

Math 4740

9/25/23



Still in topic 2

How do we make a probability function when you do two experiments in a row where the outcome of the first experiment does not influence the outcome of the second experiment?

Ex: Suppose you flip a coin and then roll a 4-sided die.

Let's make a probability space for this. [We use a normal coin & die]

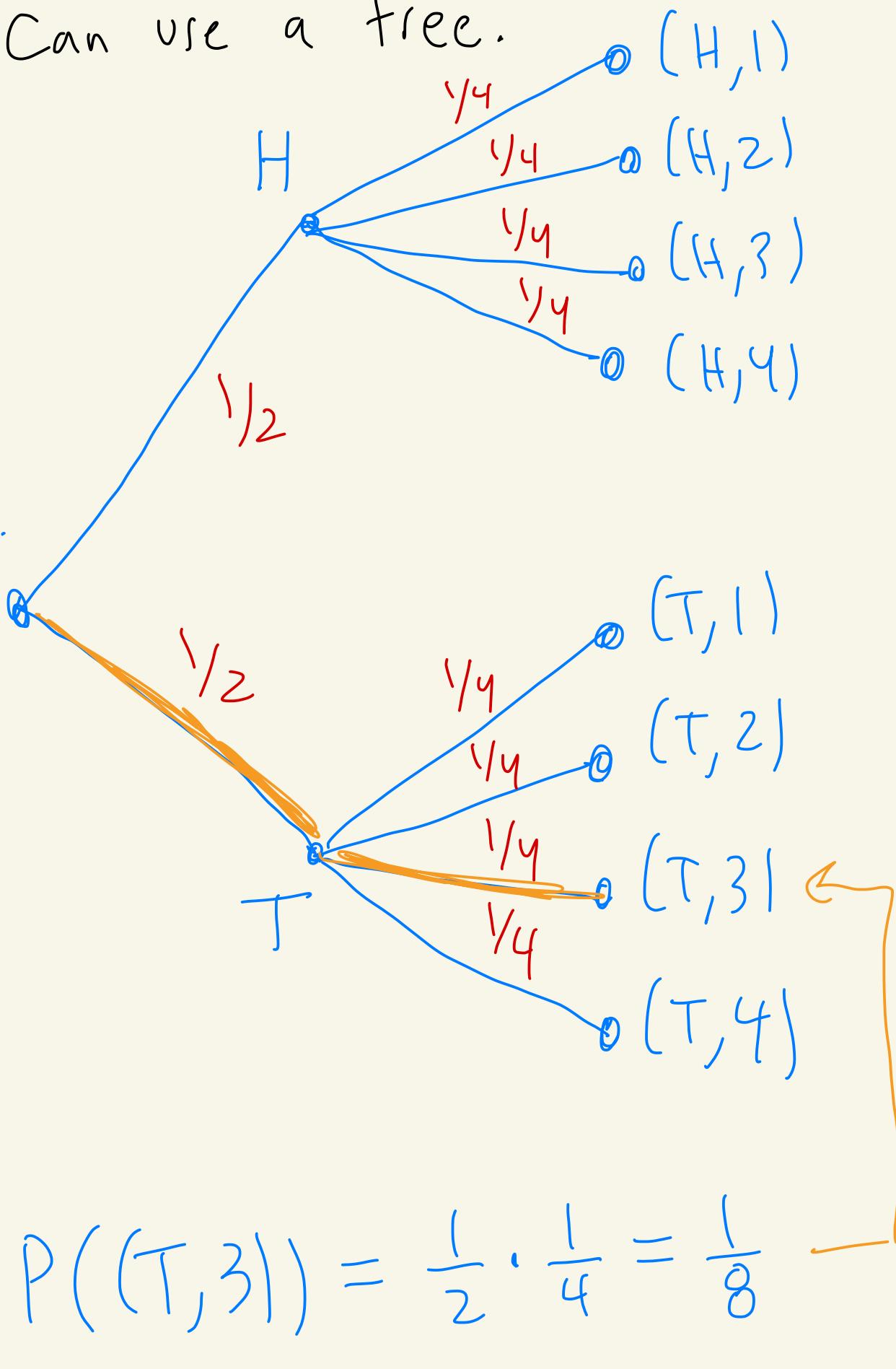
Sample space:

$$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (T, 1), (T, 2), (T, 3), (T, 4)\}$$

$$= \underbrace{\{H, T\}}_{\text{Sample space of flipping coin}} \times \underbrace{\{1, 2, 3, 4\}}_{\text{Sample space of rolling 4-sided die}}$$

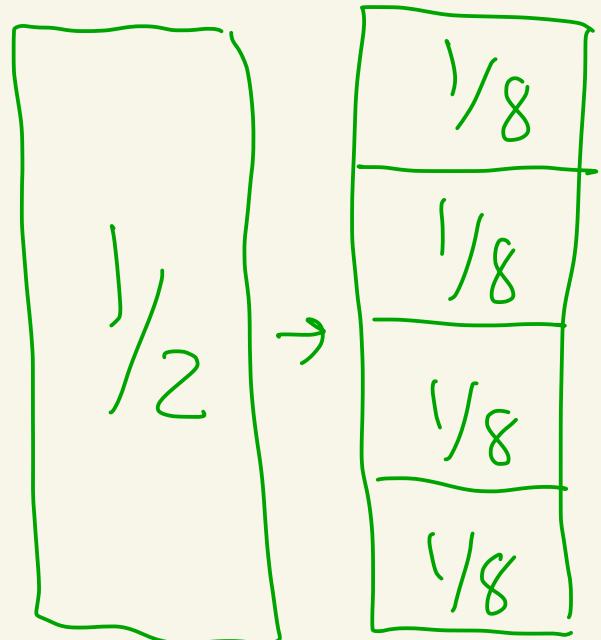
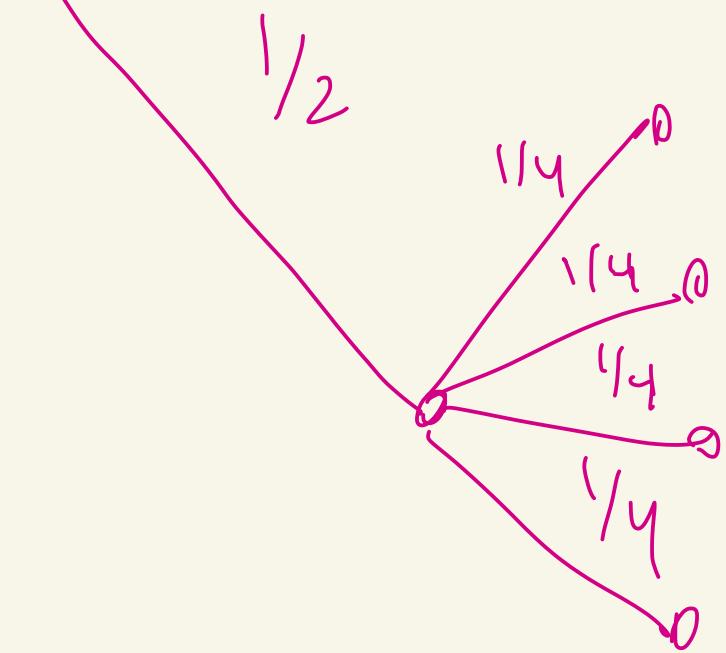
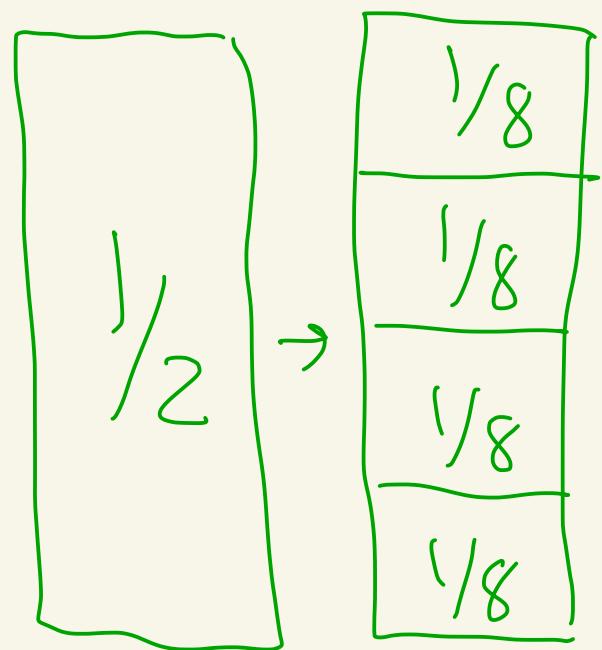
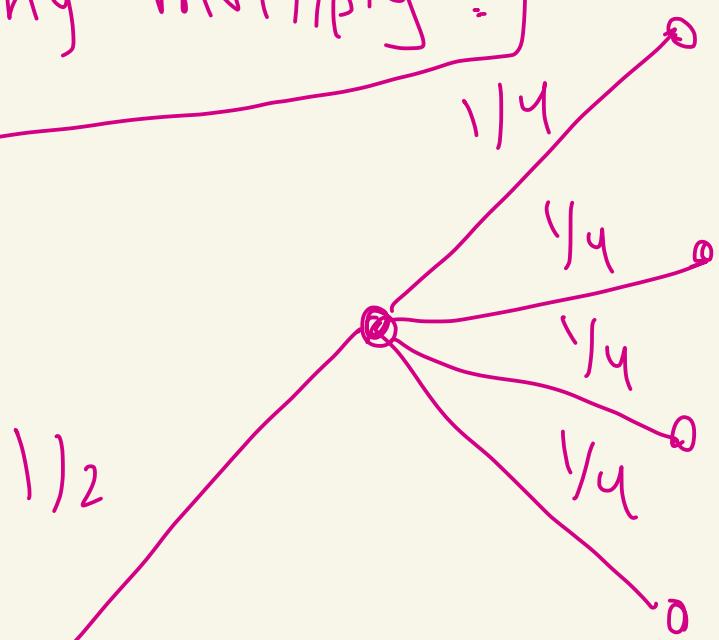
Ω = set of all subsets of S

Let's make the probability function.
Can use a tree.



multiply
probabilities
along
the
path
to
(T,3)

Why multiply?



How to do this in general

Suppose we want to do two experiments one after the other and the outcome of each experiment doesn't influence the outcome of the other.

Let (S_1, Ω_1, P_1) and

(S_2, Ω_2, P_2) be probability

spaces corresponding the first and second experiments.

Define (S, Ω, P) where

$$S = S_1 \times S_2$$

and

Ω is the smallest σ -algebra containing all subsets of S of the form $E_1 \times E_2$ where $E_1 \in \Omega_1$ and $E_2 \in \Omega_2$.

Define P on $S = S_1 \times S_2$ as follows:

$$P(\{(w_1, w_2)\}) = P_1(\{w_1\}) \cdot P_2(\{w_2\})$$

where $w_1 \in S_1$ and $w_2 \in S_2$.

If S is finite and E_1 is an event from Ω_1 , and E_2 is an event from Ω_2 then

$$P(E_1 \times E_2) = \sum_{(e_1, e_2) \in E_1 \times E_2} P(\{(e_1, e_2)\})$$

$$= \sum_{(e_1, e_2) \in E_1 \times E_2} P_1(\{e_1\}) \cdot P_2(\{e_2\})$$

$$= \sum_{e_1 \in E_1} \sum_{e_2 \in E_2} P_1(\{e_1\}) \cdot P_2(\{e_2\})$$

$$= \left(\sum_{e_1 \in E_1} P_1(\{e_1\}) \right) \cdot \left(\sum_{e_2 \in E_2} P_2(\{e_2\}) \right)$$

$$= P_1(E_1) \cdot P_2(E_2)$$

Thus,

$$P(S) = P(S_1 \times S_2)$$

$$= P_1(S_1) \cdot P_2(S_2)$$

$$= 1 \cdot 1$$

$$= 1$$

Ex: Suppose you have a 4-sided weighted die labeled 1, 2, 3, 4. From rolling the die lots of times you have determined the probabilities are:

# on die	1	2	3	4
probability	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$

Let's model first rolling this weighted die and then flipping a fair coin.

first experiment

$$S_1 = \{1, 2, 3, 4\}$$

Ω_1 = all subsets
of S_1

$$P_1(\{1\}) = 1/8$$

$$P_1(\{2\}) = 1/4$$

$$P_1(\{3\}) = 1/2$$

$$P_1(\{4\}) = 1/8$$

second experiment

$$S_2 = \{H, T\}$$

Ω_2 = all subsets
of S_2

$$P_2(\{H\}) = 1/2$$

$$P_2(\{T\}) = 1/2$$

probability space of rolling die then flipping coin

$$S = S_1 \times S_2 = \{(1, H), (2, H), (3, H), (4, H), (1, T), (2, T), (3, T), (4, T)\}$$

Ω = all subsets of S

$$P(\{(1, H)\}) = P_1(\{1\}) \cdot P_2(\{H\})$$
$$= \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$$

$$P(\{(2, H)\}) = P_1(\{2\}) \cdot P_2(\{H\})$$
$$= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(\{(3, H)\}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

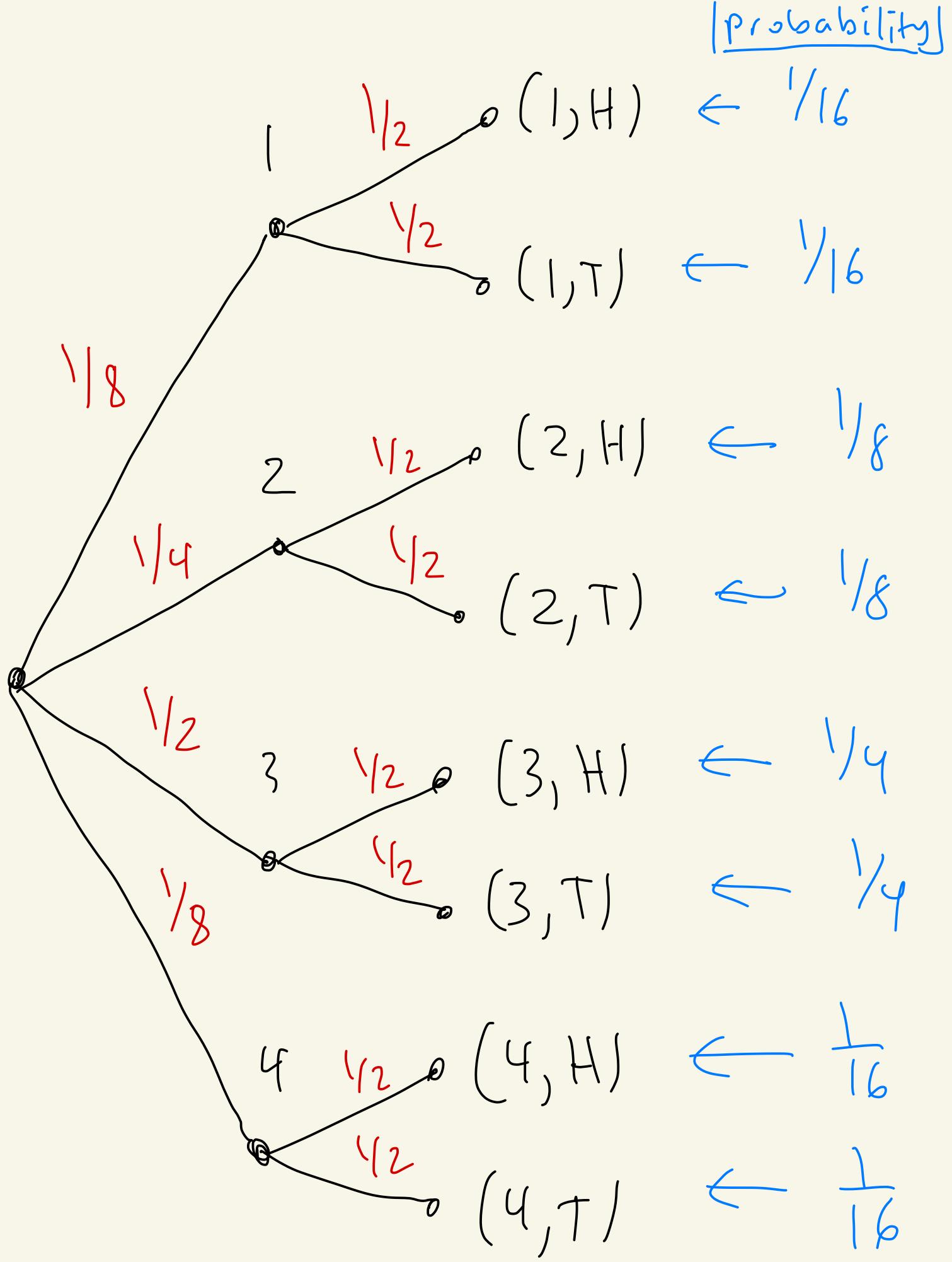
$$P(\{(4, H)\}) = \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$$

$$P(\{(1, T)\}) = \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$$

$$P(\{(2, T)\}) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(\{(3, T)\}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(\{(4, T)\}) = \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$$



HW 2

- 14 Suppose that five numbers are selected at random from the numbers 1 - 20 with no repeated #'s chosen. What's the probability that the smallest # selected is larger than 6 ?

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

#'s larger than 6

Ex:

#'s picked	smallest #	
10, 1, 20, 6, 17	1	6 < 1
10, 11, 8, 15, 19	8	6 < 8

Sample space size is # of ways to pick 5 #'s from 1-20

which is

$$\binom{20}{5} = \frac{20!}{5! \cdot 15!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15!}{(120) \cdot 15!}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$
$$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}{120}$$
$$= 15,504$$

event is # ways to select 5 #'s where smallest # is greater than 6
(ie pick 5 #'s from 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20)

$$\# \text{ ways} = \binom{20-6}{5} = \binom{14}{5}$$

$$= \frac{14!}{5! \cdot 9!} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{(120) \cdot 9!}$$

$$= \boxed{2,002}$$

$$\text{Answer} = \frac{2,002}{15,504} \approx 0.1291$$

$$\approx \boxed{12.9\%}$$