Math 4740

9/27/23

Montey Hall problem

- See Numberphile video and 21 video from website first.

- Suppose you always start by picking door #1. Then Montey Hall reveals a goat behind either door 2 or door 3. Then asks if you want to switch or stay on door 1. What do you do?

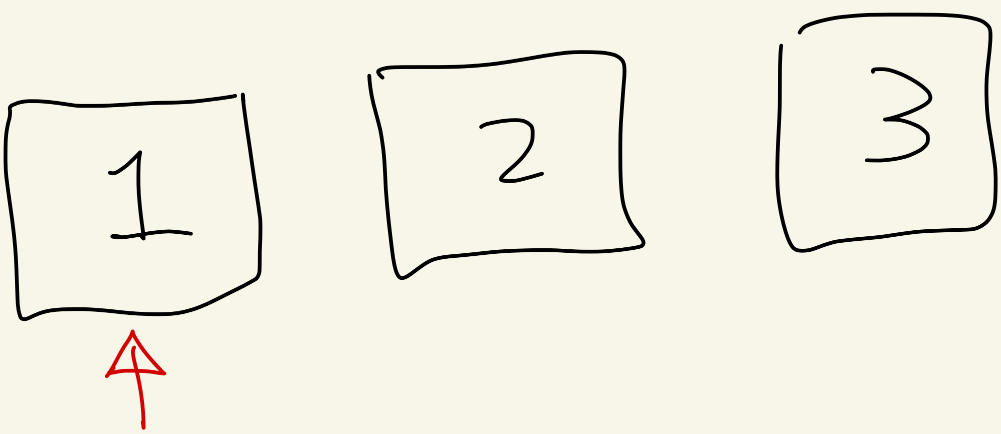


Table of possibilities

door 1	door 2	door 3	Stay w/ door 1 strategy	switch from door 1 strategy
car	goat	goat	WIN	LOSE
goat	car	goat	LOSE	WIN
goat	goat	car	LOSE	WIN

always starting with door 1 as first choice

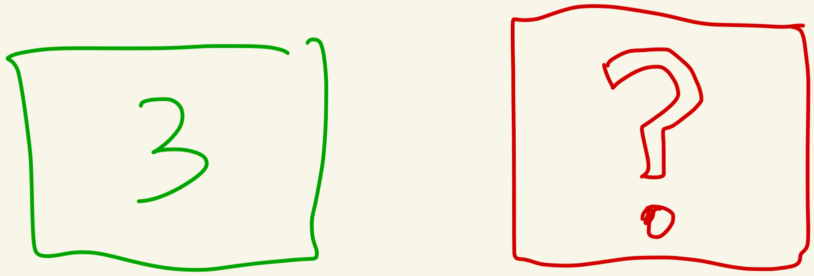
Staying You win 1/3 of the time

Switching You win 2/3 of the time

You should always switch!

Topic 3 – Conditional Probability

Ex: Suppose we roll two 6-sided dice, a green die and a red die. Suppose the green die stops rolling and lands on a 3, but the red die keeps rolling.
What's the probability that the sum of the dice is 8?



starting sample space

S

(1,1)
(1,2)
(1,3)
(1,4)
⋮

(3,1)
(3,2)
(3,3)
(3,4)
(3,5)
(3,6)

⋮
(6,5)
(6,6)

36 outcomes

(green, red)

new sample space

S'

(3,1)
(3,2)
(3,3)
(3,4)
(3,5)
(3,6)

6 outcomes

only one has sum of dice being 8.

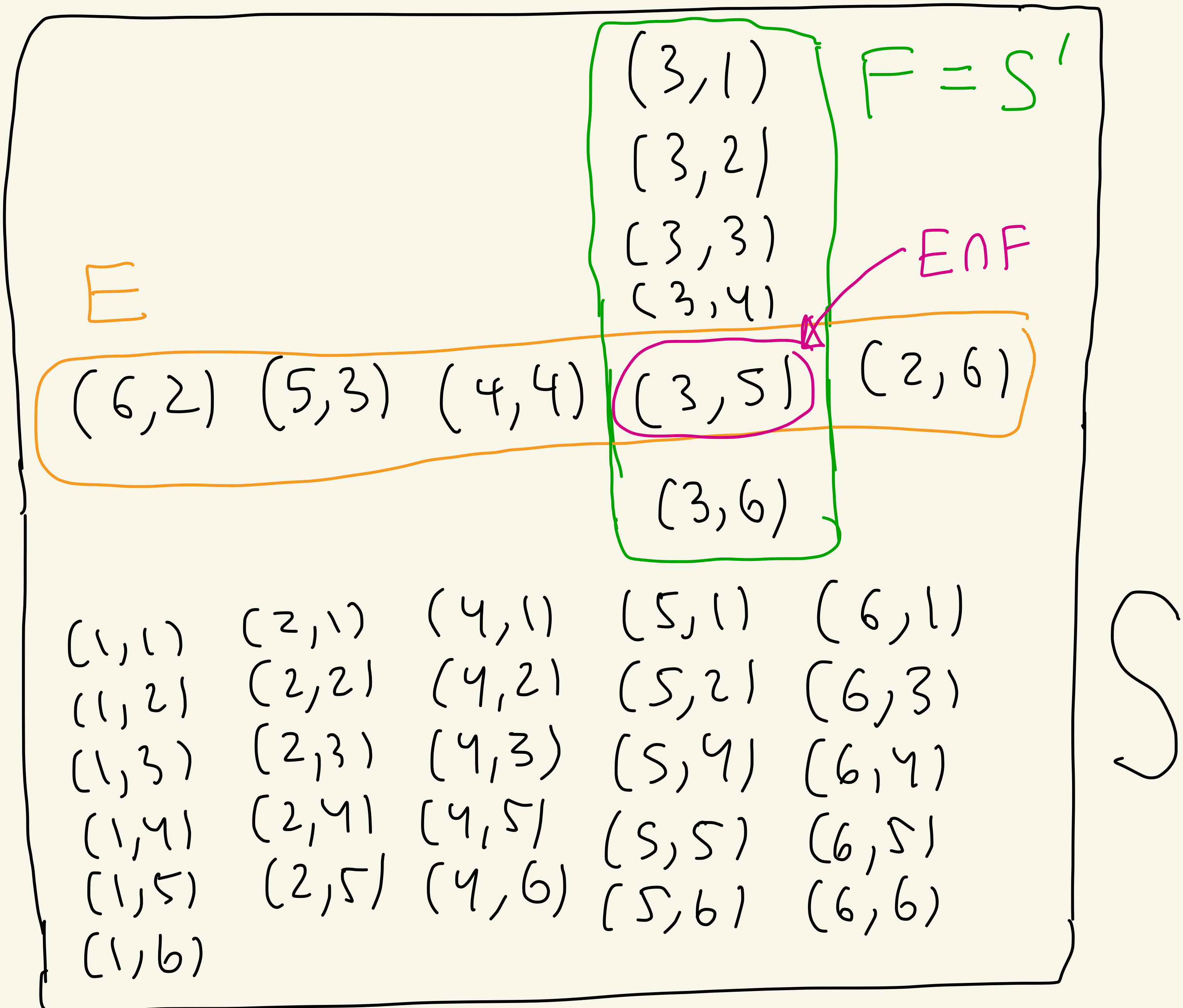
So, the probability is $1/6$.

Let's make a formula for this without having to shrink the sample space S and also a method that generalizes even to spaces where the outcomes are not equally likely.

Let E = the event in S where the sum of the dice is 8.

Let $F = S' =$ the event in S where the green die is 3.

We want to know the "conditional probability" of the event E occuring given that F has "already occured".



$$\underbrace{\frac{|E\cap F|}{|F|}}_{\text{we did this to get } 1/6} = \frac{|E\cap F|/|S|}{|F|/|S|} = \underbrace{\frac{P(E\cap F)}{P(F)}}_{\text{probability in } S} \qquad \frac{(1/36)}{(6/36)} \qquad \boxed{1/6}$$

ok since all outcomes are equally likely

$\boxed{\text{Def:}}$ Let (S, Ω, P) be a probability space. Let E and F be two events. Suppose $P(F) > 0$. Define the conditional probability that E occurs given that F occured to be

$$\underbrace{P(E|F)}_{\text{notation}} = \underbrace{\frac{P(E \cap F)}{P(F)}}_{\substack{\text{these probabilities} \\ \text{are calculated} \\ \text{in } S}}$$

Ex: (HW 3 #3 modified)

Suppose you roll two 8-sided dice. You can't see the outcome, but your friend can. They tell you that the sum of the dice is divisible by 5. What is the probability that both dice have landed on 5?

$S = \{(a,b) \mid a,b = 1,2,\dots,8\}$

$|S| = 8^2 = 64$

$F = \{(a,b) \mid a+b \text{ is divisible by } 5\}$

$E = \{(5,5)\}$

Want: $P(E|F) = \dfrac{P(E \cap F)}{P(F)}$

We have

$F = \{$