Tuesday 9/24

Previously in 5401

If N & G, then

G/N is a group

using the operation

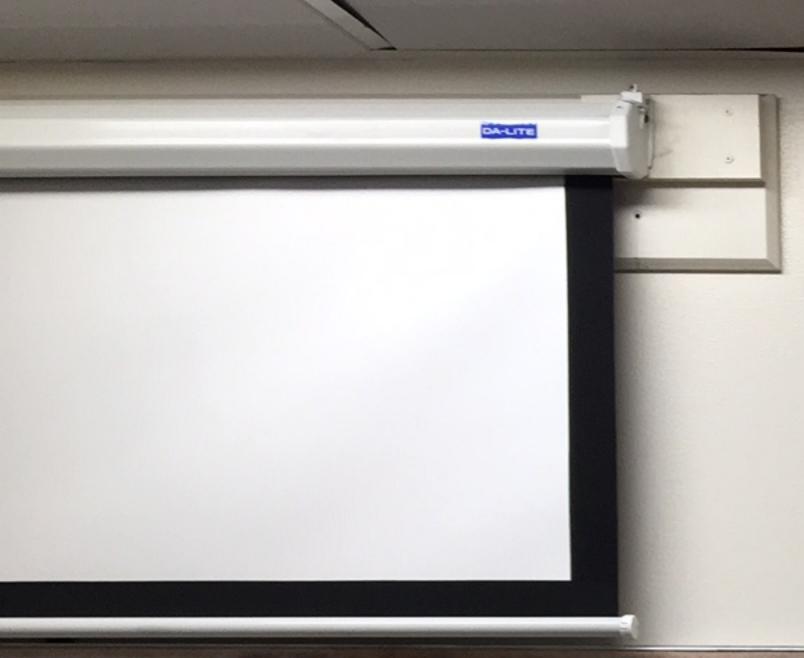
(aN)(bN)=(ab)N

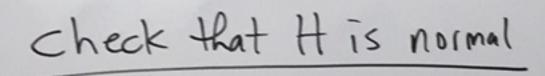
 $E_{X:}$   $D_{14} = \{1, r, r^{2}, r^{3}, r^{4}, r^{5}, r^{5}, s^{6}\}$   $= \{1, r, r^{2}, r^{3}, r^{5}, s^{7}, s^{7}, s^{7}, s^{5}, s^{6}\}$ 

 $H = \langle r \rangle = \{1, r, r, r, r^3, r^4, r^5, r^6\}$  $SH = \{s, sr, sr^3, sr^3, sr^4, sr^5, sr^6\}$ 

So, Dy/H={H, 3H}

The leave the So,





right cosets

H

Hs = 
$$\{s, rs, r^2, r^3, r^4, r^5, r^6, r^6\}$$

=  $\{s, sr^6, sr^5, sr^4, sr^3, sr^2, sr^3\}$  = sH

The left and right cosets are the same. So, H&G.

So, D/H={H,5H}?

is a group.

PH/H	H	5H
H	H	sH
5H	SH	H
-	1	

[Z]	0	Ī	
0	0	1	
T	T	0	
		-	

It's the!

Same table!

So, Phy/H= Zz.

Using +: Phy/H > Zz

H(H) = T

H(sH) = T

Theorem: Let G be

an abelian group and
H is a subgroup of G.

Then H is normal in G.

Proof: Let geG. Then

gH={gh|heH}}

= {hg|heH}=Hg.

 $Z = \{2..., -3, -2, -1, 0, 1, 2, 3, ...\}$ H= (4)=4Z= \( \frac{2}{3} - \f 1+42={--,-7,-3,1,5,9,000} 3+47={--,-5,-1,3,7,11,--} 2/42={0+42,1+42,2+42,3+42}



element)	order
0+42	1
-1+42	4
2+42	2 -
3+42	4

So,
$$\frac{\mathbb{Z}/4\mathbb{Z} = \langle 1+4\mathbb{Z} \rangle}{\text{1s cyclic.}}$$
Thus,
$$\frac{\mathbb{Z}/4\mathbb{Z} \cong \mathbb{Z}_4}{\mathbb{Z}/4\mathbb{Z} \cong \mathbb{Z}_4}$$

$$\begin{array}{c} 1+42 \neq 0+42 \\ (1+42)+(1+42)=2+42 \neq 0+42 \\ (1+42)+(1+42)+(1+42)=3+42 \neq 0+42 \\ (1+42)+(1+42)+(1+42)=4+42=0+42 \end{array}$$

## Corollary's to Lagrange's Theorem

Recalli (Lagrange's Thm)

If G is a finite group

and H < G, then 1H1

divides 1G1.

Corollary: Let G be a group of size n. Let XEG.

Then:

The order of X divides |G|=n

(3) XIGI = 1

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Proof of I and I Let I I Let I I I Let I I I Let I I I Let I Let I I Let I I Let I Let I I Let I Le

Proof of 2: By Pout 1,  $|G| = |x| \cdot k$  for some k. So,  $x^{|G|} = x^{|x| \cdot k} = (x^{|x|})^k = 1^k = 1$ .

Corollary: Let G be a group of size p

Where P is prime. Then  $G \cong \mathbb{Z}p$ . So, in particular G is cyclic. Where P is prime, p = 2.

Pf: Since P is prime, p = 2.

So we can pick some  $x \in G$  with  $x \neq 1$ .

So we can pick some  $x \in G$  with  $x \neq 1$ .

By the previous corollary,  $|\langle x \rangle| = |\langle x \rangle| = P$ .

So either  $|\langle x \rangle| = 1$  or  $|\langle x \rangle| = P$ .

But  $|\langle x \rangle| \neq 1$  since  $1 \in \langle x \rangle$  and  $x \in \langle x \rangle$  and  $1 \neq x$ .

Prime Size.

So,  $|\langle x \rangle| = P$ . Thus,  $G = \langle x \rangle$ , So, G is cyclic and  $G = \langle x \rangle$ .

Theorem: Let G be a group and

H 

G. Then H is normal in G

iff H is the kernel of some homomorphism

P: G 

G where G is a group.

Proof:

( ) Let H = ker(Q) where P: G 

G 

by proving gHg' 

Let y 

gHg'

So, y = gkg' where keH=ker(Q).

So, y = gkg' where keH=ker(Q).

Well, we see that  $\varphi(y) = \varphi(gkg')$   $= \varphi(g) \varphi(k) \varphi(g')$   $= \varphi(g) \varphi(k) \varphi(g)$   $= \varphi(g) \varphi(g)$   $= \varphi(g) \varphi(g)$   $= \varphi(g) \varphi(g')$   $= \varphi(g) \varphi(g')$   $= \varphi(g) \varphi(g')$   $= \varphi(g) \varphi(g')$   $= \varphi(g) \varphi(g')$ 

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So, y E H = ker(q). Thus, gHg'=H.

(=>) Let H = G.

Then G/H is a group.

Define  $\varphi: G \to G/H$  This is called the natural by  $\varphi(q) = gH$ . I homomorphism. by  $\varphi(g) = gH$ .

Let's verify that & 13 a homomorphism.

Pick X, y ∈ G.

def of operation in G/H Then  $\varphi(xy) = (xy)H = (xH)(yH) = \varphi(x)\varphi(y).$ So, q is a homomorphism. Furthermore,  $ker(\varphi) = \{g \in G \mid \varphi(g) = H\}$ element of G/H  $= \{g \in G \mid gH = H\}$ = {gEG|gEH} 5'a∈H

Ex: G= ZxZ,  $H = 2\mathbb{Z} \times \{\bar{o}\} = \{..., (-4, \bar{o}), (-2, \bar{o}), (0, \bar{o}), (2, \bar{o}), (4, \bar{o}), ...\}$  $(1,\overline{0})+H=\{...,(-3,\overline{0}),(-1,\overline{0}),(1,\overline{0}),(3,\overline{0}),(5,\overline{0}),...\}$  $(0,T)+H=\{...,(-4,T),(-2,T),(0,T),(2,T),(4,T),...\}$  $(1,T)+H=\{--,(-3,T),(-1,T),(1,T),(3,T),(5,T),...\}$ Fact: If G, and Gz are both abelian then G, XGz is abelian 50, ZX Zz is abelian. Thus, HOZXZ

ZxZ2/H={	(0,0)+H,(1,0)+H,(0,T)+H	to (1,T)+H} is a group.

Clement	order	
(0,0)+++	1	
H+(0,1)	2	
(O,T)+H	2	
H+(T,1)	2	-

You can check (by comparing group tables)
that  $\mathbb{Z} \times \mathbb{Z} \times /H \cong \mathbb{Z}_2 \times \mathbb{Z}_2$ 

4 ZXZZ/H is not cyclic since no element of order 4)

$$(2z/H) \text{ is not cyclic since no element of order Y}$$

$$(2,\overline{0}) \in (0,\overline{0}) + H$$

$$(1,\overline{1}) + H \neq (0,\overline{0}) + H$$

$$(1,\overline{1}) + H + (1,\overline{1}) + H = (2,\overline{0}) + H = (2,\overline{0}) + H = (6,\overline{0}) + H$$