Do you Sudoku?

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Overview

- What is Sudoku?
- Brief History
- How to play Sudoku (Strategies)
- Math Questions
- Sudoku Solvers
- Sudoku World Championship
- Sudoku Variations
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What is Sudoku?

- Standard Sudoku is a 9x9 grid with certain values filled in ("givens"; 24, 32 or 48)
- Goal: Fill remaining cells such that each row, column and 3x3 region contains the numbers 1, 2, ..., 9 exactly once
- Levels: mild, moderate, medium, devious, nasty

History

- First appeared in 1979 in New York Puzzle magazine under the name Number Place. Designed anonymously by Howard Garns, a 74-year-old retired architect and freelance puzzle constructor
- Puzzle company Nikoli introduced it in 1984 under the name Suuji wa dokushin ni kagiru (= single number) in Japan

History

- In 1997, Wayne Gould, a retired High Court Judge living in Hong Kong, discovered Sudoku and developed computer programs to produce Sudoku puzzles of varying difficulty
- In 2004, Gould sent puzzle to The Times in London, which set off a Sudoku craze
- In 2005, Sudoku spread around the world

Strategies

- Scanning (horizontal or vertical)
**Strategies**

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- Scanning combined
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- Small numbers

Pairs of numbers in two cells
Strategies

- Scanning (horizontal or vertical)
- Scanning combined
- Small numbers
- Pairs of numbers in two cells
- Missing values in rows, columns, and region
- Using region information to exclude values in rows and columns

3 9 4 \_ \_ 6 7
4 \_ \_ \_ \_ 3 9
9 7 3 4 5 8
9 4 1 5 3
3 4 \_ \_ 9
5 3 8 7 9 4
3 6 2 9 4
3 9 4 2
9 4 1 2 7 3
### Math Questions
- How many 9x9 Sudoku grids are there?
- How many givens are needed to complete puzzle in a unique way?
- How many givens can there be without a unique solution?
- How does one solve a Sudoku puzzle automatically?
- What makes a Sudoku puzzle hard?

### How many 9x9 Sudoku grids?
- **Bertram Felgenhauer (2005)** (logic and brute force computations)
  \[6,670,903,752,021,072,936,960 = 9! \times 72^2 \times 27 \times 27,704,267,971\]
- Derivation simplified by Frazer Jarvis
- Independently confirmed by Ed Russell

### Minimum number of givens
- For 9x9 Sudoku grid, lowest known is 17 givens (if no symmetry restrictions) and 18 givens if rotational symmetry is enforced
- For some of the other variants, examples with low numbers of givens have been constructed, but not for all cases.
- Only in the case of cubic Sudoku is the minimal number of givens known (=8) and examples have been constructed

### Maximum number of givens
- What is the maximal number of givens for which the puzzle cannot be completed uniquely?
- **Answer:** 77

### How many 9x9 Sudoku grids?
- **Felgenhauer & Jarvis (2006)** (using group theory)
  - Relabeling of entries
  - Reflection; rotation
  - Permutation of blocks of columns 1-3, 4-6, 7-9
  - Permutation of blocks of rows 1-3, 4-6, 7-9
  - Permutation of columns 1-3 or 4-6 or 7-9
  - Permutation of rows 1-3 or 4-6 or 7-9
  - Essentially different grids: **5,472,730,538**
Solving Sudoku Automatically

- Can express the problem as a graph coloring problem, where adjacent vertices have to be colored in a different color
- Each Sudoku cell becomes a vertex
- Vertices are connected if cells are in the same row, column, or region
- Givens represent pre-colored vertices
- NP-complete problem

Efficient programs use some of the strategies we have discussed, but in a slightly different order

- Step 1: Assign each cell all the values it can have
- Step 2: Check if any cell has just one value; if so, assign the value and eliminate it from the cells in the same row, column and region

- Step 3: Check whether a particular value occurs in only one cell of a row, column or region; if so, assign this value to its cell and eliminate it from the cells in the same row, column and region. Repeat Step 2
- Step 4: If a value occurs in only two cells in a block, and the cells are in either a row or a column, then delete the value from all other cells in that row or column. Repeat Steps 2 & 3.

Example for Step 4
Solving Sudoku Automatically

- **Step 5:** If the logic rules have been applied and no further reduction occurs, select a cell for "what if".
- **Step 6:** Try possible values until solution is found, or contradiction → back-track

Humans: Use logic rules first, since backtracking is hard

Computer: Use only a few logic rules, since backtracking is not so hard. Important: which cell is selected for going into backtracking mode

Sudoku Competition

- First World Sudoku Championship, Lucca (Italy), March 10 - 11, 2006
- 1st place: Jana Tylova, Czech Republic
- 2nd & 3rd place: USA
- Had to solve 45 puzzles, classic + 18 variations
- 85 puzzle solvers from 22 countries
- Participants ranged from 15 to 61 years
- 1/3 were women

Sudoku Variations

- Irregular Sudoku
- Diagonal Sudoku; Extra Region Sudoku
- Cubic Sudoku
- Toroidal Sudoku
- Multiple Region Sudoku
- .....

References

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- Fred Simons, Solving a Sudoku puzzle with Mathematica, Mathematica in Education and Research, 10:1, 2005
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