Math 4740

9-20-23

Ex 5-card poker hands:

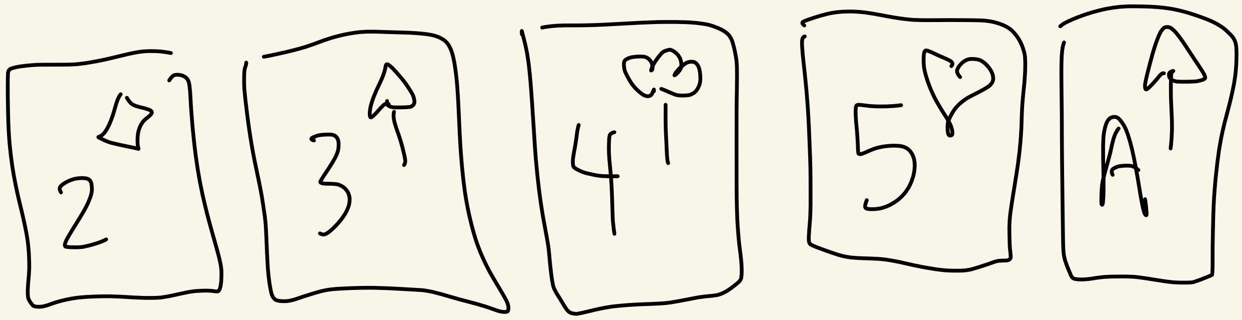
J♦ 2♣ J♥ 3♠ J♠ — 3-of-a-kind
3 jacks

2♣ 3♠ 4♥ 2♦ A♦ — pair

3♣ 3♠ 2♥ 2♦ A♠ — two pair

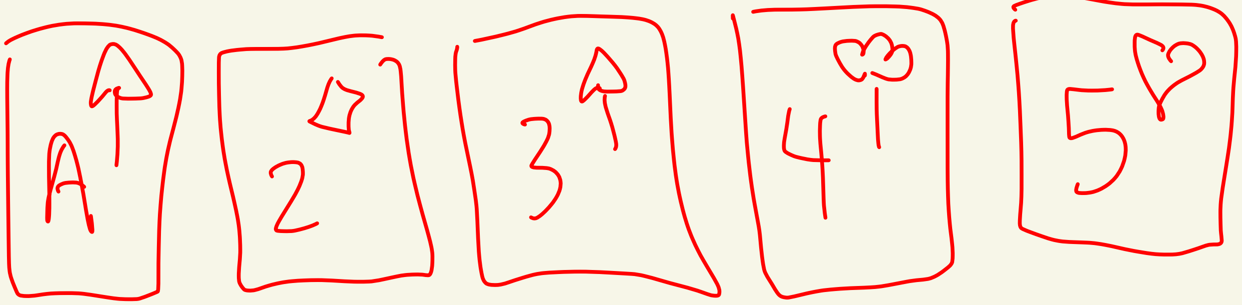
3♣ 3♠ 2♥ 2♦ 2♠ — full house

pair + 3-of-a-kind



straight

Same as:



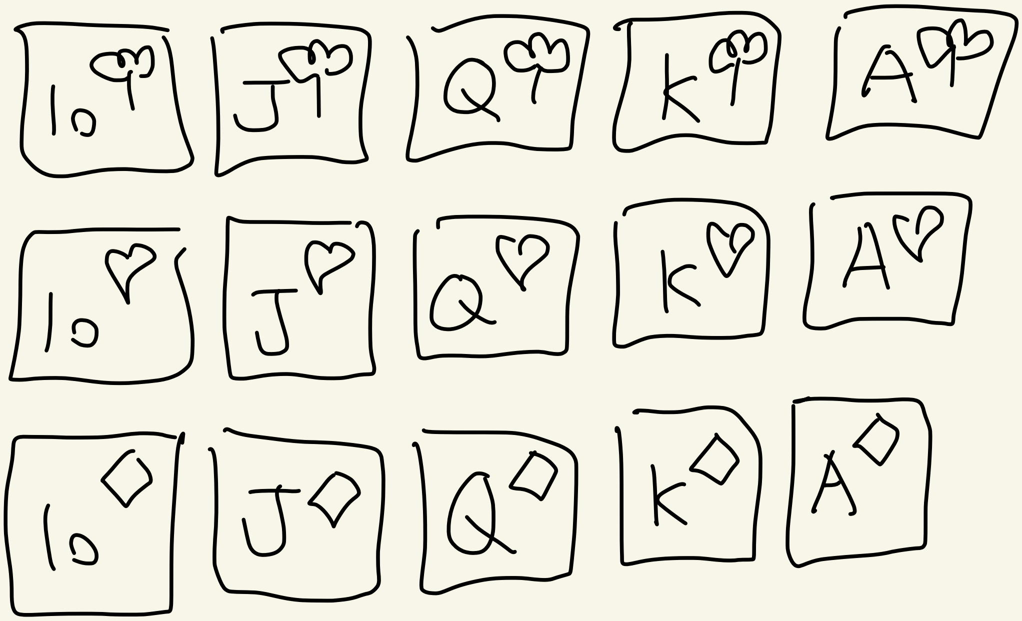
Ex: Suppose you are dealt 5 cards from a standard 52-card deck.
What's the probability that you get a royal flush?

The size of the sample space, ie the total # of possible 5-card poker hands is

$$\binom{52}{5} = 2,598,960$$

How many royal flushes are there?

$\boxed{10\spadesuit}$ $\boxed{J\spadesuit}$ $\boxed{Q\spadesuit}$ $\boxed{K\spadesuit}$ $\boxed{A\spadesuit}$



4 royal flushes

The probability of a royal flush is

$$\frac{4}{2{,}598{,}960} = \frac{1}{649{,}740}$$

$$\approx 0.000001539\ldots$$

$$\approx 0.0001539\ \%$$

<u>Ex:</u> Same setup as above, what's the probability of getting one pair and nothing better?

---

sample space size:

$$\binom{52}{5} = 2,598,960$$

---

we need to count the # of hands that make a pair and nothing better.

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K ← face value

♠, ♣, ♥, ♦ ← suit

Let's enumerate the pairs.

Step 1: Pick a face value for the pair.

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

possibilities in step 1: $\binom{13}{1} = 13$

Ex: $\boxed{2}$ $\boxed{2}$ $\boxed{}$ $\boxed{}$ $\boxed{}$

Step 2: Pick 2 suits for the pair

♣, ♠, ♢, ♡

Possibilities in step 2: $\binom{4}{2} = 6$

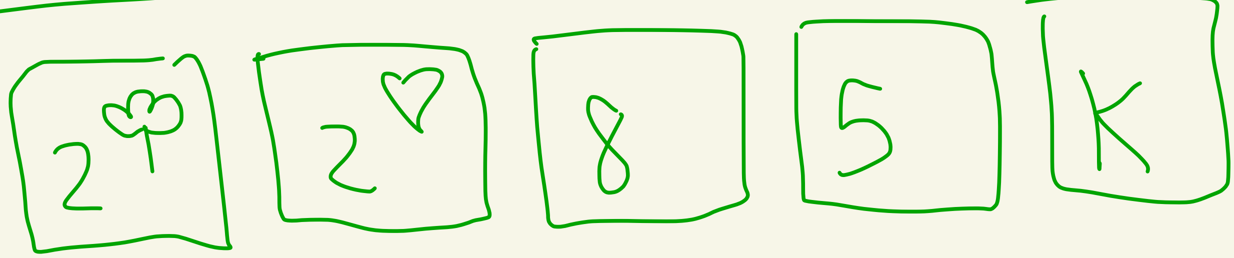
Ex:

Step 3: Pick the other 3 face values. They can't be the same as step 1, and you can't pick any duplicates.

A, ~~2~~, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

possibilities in step 3: $\binom{12}{3} = \frac{12!}{3!(12-3)!} = 220$

Ex:

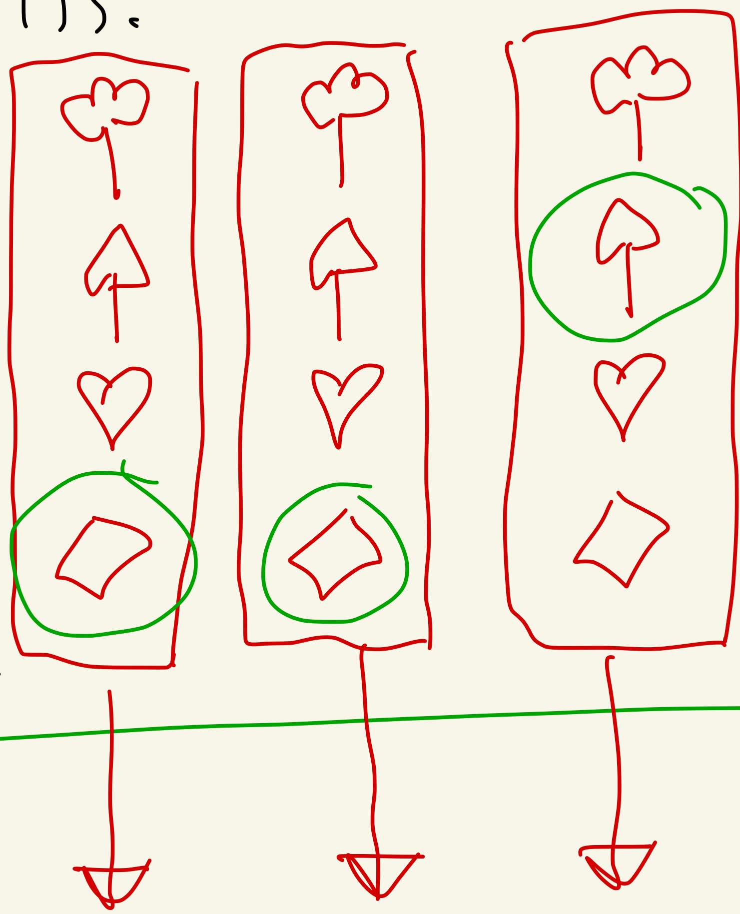


Step 4: Fill in the 3 remaining suits.

\# possibilities in step 4

$= \binom{4}{1} \cdot \binom{4}{1} \cdot \binom{4}{1}$

$= 4 \cdot 4 \cdot 4 = 64$



Ex:

2♣ 2♥ 8♦ 5♦ K♠

Thus, the total # of hands that are a pair and no better are

$$\underbrace{13}_{\text{step 1}} \cdot \underbrace{6}_{\text{step 2}} \cdot \underbrace{220}_{\text{step 3}} \cdot \underbrace{64}_{\text{step 4}}$$

$$= \boxed{1{,}098{,}240}$$

So the probability is

$$\frac{1{,}098{,}240}{2{,}598{,}960} \approx 0.422569\ldots$$

$$\approx \boxed{42\%}$$

HW 2 - Extra problems

① Suppose you are dealt 2 cards from a standard 52-card deck.

(a) What's the probability that both cards are aces?

(b) What's the probability both cards have the same face value (or rank)?

The sample space size is the total # of 2-card hands.

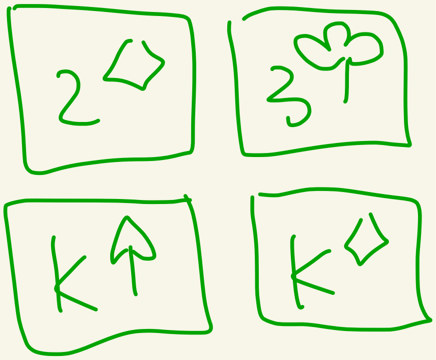
It is

$$\binom{52}{2} = \frac{52!}{2!\,(52-2)!} = \frac{52\cdot 51\cdot \cancel{(50!)}}{2\cdot \cancel{(50!)}}$$

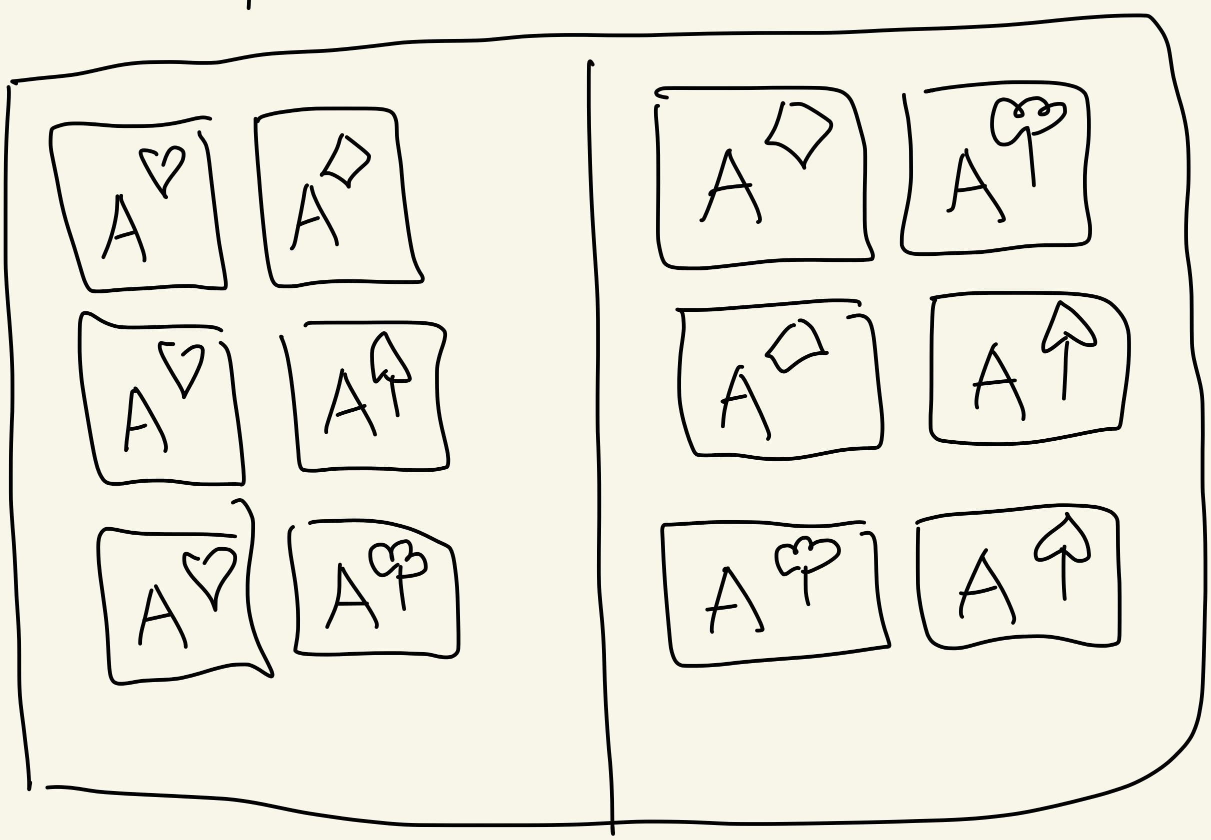
$$= 26\cdot 51$$

$$= \boxed{1326}$$

Ex:



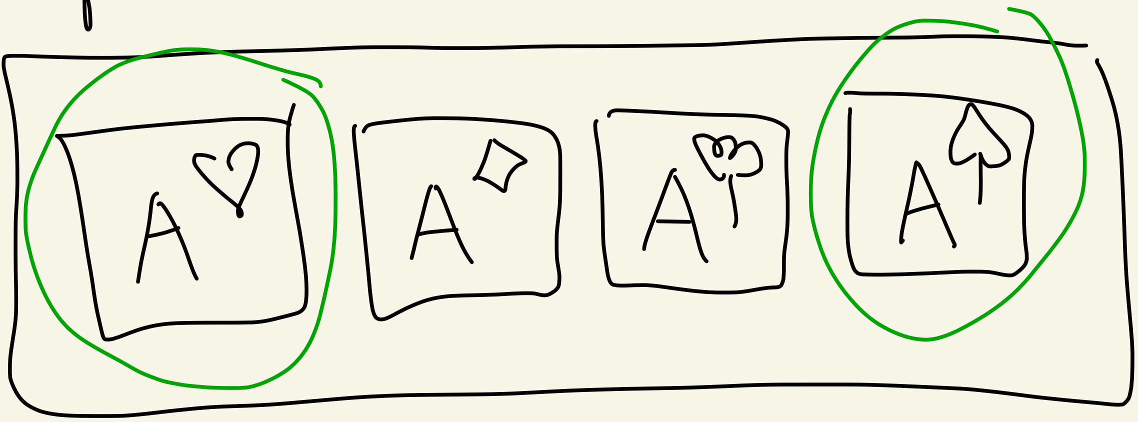
(a) How many hands have two aces?



6 hands

or use choosing.

pick 2 from:



$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 2} = 6$$

the probability of getting two aces is

$$\frac{6}{1326} = \frac{1}{221} \approx 0.00452\ldots$$

$$\approx 0.452\,\%$$

(b) Need to count the # of hands with both cards same face value.

Step 1: Pick the face value

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

possibilities in step 1: $\binom{13}{1} = 13$

Ex: 7 7

Step 2: Pick two suits

♠, ♣, ♥, ♦

$$\boxed{\# \text{ possibilities in step 2: } \binom{4}{2} = 6}$$

$\boxed{\text{Ex:}}$ $\boxed{7^{\clubsuit}}$ $\boxed{7^{\diamondsuit}}$

\# of hands with same face value on both cards is

$$\underbrace{13}_{\text{step 1}} \cdot \underbrace{6}_{\text{step 2}} = \boxed{78}$$

Probability is $\dfrac{78}{1326} = \dfrac{1}{17}$

$\approx 0.0588\ldots$

$\approx 5.88\%$