \) and \( x = 4 \):
   a. Must be equal to 5.
   b. Must be greater than 5.
   c. Must be less than 5.
   d. Cannot be less than 5.
   e. Cannot be greater than 5.

4. Let \( f(x, y) = \frac{xy}{\sqrt{4-x^2-y^2}} \), \((x \in \mathbb{R}, \ y \in \mathbb{R})\). The domain of \( f(x, y) \) is:
   a. All \( (x, y) \) in the xy-plane.
   b. All \( (x, y) \) in the xy-plane except for the origin.
   c. All \( (x, y) \) inside the circle \( x^2+y^2 = 4 \).
   d. All \( (x, y) \) outside the circle \( x^2+y^2 = 4 \).
   e. All \( (x, y) \) except for those points on the x- and y-axes.

5. The sum of the infinite series
   \[
   1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} + \ldots
   \] is
   a. 2.
   b. 1.
   c. 3/2.
   d. \( \infty \).
   e. The sum does not exist.

6. Which of the following pairs of planes are perpendicular
   a. \( x - y + z = 2 \) and \( -x + y - z = 1 \).
   b. \( x - 2y + z = 2 \) and \( x + y + z = 1 \).
   c. \( 2x - y + z = 2 \) and \( x + \frac{1}{2}y - z = 0 \).
   d. \( x - y + z = 0 \) and \( -x + y - z = 1 \).
   e. \( x - y + 2z = 2 \) and \( -x + y - z = 0 \).

7. The tangent lines to the graphs of \( xy = 1 \) and \( x^2 - y^2 = 1 \) at their point of intersection are
   a. At right angles
   b. Parallel
   c. None of the above

8. The best way to evaluate the indefinite integral \( \int (3x^2 + 1)(5x^3 + 5x - 12)^5 \, dx \) is
   a. By expanding and applying the power rule
   b. By substitution \( 5x^3 + 5x - 12 = u \)
   c. By substitution \( 3x^2 + 1 = u \)
9. The additional information that will make the function \( f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ x & \text{if } -1 < x < 1 \\ 1 - x & \text{if } x \geq 1 \end{cases} \)

continuous is
a. \( f(-1) = 0 \)
b. \( f(1) = 1 \)
c. \( f(t) = 0 \)
d. \( f(-1) = 1 \)
e. None of the above

10. The area of the region below enclosed by the two graphs and the \( x \)-axis is given by

\[
\int_{-3}^{2} (y = \sqrt{x}) - (y = x + 6) dy
\]

a. \( \int_{0}^{2} -x + 6 - \sqrt{x} dx \)
b. \( \int_{-3}^{2} -x + 6 - \sqrt{x} dx \)
c. \( \int_{0}^{2} 6 - y - y^2 dy \)
d. \( \int_{-3}^{2} y^2 - (6 - y) dy \)

11. If \( f(x) = \int_{2x}^{3x} \frac{1}{t} dt \), then \( f \) is
a. A constant function on \((0, \infty)\)
b. Increasing on \((0, \infty)\)
c. Unbounded.
12. The function(s) to which you can apply L'Hospital's rule is/are

(i) \( \lim_{x \to \infty} x^x \)  
(ii) \( \lim_{x \to -\infty} (e^{-x} - x) \)  
(iii) \( \lim_{x \to 0} \left( \frac{1}{x-1} - \frac{x}{\ln x} \right) \)

(a) (i) and (ii)  
(b) (iii) only  
(c) (i) only  
(d) All three of them

13. If \( f(c) \) is not defined then

(a) \( \lim_{x \to c} f(x) \) does not exist.  
(b) \( f(x) \) is not continuous at \( x=c \)  
(c) Either \( \lim_{x \to c^-} f(x) \) or \( \lim_{x \to c^+} f(x) \) does not exist  
(d) Both (a) and (c) hold.

14. The following function possesses the attributes.

(a) Infinite limit at \( x=1 \), no limit at \( x=8 \)  
(b) Infinite limit at \( x=2 \), no limit at \( x=-5 \)  
(c) Infinite limit as \( x \to 2^+ \), limit at infinity is equal to 1.  
(d) Not continuous at \( x=-5 \), no limit at \( x=8 \)  
(e) Both (c) and (d)
**Topics for the Math 248 Assessment Exam**

1. Binary, Octal, Hexadecimal Number Systems

2. Relations on a set: reflexive, symmetric, transitive, antisymmetric; equivalence relation, partial order, composition of relations matrices of relations

3. Counting problems: permutations, combinations, generalized permutations and combinations

4. Paths, cycles in graphs, Euler cycle, Hamiltonian cycle, adjacency matrix of a graph.

5. Terminology and characterization of trees.
1. Let Z denote the integers. For \(a, b, \in Z\), \(aRb\) means \(a \leq b + 1\).

   The relation R is (check all that are true)
   (a) reflexive  (b) symmetric  (c) transitive  (d) antisymmetric

2. Find the number of integer solutions to
   \[x_1 + x_2 + x_3 + x_4 = 21\]
   where \(x_i \geq 0, \ i = 1,2,3,4.\)
   (a) \(C(21,4)\)  (b) \(C(25,4)\)  (c) \(C(24,3)\)  (d) \(C(24,4)\)

3. Find the coefficient of \(x^2y^3z^5\) in the expansion of \((2x+y-z)^{10}\).
   (a) -10080  (b) -10,800  (c) 2520  (d) -2520  (e) -5040

4. Find the base 16 representation of the base ten integer 1548.
   (a) F11  (b) C06  (c) E0F  (d) F0B  (e) 60C

5. Find the sum of the base 16 integers; leave answer in base 16.
   \((5AFB)_{16} + (B046)_{16} =\)
   (a) \((10B41)_{16}\)  (b) \((16B41)_{16}\)  (c) \((16B47)_{16}\)  (d) \((6B477)_{16}\)  (e) \((17B51)_{16}\)

6. The graph in the figure below has:

   (a) An Euler cycle and no Hamiltonian cycle.
   (b) An Euler cycle and a Hamiltonian cycle.
   (c) No Euler cycle and a Hamiltonian cycle.
   (d) No Euler cycle and no Hamiltonian cycle.
7. Find $M^2$ where $M$ is the adjacency matrix $M$ of the graph shown below with alphabetical ordering of vertices.

![Graph Image]

(a) $M^2 = \begin{pmatrix} 4 & 2 & 2 & 2 & 2 \\ 2 & 3 & 1 & 3 & 1 \\ 2 & 1 & 3 & 1 & 3 \\ 2 & 3 & 1 & 3 & 1 \\ 2 & 1 & 3 & 1 & 3 \end{pmatrix}$  
(b) $M^2 = \begin{pmatrix} 4 & 2 & 3 & 2 & 2 \\ 2 & 3 & 1 & 3 & 1 \\ 3 & 1 & 3 & 1 & 3 \\ 2 & 3 & 1 & 3 & 1 \\ 2 & 1 & 3 & 1 & 3 \end{pmatrix}$  
(c) $M^2 = \begin{pmatrix} 4 & 3 & 3 & 2 & 2 \\ 3 & 3 & 1 & 3 & 1 \\ 3 & 1 & 3 & 1 & 3 \\ 2 & 3 & 1 & 3 & 1 \\ 2 & 1 & 3 & 1 & 3 \end{pmatrix}$  
(d) $M^2 = \begin{pmatrix} 4 & 2 & 2 & 2 & 2 \\ 2 & 3 & 1 & 3 & 1 \\ 2 & 1 & 3 & 1 & 3 \\ 2 & 3 & 1 & 3 & 2 \\ 2 & 1 & 3 & 1 & 3 \end{pmatrix}$  
(e) $M^2$ is not defined for this graph

8. How many 8-bit binary strings (that is, each bit is either 0 or 1) contain exactly three 1’s?

(a) $8!$  
(b) $\frac{8!}{(3!)(5!)}$  
(c) $8 \times 7 \times 6$  
(d) $8! - 3!$  
(e) $\frac{8!}{3!}$

9. Let $G$ be a graph with $n$ vertices. Which of the following statements is not always true?

a. The number of odd degree vertices is always odd.  
b. If $G$ is a tree, then $G$ has $n - 1$ edges.  
c. If $G$ is a tree, then $G$ is connected.  
d. The sum of degrees of all vertices of $G$ is always even.
Topics for Linear Algebra

1. Definition and examples of subspaces of $\mathbb{R}^n$.

2. For a given set of vectors in a subspace of $\mathbb{R}^n$, determine if it is linearly independent, if it spans the subspace, and if it is a basis for the subspace.

3. Determine the dimension of, and a basis for, a given subspace of $\mathbb{R}^n$.

4. Definition and examples of linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$.

5. Determine the matrix and nullspace of a given linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.

6. Definition of, and the technique for determining, the eigenvalues and eigenvectors of a linear transformation.

7. Examples of vector spaces and subspaces other than $\mathbb{R}^n$ and its subspaces.
Sample Problems for Linear Algebra

1. Let \( T = \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5 \} \) be a set of five distinct vectors in \( \mathbb{R}^4 \). Which of the following statements is true?
   a. \( T \) must be linearly independent.
   b. \( T \) must span \( \mathbb{R}^4 \).
   c. \( T \) cannot be a basis for \( \mathbb{R}^4 \).
   d. \( T \) is linearly independent if and only if it spans \( \mathbb{R}^4 \).
   e. None of the above statements is true.

2. Let \( S \) be the subspace of \( \mathbb{R}^4 \) consisting of all vectors with first and second components equal to 0. A basis for \( S \) is the set ___________________________

3. Let \( L \) be a linear transformation from \( \mathbb{R}^2 \) to \( \mathbb{R}^3 \) defined by \( L(x, y) = (x, y, x + y) \). The matrix associated with \( L \) is given by _______________

4. The eigenvalues of the matrix \[
\begin{pmatrix}
1 & 2 \\
3 & 2
\end{pmatrix}
\] are:
   a. 1 and 2
   b. -1 and -2
   c. -1 and 4
   d. 2 and 3
   e. None of these answers is correct.

5. Give an example of a vector space (describing the underlying set and operations on that set) that is not \( \mathbb{R}^n \) or one of its subspaces.
Topics for Math 455

Definition of a group/subgroup/ring/field
Order of an element in a group
Order of a group/subgroup/quotient group
Cosets
Quotient groups
Cyclic groups, \( \mathbb{Z}_n \)
Permutation Groups, \( S_n \)
Generators of a group
Homomorphism
Isomorphism
Definition of abelian
Direct products of groups
Cycle notation for permutations

Sample problems:

1. \( \mathbb{Z}_{24}/\mathbb{Z}_4 \) is isomorphic to which of the following groups?
   a) \( \mathbb{Z} \)  
   b) \( S_6 \)  
   c) \( \mathbb{Z}_2 \times \mathbb{Z}_3 \)  
   d) \( \mathbb{Z}_{96} \)  
   e) None of these

2. Let \( \mathbb{Z}_n \) be the integers mod n under addition with members \{0, 1, 2, 3, …, [n-1]\}.
   Define \( \varphi: \mathbb{Z}_n \rightarrow \mathbb{Z} \) by \( \varphi([a]) = a \). Then (mark all that apply)
   a) \( \varphi \) is an onto function  
   b) \( \varphi \) is a 1-1 function  
   c) \( \varphi \) is not a function  
   d) \( \varphi \) is a homomorphism  
   e) \( \varphi \) is an isomorphism

3. If \( G \) is a group of order 8 then (mark all that must be true.)
   a) \( G \) is cyclic  
   b) \( G \) is abelian  
   c) \( a \in G \implies |a| = 8 \)  
   d) \( a \in G \implies |a| \divides 8 \)  
   e) \( G \) is non-abelian

4. Which of the following are homomorphisms of the group of Integers under addition?
   \( \varphi: \mathbb{Z} \rightarrow \mathbb{Z}, \varphi(z) = 6z \)  
   \( \psi: \mathbb{Z} \rightarrow \mathbb{Z}, \psi(z) = z + 6 \)
   a) neither  
   b) \( \psi \) only  
   c) \( \varphi \) only  
   d) \( \varphi \) and \( \psi \)

5. In \( \mathbb{Z}_{24} \), what is the order of 18? \( \mathbb{Z}_{24} = \{0,1,2,\ldots,23\} \) with modular arithmetic
   a) 4/3  
   b) 4  
   c) 6  
   d) 18  
   e) 24

6. Consider \( \mathbb{Z}_6 \) with the usual modular operations +, \( \cdot \). Which of the following is/are true (mark all that apply)
   a) \( (\mathbb{Z}_6, +) \) is a group  
   b) \( (\mathbb{Z}_6, \cdot) \) is a group  
   c) \( (\mathbb{Z}_6, +, \cdot) \) is a ring  
   d) \( (\mathbb{Z}_6, +, \cdot) \) is a field.
Topics for Math 465 Advanced Calculus

**Sequences:** completeness of the real line, convergence of sequences, supremum, infimum, Cauchy sequences, cluster points (limit points), sup, inf.

**Topology:** open sets, neighborhoods, interior of a set, closed sets, closure of a set, boundary of a set, accumulation points (limit points), compact sets, Heine-Borel theorem.

**Series:** power series.

**Continuous functions:** intermediate value theorem, uniform continuity, pointwise convergence, uniform convergence.
Sample Problems for Advanced Calculus

1) Define the sequence \( \{x_n\}_{n=1}^{\infty} \) on the real line by \( x_n = 2 + \frac{1}{n} \) for \( n \) even and \( x_n = n \) for \( n \) odd. What are \( \inf x_n \) and \( \sup x_n \)?
   (a) 2 and \( n \)
   (b) 1 and \( \infty \)
   (c) 2 and \( \infty \)
   (d) \( \frac{1}{n} \) and \( n \)

2) A sequence \( \{x_n\} \) on the real line converges to a limit \( L \) if for every positive \( \varepsilon \)
   (a) there is an \( N > 0 \) with \( |x_n - L| < \varepsilon \) whenever \( n > N \)
   (b) \( |x_n - L| < \varepsilon \) for some \( n > N \)
   (c) there is an \( N > 0 \) with \( |x_n| < L + \varepsilon \) whenever \( n > N \)
   (d) none of the above

3) Every convergent sequence on the real line is
   (a) Cauchy
   (b) monotone
   (c) bounded
   (d) (a) and (c)

4) Which of the following sets are open
   (a) \( \bigcap_{n=1}^{\infty} (1 - \frac{1}{n}, 2 + \frac{1}{n}) \)
   (b) \( \{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 \leq 1\} \)
   (c) \( \bigcup_{n=2}^{\infty} [1 + \frac{1}{n}, 2 - \frac{1}{n}] \)
   (d) (a) and (c)

5) The closure of a subset \( A \subset \mathbb{R}^n \) is
   (a) the union of \( A \) with all its interior points
   (b) the intersection of all the closed sets containing \( A \)
   (c) any closed set containing \( A \)
   (d) the intersection of \( A \) with its complement

6) Let \( f_n(x) \) be a sequence of continuous functions defined on \([0,1]\). State what it means for the sequence \( f_n(x) \) to converge uniformly to a function \( f(x) \) on \([0,1]\).